Online Adaptive Traffic Signal Coordination

with a Game Theoretic Approach

by

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ABSTRACT

In the coming era of Connected and Automated Vehicles (CAV), there is a pressing need to develop online adaptive traffic signal control algorithms given rich real-time data from CAVs. The occurrence of CAV technologies brings an opportunity to develop a real-time data-driven signal coordination method which cooperatively optimize the network performance at a network level. This thesis proposes a game theoretic approach, epsilon-equilibrium algorithm, to achieve online adaptive coordination. Each intersection of the traffic network is just like a player in a game. Different intersections pursue their own benefit maximization. This thesis also compares the network delay performances between CAVs and Human-driven Vehicle (HDV). A simulation platform is built using Matlab to code and evaluate the proposed algorithms and models.

The effect, applicability and efficiency of the game theoretic approach in signal coordination are examined. The game theoretical approach is proven to outperform the systematical optimization on vehicle delay at intersection level in terms of delay equity. The variances of vehicle delay among different intersections are significantly decreased by the proposed game theoretic algorithm. Thus no intersection needs to sacrifice its own delay performance to achieve system optimal. The results also show that the CAVs can achieve better delay performances compared to HDVs.

Key words: Online signal control; Game theory; Signal coordination; CAV
CHAPTER 1: INTRODUCTION

1.1 Background

In the design of traffic signal control, there are several signal timing parameters: cycle length, splits and offsets. Offsets are defined as the starting times of green phases. The design of offsets is vital to the performance of arterial operation for offsets provide coordination between adjacent intersections with respect to a fixed reference point (FHWA 2015). Offsets are also a very sensitive timing parameter for the wrong selection of an offset at one intersection can effect the delay of the whole arterial system (HCM 2010).

In terms of offset optimization, the offsets are typically determined by the average traffic volumes for each intersection and average travel times. Many methods have been developed based on this and they use an offline package to implement the settings. These methods can be divided into two groups for different objectives. One kind of objective is progression bandwidth optimization and the other is minimizing system delay.

With the development of vehicle identification and telecommunication technology, many online offset optimization models have been developed to better adjust to the complex field traffic. High resolution operation data can also be obtained and these data makes the performance of offline models greatly improved.

However, current practice for offset design focus either on progression quality for coordinated phases or green time occupancy and most of them are designed to improve the efficiency of one main direction. To achieve the best system performances, these centralized signal control methods only consider the benefits of some specific parts. Other parts of the arterial need to sacrifice a lot of their own benefits. With the rapid development of Connected and Automated Vehicle (CAV)
technologies, it is possible to track or control the movement and trajectory of vehicles. The occurrence of CAV technologies brings an opportunity to develop an online adaptive signal coordination method which aims to achieve equity for the arterial. To achieve the equity and stability for signal coordination, it is required to consider performances or progressions for different, sometimes even conflicted phases simultaneously.

In the problem of multiple agents, game theory studies the behaviors between the different decision makers who intend to pursue their own benefit maximization and the game theoretic approach has been gradually applied in different traffic control areas. The phases or intersections of traffic network are just like the different decision makers in game theory, which provides an attractive analytical method to achieve the equity and stability for online signal coordination.

1.2 Research Objectives

The objective of this thesis is to study and discuss the application of game theory in offset refinement for multiple intersections. This research will develop an algorithm which utilizes the game theoretic approach to improve the delay performance of multi-intersections. A simulation platform will be built using Matlab to code and evaluate the algorithm. The effect, applicability and efficiency of the game theoretic approach in offset refinement will be examined.

This thesis will also study the effect of CAV in the performances of traffic network. This research will contribute to researches in developing stable signal coordination for multiple agents in complex traffic arterial.

1.3 Thesis Organization

The rest of this thesis is structured as follows. In Chapter 2, a literature review is conducted to summarize previous work related to offset optimization, the applications of game theory in traffic
signal control and coordination effects of Connected and Automated Vehicles. In Chapter 3, the methodology of modeling and simulation is described. In Chapter 4, simulation results are presented and analyzed. Conclusions and future works are included in Chapter 5.
CHAPTER 2: LITERATURE REVIEW

The literature review of this research is composed of three parts. The first part is about previous studies towards traffic signal offset and coordination optimization. The second part reviews existing studies that applied game theory approach in traffic signal control. In the third part, the coordination effects of Connected and Automated Vehicles (CAVs) are reviewed.

