*a* is algebraic over *F*

*E* is an extension of a field *F*, .

*a*  is the zero of some nonzero polynomial in  .

*a* is transcendental over *F*

*a* is not algebraic over *F*.

algebraic extension of a field *F*

An extension *E* of a field *F* for which every element of *E* is algebraic over *F*.

Transcendental extension of a field *F*

An extension *E* of a field *F* that is not algebraic.

Simple extension of the field *F*

An extension of a field *F* of the form .

Characterization of Extensions

Let *E* be an extension of the field *F*, and let . If *a* is transcendental over *F*, then . If *a* is algebraic over *F*, then , where is a polynomial in  of minimum degree such that . Moreover,  is irreducible over *F*.

Uniqueness Property

If *a* is algebraic over a field *F*, then there is a unique monic irreducible polynomial  in  such that .

Divisibility Property

Let *a* be algebraic over *F*, and let  be the minimal polynomial for *a* over *F*. If  and , then  divides  in .

*E* has degree *n* over *F*

*E* is an extension over a field *F*.

*E* has dimension *n* as a vector space over *F*.

Notation: .

Finite extension of *F*

An extension *E* of a field *F* for which  is finite.

Infinite extension of *F*

An extension *E* of a field *F* for which  is infinite.

Finite Implies Algebraic

If *E* is a finite extension of *F*, then *E* is an algebraic extension of *F*.



Let *K* be a finite extension field of the field *E* and let *E* be a finite extension of the field *F*. Then *K* is a finite extension field of *F* and .

Primitive Element Theorem

If *F* is a field of characteristic 0, and *a* and *b* are algebraic over *F*, then there is an element *c* in  such that .

Primitive element of *E*

*E* is an extension of a field *F*.

An element *a* with the property that .

Algebraic Over Algebraic Is Algebraic

If *K* is an algebraic extension of *E* and *E* is an algebraic extension of *F*, then *K* is an algebraic extension of *F*.

Subfield of Algebraic Elements

Let *E* be an extension field of the field *F*. Then the set of all elements of *E* that are algebraic over *F* is a subfield of *E*.

Algebraic closure of *F* in *E*

*E* is an extension of the field *F*.

The subfield of *E* of the elements that are algebraic over *F*.

Algebraically closed field

A field that has no proper algebraic extension.

Algebraic Closure of *F*

The unique (up to isomorphism) algebraic extension of a field *F* that is algebraically closed.