 *a* and *b* are conjugate in *G*

*G* is a group; .

For some , .

Conjugacy class of *a*

*G* is a group; .

The set .

The number of Conjugates of *a*

Let *G* be a finite group and let *a* be an element of *G*. Then, .

The Class Equation

For any finite group *G*,



where the sum runs over one element *a* from each conjugacy class of *G*.

*p*-group

A group of order , where *p* is prime.

*p*-Groups Have Nontrivial Centers

Let *G* be a finite group whose order is a power of a prime *p*. Then  has more than one element.

Groups of Order  are Abelian

If , where *p* is prime, then *G* is Abelian.

Existence of Subgroups of Prime-Power Order

(Sylow’s First Theorem)

Let *G* be a finite group and let *p* be a prime. If  divides , then *G* has at least one subgroup of order .

Sylow *p*-Subgroup

*G* is a group and *p* is a prime divisor of .  divides  but  does not.

A subgroup of order .

Cauchy’s Theorem

Let *G* be a finite group and *p* a prime that divides the order of *G*. Then *G* has an element of order *p*.

Conjugate Subgroups

Two subgroups *H* and *K* of a group *G* that are related by , where .

Sylow’s Second Theorem

If *H* is a subgroup of a finite group *G* and  is a power of a prime *p*, the *H* is contained in some Sylow

*p*-subgroup of *G*.

Sylow’s Third Theorem

The number of Sylow *p*-subgroups of *G* is equal to 1 modulo *p* and divides . Furthermore, any two Sylow *p*-subgroups are conjugate.

A Unique Sylow *p*-Subgroup Is Normal

A Sylow *p*-subgroup of a finite group *G* is a normal subgroup of *G* if and only if it is the only Sylow

*p*-subgroup of *G*.

Cyclic Groups of Order *pq*

If *G* is a group of order *pq*, where *p* and *q* are prime, , and *p* does not divide , then *G* is cyclic. In particular, *G* is isomorphic to .