2.1 Traffic Signal Offset and Coordination Optimization

Signal offsets are introduced in traffic signal control to achieve coordination between different intersections. In the very beginning, the offsets are simply determined by the road lengths and free flow speeds between coordinated intersections.

Traditionally, offsets between different traffic signals are typically determined by the average traffic volumes for each intersection and average travel times. There are mainly two kinds of algorithms. One kind focuses on progression bandwidth optimization along arterials, including PASSER (Messer et al., 1974), MAXBAND (Little et al., 1981) and MULTIBAND (Gartner et al., 1990). The other kind focuses on minimizing system delay, including TRANSYT (Robertson, 1967) and Synchro (Trafficware, 2001).

Most of these algorithms do not consider or fully consider traffic flow fluctuations. They are not consistent with actual traffic conditions thus not applicable to field traffic.

In the coordinated-actuated control systems, traffic flow variations on uncoordinated approaches can lead to variations of green times. Starting times and ending times of green times are not fixed. This problem is called “early return to green”. To address the problem of “early return to green”, Skabardonis (1996) developed several approaches to determine yield points, force-offs and
maximum greens. All these methods are based on making the estimation for average starting point of the synchronization phase first, then determining offsets. Shoup and Bullock (1999) presented an offline procedure to optimize offsets using the link travel times. This kind of optimization is based on that the arterial deploys vehicle re-identification technologies. Yin et al. (2007) presented an offline offset refiner. The goal of this refiner is to address the problems of uncertain starts and ends of green. The refiner adopts a bandwidth-maximizing approach using archived signal status data.

Even with the problem of “early return to green” solved, all of these studies deal with offline optimizations and few of them rely on field data which specifies how traffic networks actually perform. The traffic conditions can be very complex and these algorithms are not representative enough for real traffic.

Some efforts have been made to develop real-time offset optimization models. Abbas et al. (2001) proposed an online real-time offset adjusting algorithm for coordinating traffic signals. The smooth progression of a platoon of one direction is mainly optimized. So the algorithm is designed to adjust the offsets continually by moving the green window so that more vehicles can pass during green. A greedy search method is used in this algorithm for determining the optimal shift of green window. Both “early return to green” and waiting queues lengths problems are addressed in this proposed algorithm. Gettman et al. (2007) proposed a data-driven real-time control algorithm to tune intersection offsets in a coordinated traffic signal system based on the signal phase and detector data from the last several cycles. The proposed algorithm mainly considers green occupancy maximization. It considers the local and downstream impacts of incremental changes to offsets at each signal by assessing the amount of traffic captured by the green time of the coordinated phases. Day et.al (2010) introduced the Purdue Coordination Diagram (PCD) to adjust
offsets and assess arterial signal coordination. The PCD model can be utilized to visualize arrival patterns at intersections and predict the effects of different offsets by assessing the quality of progression; then the best offset combinations can be determined. The authors also introduced a technique for evaluating the effects of implemented offsets. This technique uses Bluetooth MAC address matching to measure actual arterial travel time. When calculating the effects of offsets, the value of the offset is changing by moving forward in time by 25% of the cycle length.

Besides real-time models, offline models that heavily rely on historical data was developed by Hu and Liu (2013). As an arterial offset optimization model, it is data-driven for it uses archived high-resolution traffic signal data, including both signal status data and vehicle actuation data as the inputs of the model. The proposed model is designed to solve mainly two problems produced by the stochastic nature of traffic. It adopts the conditional probability of the green start times to solve the problem of “early return to green” and uses a scenario-based optimization which uses a series of traffic demand scenarios as input, to solve the problem of the queue lengths variation for coordinated directions. The model mainly optimizes the main direction traffic and the opposite directions yields to the main directions. Liu et al. (2015) improved model heavily relies on historical data by proposing a two-stage signal control strategy. At the first (offline) stage, a linear decision rule used to map real-time traffic data to optimal signal control policies is formulated. A distributional robust optimization is then solved to address the presence of uncertainties associated with the real-time traffic. The second (online) stage implements the efficient linear decision rule and distributional robust optimization computed by the offline stage.

These real-time or data-driven models focus either on progression quality or green time occupancy and most of them are designed to improve the efficiency of one main direction. As indicated by Day and Bullock (2016), connected vehicle data has the potential to transform traffic signal
operation. With the development of Connected and Automated Vehicle (CAV) technologies, there is a pressing need to develop real-time data-driven stable signal coordination methods which cooperatively optimize the network performance at intersection level.

