(Two-sided) Ideal

A subring *A* of a ring *R* such that for every and , both and are in *A*.

Proper ideal

An ideal *A* of a ring *R* that is a proper subset of *R*.

Ideal Test

A nonempty subset *A* of a ring *R* is an ideal of *R* if

1. whenever .
2. whenever and .

Trivial ideal

{0}

Principal ideal generated by

*R* is a commutative ring with unity.

The set .

Ideal generated by

*R* is a commutative ring with unity.

The set .

Existence of Factor Rings

Let *R* be a ring and let *A* be a subring of *R*. The set of cosets is a ring under the operations and

if and only if *A* is an ideal of *R*.

Prime Ideal

A proper ideal of a commutative ring *R* such that and implies or .

Maximal Ideal

A proper ideal of *R* such that if *B* is an ideal of *R* and **, then or .

*R/A* is an Integral Domain if and Only if *A* is Prime

Let *R* be a commutative ring with unity and let *A* be an ideal of *R*. Then *R/A* is an integral domain if and only if *A* is prime.

*R/A* is a Field if and Only if *A* is Maximal

Let *R* be a commutative ring with unity and let *A* be an ideal of *R*. Then *R/A* is a field if and only if *A* is maximal.