Simple group

A group whose only normal subgroups are the identity subgroup and the group itself.

Composition factors of a group

Let *G* be a group and .

The simple groups , , and so on up to , where  and the  are normal groups of  of the largest possible order.

Sylow Test for Nonsimplicity

Let *n* be a positive integer that is not prime, and let *p* be a prime divisor of *n*. If 1 is the only divisor of *n* that is equal to 1 modulo *p*, then there does not exist a simple group of order *n*.

2 Times Odd Test

An integer of the form , where *n* is an odd number greater than 1, is not the order of a simple group.

Generalized Cayley Theorem

Let *G* be a group and *H* a subgroup of *G*. Let *S* be the group of all permutations of the left cosets of *H* in *G*. Then there is a homomorphism from *G* into *S* whose kernel lies in *H* and contains every normal subgroup of *G* that is contained in *H*.

Index Theorem

If *G* is a finite group and *H* is a proper subgroup of *G* such that  does not divide , then *H* contains a nontrivial normal subgroup of *G*. In particular, *G* is not simple.

Embedding Theorem

If a finite non-Abelian simple group *G* has a subgroup of index *n*, then *G* is isomorphic to a subgroup of .