2.2 The Applications of Game Theory in Traffic Signal Control

Transportation researchers have a long history in using Game theory for the development of User Equilibrium models in traffic assignment (Sheffi, 1985).

Recently, the game theoretic approach has been gradually applied in different traffic control areas. In traffic flow control area, Purohit and Mantri (2013) utilized the game theory in a dynamic transportation environment to control the traffic flow and improve the disaster management for both non-cooperative and cooperative problems. The goal of the research is to select the optimal path for the vehicles to optimize the queuing result.

Besides, game theory is also popular in the area of traffic signal control. Alvarez et al. (2008) developed a framework to minimize the delay of a simple isolated intersection and optimize the congestion by using game theory and Markov chain model. For an intersection, each street is considered as a player. Different types of intersections are studied to analyze the delay, such as two player intersection, three player intersection, the four flow intersection (two players) and the intersection with English turn. The intersection is seen as a non-cooperative game where each player tries to minimize its own queue. Thus, the Nash’s equilibrium and Stackelberg equilibrium are the solutions. Compared with the adaptive control, the results of game theory are more efficient. Clempner and Poznyak (2015) developed a Markov chains game theory approach to model the multi-traffic signal-control synchronization. It is proven that the Nash equilibrium can find an optimal signal timing strategy for a signal controller. Thus the signal settings can be designed to
minimize the queuing delay according to a specific local policy. Hossam et al. (2016) modeled a signalized intersection considering four phases and applied the Nash bargaining solution to obtain the optimal strategy.

The above studies about traffic signal control mainly focus on the applications of game theory in one intersection or local situations but are not applicable for the whole traffic arterial. However, with the increasing complexity of urban transportation system, a functional and systematic coordination is required to improve the efficiency of a traffic network.

Camponogara and Kraus (2003) tried to study the coordination of only two intersections by using stochastic game theory and reinforcement learning. The results are better than a random policy and also better than Q-learning. Bazzan (2005) aimed to develop a series of techniques which allow to control a traffic arterial (multi-agent system) coordinately through a distributed approach. In this research, each agent is a player and own its local condition. The global pattern of traffic flow is known by the network system but remains unknown to the local agents. The invented algorithm is able to modify the network-level parameters and agent-level parameters of their objective functions automatically due to the local change in traffic situations. This algorithm needs the game theory to determine the Nash equilibria of the multi-agent (multi-player) and then uses the learning algorithm to improve the local objective functions timely. It seems that each agent has the incentive to coordinate towards the strategy which makes its own traffic flow perform better instead of considering the traffic network. However, the drawback of this algorithm is obvious. The more agents (intersections) included, the more uncertainty will occur, which may make the computing time increase exponentially with the rising number of intersections. Thus the participants should be able to distinguish between the durable changes and the ephemeral changes to increase the efficiency of the algorithm, but this is a totally new topic and another challenge. In the following
research, Bazzan (2009), Dr. Bazzan discovered the utilization of artificial intelligence and reinforcement learning in the area of the multi-agent system. Same idea as her previous research, but this time Dr. Bazzan developed a more general and detailed framework of the game-theoretic approach to control a coordinated system. Although many neoteric ideas are mentioned to control multi-agent system (MAS) in this paper, these ideas are still in an early stage before practical application. Moreover, some open challenges are waiting to be solved as the author said. One unavoidable problem with using MAS is that the joint actions increase exponentially with the rising number of agents. To resolve these challenges from arbitrarily big traffic network, Bazzan et al. (2010) partitioned a multi-agent environment into several smaller multi-agent systems. These groups of agents are then supervised by further agents. This approach is a good compromise between complete distribution and complete centralization.

