Zero-divisor

A nonzero element *a* of a commutative ring *R* such that there is a nonzero element $b\in R$ with $ab=0$.

Integral domain

A commutative ring with unity and no zero-divisors.

Cancellation

Let *a*, *b*, and *c* belong to an integral domain. If $a\ne 0$ and $ab=ac$, then $b=c$.

Field

A commutative ring with unity in which every nonzero element is a unit.

Finite Integral Domains are Fields

A finite integral domain is a field.

$Z\_{p}$ is a Field

For every prime *p*, $Z\_{p}$, the ring of integers modulo *p*, is a field.

Characteristic of a Ring, denoted *char R*

The least positive integer *n* such that $nx=0$ for all *x* in *R*. If no such integer exists, the number 0.

Characteristic of a Ring with Unity

Let *R* be a ring with unity 1. If 1 has infinite order under addition, then the characteristic of *R* is 0. If 1 has order *n* under addition, then the characteristic of *R* is *n*.

Characteristic of an Integral Domain

The characteristic of an integral domain is 0 or prime.