Associates

Let *D* be an integral domain.

Two elements  related by , where *u* is a unit of *D*.

Irreducible

Let *D* be an integral domain.

A nonzero element  that is not a unit and for which whenever  and , *b* or *c* is a unit.

Prime

Let *D* be an integral domain.

A nonzero element  that is not a unit and for which  implies  or .

Prime Implies Irreducible

In an integral domain, every prime is an irreducible.

PID Implies Irreducible Equals Prime

In a principal ideal domain, an element is an irreducible if and only if it is a prime.

Unique Factorization Domain (UFD)

An integral domain *D* such that

1. Every nonzero element of *D* that is not a unit can be written as a product of irreducibles of *D*.
2. The factorization into irreducibles is unique up to associates and the order in which the factors appear.

Ascending Chain Condition for a PID

In a principal ideal domain, any strictly increasing chain of ideals  must be finite in length.

PID Implies UFD

Every principal ideal domain is a unique factorization domain.

Noetherian Domain

An integral domain *D* for which there is no infinite, strictly increasing chain of ideals in *D*.

 Is a UFD

Let *F* be a field. Then  is a unique factorization domain.

Euclidean Domain

An integral domain *D* for which there is a function *d* (called the measure) from the nonzero elements of *D* to the non-negative integers such that

1.  for all nonzero .
2. If , then there exist elements *q* and *r* in *D* such that , where  or.

ED (Euclidean Domain) Implies PID

Every Euclidean domain is a principal ideal domain.

ED Implies UFD

Every Euclidean domain is a unique factorization domain.

*D* a UFD Implies  a UFD

If *D* is a unique factorization domain, then  is a unique factorization domain.