2.3 Coordination Effects of Connected and Automated Vehicles

With the rapid development of Connected and Automated Vehicles, vehicle-to-infrastructure and vehicle-to-vehicle communications are available, and it is also possible to track or control the movement and trajectory of vehicles in traffic networks. The efficiency of intersection control strategies can thus be promoted. Lee and Park (2012) developed a fully cooperative vehicle intersection control (CVIC) algorithm to manage connected vehicles at an intersection without any traffic signals. The researchers assumed that all vehicles are totally automated and able to communicate with other vehicles and infrastructure. Although the stopped delay has been reduced 99% by the algorithm, the simulation is only applied to a four-way intersection with a single through lane at each approach in the simulation. To consider multiple intersections along the corridor, Lee et al. (2013) expanded the algorithm to a corridor consisting of multiple intersections. The CVIC system is proven to dramatically reduce the total delay times and the rear-end crash
events at all volume cases. It also contributes to improving the air quality and saving fuel consumptions. Goodall et al. (2013) proposed a rolling-horizon traffic signal control algorithm, PMSA. It uses instantaneous vehicle data obtained by connected vehicles, such as locations, headings, and speeds, to predict an objective function over a 15-s period in the future through the use of microscopic simulation. The algorithm maintained or improved performance compared with coordinated actuated timing plan at low and mid level volumes. Feng et al. (2015) developed a real-time adaptive signal control strategy which optimizes the phase sequence and duration in a connected vehicle environment. The strategy is tested in a real-world intersection and reduces total delay significantly under high penetration rates of connected vehicles. Guler et al. (2014) proposed a signal control algorithm utilizing connected vehicle technology for one intersection. The algorithm enumerates sequences of cars discharging from the intersection by incorporating information from equipped cars to minimize the total delay. With the penetration rates increasing up to 60%, the average delay can be reduced up to 60%. The author also studied the effect of automated vehicles by allowing for priority to switch between approaches rapidly and found only small decrease of delay for low demand scenarios. Benefits of platooning and controller flexibility are also studied. Yang, Guler and Menedez (2016) extended this research by incorporating trajectory design for automated vehicles. By varying information level (the ratio of automated and connected vehicles to all vehicles) from 0.2 to 1, automated level (the ratio of automated vehicles to all vehicles) varying from 0 to 1, the revised model studies the performance of complex systems consisting of regular, connected and automated vehicles. It found that connected vehicle algorithm can evidently decrease the total number of stops and delay, with the information level as low as 50%. And the benefit of trajectory tuning is greater in low or high demand scenarios with high information levels. Talebpour et al. (2016) developed an acceleration framework to model a
driving environment with regular, connected and autonomous vehicles. Unlike conventional vehicles and connected vehicles, automated vehicles can react almost instantaneously to any changes in the driving environment, whose reaction time only results from sensing delay and mechanical delay. Thus, the research found the automated vehicles can result in higher throughput compared to connected vehicles at similar penetration rate.

The CAV technologies also provide a significantly improved opportunity for multi-modal traffic signal control. He et al. (2011) proposed a heuristic algorithm for priority traffic signal control based on connected vehicle technology. The algorithm can achieve near-optimal signal timing when all simultaneous requests are considered and reduce average bus delay in congested conditions by about 50%. He et al. (2012) created a platoon-based arterial multi-modal signal control formulation, also called PAMSCOD. With the connected vehicle technology, travelers’ travel mode, position, speed and requested traffic signal phase can be sent to the intersection traffic controller. Thus platoons can be divided precisely and multi-modal traffic control can be realized by using mixed-integer linear program. The model is also extended to realize dynamic coordination between upstream and downstream intersections. Compared with state-of-practice signal control approaches, PAMSCOD significantly decreases both overall average vehicle delay and also bus delay. In the following research, He et al. (2014) considered signal coordination and signal priority control simultaneously by treating signal coordination as a virtual priority request. Enabled by vehicle-to-infrastructure communication in connected vehicle systems, a mixed-integer linear program is formulated to address multiple priority requests from different modes of vehicles and pedestrians while considering signal coordination at the same time. Hu et al. (2015) developed a person-delay-based optimization method for transit signal priority that enables bus and signal coordination along a corridor under the connected vehicle environment. The criterion
of the model is minimizing per person delay. A binary mixed integer linear program is utilized to formulate the problem. Christofa et al. (2016) developed a real-time signal control system based on minimization of person delay on arterials. The model can provide signal priority to transit vehicles and maintains auto vehicle progression.

Connected and Automated Vehicles can significantly improve the efficiency of traffic signal control methods, including the signal coordination. The effects of Connected and Automated Vehicles on signal coordination can be further studied to improve the efficiency of arterial traffic signal.
CHAPTER 3: METHODOLOGY

This chapter will first introduce the modeling description of the online coordination problem.
The second part presents the calculation of delay. The third part proposes the game theoretic
solution algorithm. It will then proceed with the settings and platforms of Human-driven Vehicle
simulation. In the last part, the modeling of Automated Vehicle is introduced.

