(Two-sided) Ideal

A subring *A* of a ring *R* such that for every $r\in R$ and $a\in A$, both $ar$ and $ra$ are in *A*.

Proper ideal

An ideal *A* of a ring *R* that is a proper subset of *R*.

Ideal Test

A nonempty subset *A* of a ring *R* is an ideal of *R* if

1. $a-b\in A$ whenever $a,b\in A$.
2. $ar, ra\in A$ whenever $a\in A$ and $r\in R$.

Trivial ideal

{0}

Principal ideal generated by $a\in R$

*R* is a commutative ring with unity.

The set $\left〈a\right〉=\left\{ra|r\in R\right\}$.

Ideal generated by $a\_{1},a\_{2},…,a\_{n}\in R$

*R* is a commutative ring with unity.

The set $\left〈a\_{1},a\_{2},…,a\_{n}\right〉=\left\{r\_{1}a\_{1}+r\_{2}a\_{2}+…+r\_{n}a\_{n}|r\_{i}\in R\right\}$.

Existence of Factor Rings

Let *R* be a ring and let *A* be a subring of *R*. The set of cosets $\left\{r+A|r\in R\right\}$ is a ring under the operations $\left(s+A\right)+\left(t+A\right)=s+t+A$ and

$\left(s+A\right)\left(t+A\right)=st+A$ if and only if *A* is an ideal of *R*.

Prime Ideal

A proper ideal of a commutative ring *R* such that $a,b\in R$ and $ab\in A$ implies $a\in A$ or $b\in A$.

Maximal Ideal

A proper ideal of *R* such that if *B* is an ideal of *R* and **, then $B=A$ or $B=R$.

*R/A* is an Integral Domain if and Only if *A* is Prime

Let *R* be a commutative ring with unity and let *A* be an ideal of *R*. Then *R/A* is an integral domain if and only if *A* is prime.

*R/A* is a Field if and Only if *A* is Maximal

Let *R* be a commutative ring with unity and let *A* be an ideal of *R*. Then *R/A* is a field if and only if *A* is maximal.