Word

 is a set; define .

A formal finite string of the form  , where .

Empty Word

The string with no elements, denoted *e*.

*W(S)*

The set of all words constructed from *S*, including the empty word.

Equivalence Classes of Words

 are related if *v* can be obtained from *u* by a finite sequence of insertions or deletions of the form  or , where .

Equivalence Classes Form a Group

Let *S* be a set of distinct symbols. For any word *u* in *W(S)*, let  denote the set of words in *W(S)* equivalent to *u* (that is,  is the equivalence class containing *u*). Then the set of all equivalence classes of elements of *W(S)* is a group under the operation .

Free group on *S*

The group of equivalence classes of elements of *W(S)*, with the group operation being equivalence class of concatenation.

The Universal Mapping Property

Every group is a homomorphic image of a free group.

Universal Factor Group Property

Every group is isomorphic to a factor group of a free group.

Generators and Relations

Let *G* be a group generated by some set  and let *F* be the free group on *A*. Let  be a subset of *F* and let *N* be the smallest normal subgroup of *F* containing *W*. We say that *G* is *given by the generators*  *and the relations*  if there is an isomorphism from *F/N* onto *G* that carries  to .

Notation: .

Presentation of a Group

A specification of a group in terms of generators and relations.

Dyck’s Theorem

Let  and . Then  is a homomorphic image of *G*.

Largest Group Satisfying Defining Relations

If *K* is a group satisfying the defining relations of a finite group *G* and , then *K* is isomorphic to *G*.

Quaternions

The 8-element group .

Classification of Groups of Order 8

Up to isomorphism, there are only five groups of order 8: , , and the quaternions.

Characterization of Dihedral Groups

Any group generated by a pair of elements of order 2 is dihedral.