3.1 Modeling Description

This thesis makes the following assumptions with regards to the signals of traffic networks we
simulate and study:

- **Assumption 1.** All intersections in arterials have only two conflicting signal phases.
- **Assumption 2.** Only through traffic is considered, no turns are permitted.
- **Assumption 3.** The sequence and durations of phases is fixed.

The delay refinement in traffic networks are considered as simultaneous non-cooperative games
in this thesis. The games include the following essential components:

1) **Players** – intersections \(i\). Each intersection in the network is considered as a player in this
game. Based on different networks, 2 to 5 players in arterial are studied.

2) **Action Set** (\(S\)) – a set of available actions for players to choose. Given the assumptions
about fixed phases, offset (\(O\)) of each intersection is the only decision variable for each
intersection. Each player can choose an offset to maximize its payoff given that other
players also do the same. The number of possible actions for each player is determined by
the cycle length.

\[
S_i = \{0, 1, 2, \ldots, O_i \ldots, Cycle\ length - 1\}, \forall i \in I
\]
3) Payoffs – Delay for each player. For different offset combinations \((k)\), payoffs to players are different. Each offset combination is determined by the choices of all the players. Every player aims to minimize its own delay in a period of time in these games.

\[ k = \{O_1, O_2, \ldots, O_I\} \]  

(2)

The process of solving the game is shown in the Figure 1.
3.2 Delay Calculation

Traffic delay is one of the most significant indicators for evaluating the efficiency of signalized intersections. For vehicles entering an intersection, their delay in the intersection can be divided into two categories: signal delay and queue delay. With the development of Connected Vehicle technology, it is possible to track individual vehicle’s trajectory thus the accurate delay on individual vehicle level can be estimated. Then the accurate total delay of an intersection can be calculated by summing the individual delay.

To calculate the delay for individual vehicles, two assumptions are made:

- **Assumption 1.** The predicted time of vehicles entering (TE) the studied networks in a period of time can be obtained by vehicle-to-infrastructure and vehicle-to-vehicle systems.

- **Assumption 2.** All vehicles at an intersection can be served in two cycles.

Given an offset combination (k), the traffic signals in networks can be determined. The trajectory and delay of all vehicles in the studied networks in a period of time can thus be estimated.

The delay under each specific offset combination is calculated respectively. Each vehicle is identified by its direction (d) and its sequence (n) in the direction. The predicted arrival time (A) describes the time each vehicle arrives specific intersection (i).

\[
A(i, d, n) = TE + \frac{L}{v_f} \text{ or }
\]

\[
= D(i', d, n) + \frac{L}{v_f}
\]

where \(L\) is the road length and \(v_f\) is the free flow speed. For the first intersection the vehicles arrive, the predicted arrival time is determined by the time the vehicles entering the network. For the rest of intersections those vehicles needing to cross, the predicted arrival time is determined by the time of those vehicles departing the last intersection.
The predicted departure time \((D)\) describes the time each vehicle departs specific intersection. For the first vehicle in its direction,

\[
D(i, d, 1) = A(i, d, 1), \quad \text{if } A(i, d, 1) \text{ meets green} \\
= T_{rg} + t_{acc}, \quad \text{if } A(i, d, 1) \text{ meets red}
\]  

(4)

where \(r_g\) is the time of the current red switches to green, \(t_{acc}\) is the acceleration time of a stopped vehicle. For other vehicles in the same direction,

\[
D(i, d, n) = \begin{cases} 
\max \left( A(i, d, n); D(i, d, n') + \frac{1}{S_m} + t_{acc} \right), & \text{if } A(i, d, n) \text{ meets green and is served in this cycle} \\
\max \left( D(i, d, n') + \frac{1}{S_m} + t_{acc}; T_{rg} + t_{acc} \right), & \text{if } A(i, d, n) \text{ meets red and is served in this cycle} \\
\max \left( D(i, d, n') + \frac{1}{S_m} + t_{acc}; T_{ngr} + t_{acc} \right), & \text{if } A(i, d, n) \text{ meets green and is served in next cycle} \\
\max \left( D(i, d, n') + \frac{1}{S_m} + t_{acc}; T_{rgr} + t_{acc} \right), & \text{if } A(i, d, n) \text{ meets red and is served in next cycle}
\end{cases}
\]

(5)

where \(S_m\) is the saturation flow, \(T_{ngr}\) is the time switching to the next cycle green from the current green, \(T_{rgr}\) is the time switching to the next cycle green from the next green.

