Ring of Polynomials over *R* in the Indeterminate *x*

Let *R* be a commutative ring. The set of formal symbols



Addition in *R[x]*

Let *R* be a commutative ring and let

 

and



belong to *R[x]*. Then



where *s* is the maximum of *m* and *n*, ,  for , and  for .

Multiplication in *R[x]*

Let *R* be a commutative ring and let

 

and



belong to *R[x]*. Then



where

 

and  for , and  for .

Monic polynomial

A polynomial with a leading coefficient of 1.

*D* an Integral Domain Implies *D[x]* Is

If *D* is an integral domain, then *D[x]* is an integral domain.

Division Algorithm for *F[x]*

Let *F* be a field and let  with . Then there exist unique polynomials  and  in  such that  and either  or .

The Remainder Theorem

Let *F* be a field, , and . Then  is the remainder in the division of  by .

The Factor Theorem

Let *F* be a field, , and . Then *a* is a zero of  if and only if  is a factor of .

Polynomials of Degree *n* Have at Most *n* Zeros

A polynomial of degree *n* over a field has at most *n* zeros counting multiplicity.

Principal Ideal Domain

An integral domain *R* in which every ideal has the form  for some .

*F[x]* is a PID

Let *F* be a field. Then *F[x]* is a principal ideal domain.

Criterion for 

Let *F* be a field, *I* a nonzero ideal in *F[x]*, and  an element of *F[x]*. Then  if and only if  is a nonzero polynomial of minimum degree in *I*.