Once the predicted departure time and arrival time for each vehicle at different intersection are calculated for each offset combination, the delay for each vehicle at different intersections are obtained by their difference. Then the delay of each intersection \((i)\) under specific offset combination \((k)\) can be obtained by taking the summation of delay of vehicles \((n)\) from all directions \((d)\).

\[
\text{Delay}(i, k) = \sum_d \sum_n (D(i, d, n) - A(i, d, n))
\]

(6)
3.3 Game Theoretic Solution

For simultaneous non-cooperative games, actions and strategies are the same. Each player chooses its action without knowing of the actions chosen by other players. Nash equilibrium is the method to solve this kind of problem. In a traffic network, each intersection can be a player aiming to minimize its own delay. Then the traffic system will reach to the Nash equilibrium point and no intersection could get a better condition through changing their offsets. Assuming there are two intersections, the formulas for Nash equilibrium can be expressed as,

\[
\min_{O_1} D_1(O_1, O_2, ..., O_n)
\]

\[
\hat{O}_1(O_2, ..., O_n) = \arg \min D_1(O_1, O_2, ..., O_n)
\]

\[
\min_{O_2} D_2(O_1, O_2, ..., O_n)
\]

\[
\hat{O}_2(O_1, O_3, ..., O_n) = \arg \min D_2(O_1, O_2, ..., O_n)
\]

\[
...\]

\[
\min_{O_n} D_n(O_1, O_2, ..., O_n)
\]

\[
\hat{O}_n(O_1, O_2, ..., O_{n-1}) = \arg \min D_n(O_1, O_2, ..., O_n)
\]

where \(D_1\) is the delay of intersection 1, \(D_2\) is the delay of intersection 2, \(O_1\) is the offset of intersection 1 and \(O_2\) is the offset of intersection 2, same as to \(D_n\) and \(O_n\). At the Nash equilibrium point, no one can gain more benefit of delay by changing its action unilaterally.

However, in some situations, players can not get the Nash equilibrium. There are many payoffs matrices. It is difficult to statistically model this uncertainty. So each player is allowed to have a little bit of bias to get an approximate Nash equilibrium. With more and more intersections in a traffic network, the Nash equilibrium would be hard to get. Thus it is necessary to utilize the approach of epsilon-equilibrium, which is also called near-Nash equilibrium.
The results from epsilon-equilibrium are not unique. Meanwhile, in order to satisfy the equity of all intersections, it is reasonable to choose the point that the range of delays between intersections is minimum. The epsilon-equilibrium can be expressed as,

\[ G = (N, O = O_1 \times O_2 \ldots \times O_N, u: A \in R^N) \]

\[ u_i(\sigma) \geq u_i(\sigma'_i, \sigma_{-i}) - \epsilon \text{ for all } \sigma'_i \in \Delta_i, i \in N \]

Where,

G = an \( \epsilon \)-Nash equilibrium

\( \epsilon \) = the bias of the \( \epsilon \)-Nash equilibrium, \( \epsilon \in [0, 1] \)

N = the number of players in the game

O_i = action sets

\( u_i(s) \) = utility function of the payoff (delays) to player i when strategy profile s is played.

\( \sigma \) = a vector of strategies, for \( \sigma \in \Delta = \Delta_1 \times \ldots \times \Delta_N \)

3.4 Modeling of CAV

The automated vehicles could be considered as intelligent driver model (IDM). Thus the acceleration and the desired gap could be expressed as, (Rachel, 2014)

\[ v'_\alpha = (s_\alpha, v_\alpha, \Delta v_\alpha) = a \left[ 1 - \left( \frac{v_\alpha}{v_0} \right)^\delta - \left( \frac{s^*(v_\alpha \Delta v_\alpha)}{s_\alpha} \right)^2 \right] \]  

\[ s^*(v_\alpha, \Delta v_\alpha) = s_{0,\alpha} + s_{1,\alpha} \sqrt{\frac{v_\alpha}{v_{0,\alpha}}} + T_\alpha v_\alpha + \frac{v_\alpha \Delta v_\alpha}{2 \sqrt{a \alpha b_\alpha}} \]

where, \( \alpha \) = the \( \alpha^{th} \) vehicle in a platoon;

\( v_0 \) = Desired Velocity; \( \delta \) = Acceleration Exponent;

\( T \) = Safe Time Headway; \( l \) = Length of car;

\( a \) = Maximum Acceleration; \( s_{0,\alpha} \) = Linear Jam Distance;

\( b \) = Comfortable Deceleration; \( s_1 \) = Non-Linear Jam Distance;
The platoon of automated vehicles can be seen as a tight vehicle following model ($\Delta v_{\alpha} = 0$). This thesis only studies the under-saturated cases, so no jam exist ($s_{0,\alpha} = 0, s_1 = 0$). Thus the desired gap becomes,

$$s^*(v_{\alpha}, \Delta v_{\alpha}) = T_{\alpha} v_{\alpha}$$

where $v_{\alpha}$ is in m/s. Using the spacing of one vehicle length $l$ in meters for every 10 mph, $s^*$ becomes

$$s^*(v_{\alpha}) = T_{\alpha} v_{\alpha} = \frac{v_{\alpha} l}{10} \frac{3600}{1600} = 0.225 l v_{\alpha} \tag{10}$$

$$T_{\alpha} = 0.225 l \tag{11}$$

Assuming the length of automated vehicle is 4 meter, the safe time headway $T_{\alpha} = 0.9$ s.

Different with the human driver, the reaction time $t_r$ of human driver becomes to the sensor communication delays for automated vehicles, so $t_r$ could be very small. For automated vehicle, we can set $t_r = 0.1$ s (Ioannou, Petros A, 1993).

The arrivals of CAVs are generated randomly assuming an exponential headway distribution. The expected headway equals the inverse of the flow for a given approach, (K. Yang, 2016).
CHAPTER 4: SIMULATION RESULTS AND ANALYSIS

According to the number of intersections, the level of traffic flow and the road type in a traffic system, four different models are considered in the simulation. The size of offset combination matrix is relevant to the number of intersections. With the increase of the number of intersections, the number of offset combination will increase exponentially. Due to this computational limitation, the running time of the Model 2, Model 3 and Model 4 are unpractical for online control. Setting steps for offset is one effective way to reduce the computing time.

Arrivals are randomly generated assuming a Poisson headway distribution. The settings and inputs to the simulation are assumed as,

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Initial Parameters and Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation Duration</td>
<td>360 s</td>
</tr>
<tr>
<td>Cycle Length</td>
<td>60 s</td>
</tr>
<tr>
<td>Green Phase Duration</td>
<td>30 s</td>
</tr>
<tr>
<td>Number of Lanes of Each Direction</td>
<td>1</td>
</tr>
<tr>
<td>Road Length Between Intersections</td>
<td>1000 ft</td>
</tr>
<tr>
<td>Saturation Flow Rate</td>
<td>1800 veh/h</td>
</tr>
<tr>
<td>Free Flow Speed</td>
<td>50 ft/s</td>
</tr>
<tr>
<td>Steps for Offset for Model 2, 3</td>
<td>4 s</td>
</tr>
<tr>
<td>Steps for Offset for Model 4</td>
<td>10 s</td>
</tr>
<tr>
<td>Epsilon ε</td>
<td>15%</td>
</tr>
</tbody>
</table>

In order to determine the value of epsilon, the value of epsilon from 1% to 20% have been tried to simulate the models. When the epsilon below 10%, over half of the seeds cannot find epsilon-equilibrium solution. Thus a fixed value of epsilon is needed to simulate the models for the analysis of the approach of epsilon equilibrium for all of the seeds. In order to satisfy the equity and stability of the intersections in a model, the sensitivity of the value of epsilon respect to the variance of the
delays of each intersection in a model have been analyzed. Figure 2 shows that the sensitivity of the value of epsilon respect to the variance of delays in Model 4. As the Figure 2 shown, the variance of delays decreases with the increase of the epsilon. Thus epsilon of 15% is used to simulate the models not only for the availability of the game theoretic results but also for ensuring the equity of each intersection in a traffic network.

![Figure 2. Sensitivity of Epsilon Respect to Variance](image)

**4.1 Results and Analysis of HVs**

**4.1.1 Simulation of Model 1**

The Model 1 has two intersections. The layout of the Model 1 is shown in Figure 3. The traffic flow of westbound and eastbound is 400 veh/h. The traffic flow of northbound is 200 veh/h to each intersection.

Taking one example of Model 1, the results are shown in Table 2. There is no Nash equilibrium
point in this example, like in Figure 4. So taking 15% epsilon to find approximate solutions. It is reasonable to choose the point as the best epsilon solution that the range of delays between intersections is minimum. The red point in Figure 5 is the best point of epsilon-equilibrium. The delays of each intersection become close to each other after using epsilon-equilibrium.
Figure 5. Finding Nash equilibrium with 15% epsilon

Table 2. Example of Model 1

<table>
<thead>
<tr>
<th></th>
<th>Delay 1 (s)</th>
<th>Delay 2 (s)</th>
<th>Total Delay (s)</th>
<th>Offset 1 (s)</th>
<th>Offset 2 (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>1867</td>
<td>2405</td>
<td>4272</td>
<td>29</td>
<td>7</td>
</tr>
<tr>
<td>Optimal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epsilon</td>
<td>2245</td>
<td>2245</td>
<td>4490</td>
<td>28</td>
<td>57</td>
</tr>
<tr>
<td>Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The average results of Model 1 are calculated from running this model 20 times. Delay 1 and Delay 2 represent the delay of intersection 1 and intersection 2 respectively. The total delay is the summation of the delays of the two intersections. Based on the two different methods, the results are summarized in the Table 3.

Table 3. Average Delay Results of Model 1

<table>
<thead>
<tr>
<th></th>
<th>Delay 1 (s)</th>
<th>Delay 2 (s)</th>
<th>Total Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>2019.133</td>
<td>2051.400</td>
<td>4070.533</td>
</tr>
<tr>
<td>Optimal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epsilon</td>
<td>2256.733</td>
<td>2262.633</td>
<td>4519.367</td>
</tr>
<tr>
<td>Algorithm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Comparisons of the Systematic Approach and the Game Theoretic Approach of Model 1

<table>
<thead>
<tr>
<th></th>
<th>Total Delay</th>
<th>Delay Max Difference</th>
<th>VAR of Delay Max Difference</th>
<th>VAR of Delay 1, 2 and 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>4070.533</td>
<td>521.000</td>
<td>125744.276</td>
<td>520.569</td>
</tr>
<tr>
<td>Epsilon-Equilibrium</td>
<td>4519.367</td>
<td>18.233</td>
<td>1828.116</td>
<td>17.405</td>
</tr>
<tr>
<td>Difference</td>
<td>448.833</td>
<td>-502.767</td>
<td>-123916.160</td>
<td>-503.164</td>
</tr>
<tr>
<td>Change (%)</td>
<td>11.0%</td>
<td>-96.5%</td>
<td>-98.5%</td>
<td>-96.7%</td>
</tr>
</tbody>
</table>

From the Table 4, we can find that compared with the systematical optimization, the epsilon algorithm increases the total delay of the model by 11.0%. However, the approach of epsilon-equilibrium decreases the variance of the delay max difference significantly by 98.5%. And the variance of the delay 1 and delay 2 decreases 96.7%, which means the delays of the two intersections are distributed much more equally after using epsilon-equilibrium.

### 4.1.2 Simulation of Model 2

The Model 2 has three intersections. The layout of the Model 2 is shown in Figure 6. The traffic flow of westbound and eastbound is 400 veh/h. The traffic flow of northbound is 200 veh/h to each intersection.

![Figure 6. Network 2 (Three intersections)](image)

The average results of Model 2 are calculated from running this model 20 times. Delay 1, Delay 2 and Delay 3 represent the delay of intersection 1, intersection 2 and intersection 3 respectively.
The total delay is the summation of the delays of the three intersections. Based on the two different methods, the results are summarized in the Table 5 and Figure 7.

<table>
<thead>
<tr>
<th></th>
<th>Delay 1 (s)</th>
<th>Delay 2 (s)</th>
<th>Delay 3 (s)</th>
<th>Total Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic</td>
<td>925.281</td>
<td>1974.188</td>
<td>2046.875</td>
<td>4946.344</td>
</tr>
<tr>
<td>Optimal</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epsilon</td>
<td>1523.844</td>
<td>2233.344</td>
<td>2299.188</td>
<td>6056.375</td>
</tr>
<tr>
<td>Algorithm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the Table 6, we can find that compared with the systematical optimization, the epsilon algorithm increases the total delay of the model by 22.4%. However, the approach of epsilon-equilibrium decreases the variance of the delay max difference significantly by 72.4%. And the variance of the average delay 1, delay 2 and delay 3 decreases 53.1%, which means no intersection in this traffic network would sacrifice too much. Under the control of the epsilon-equilibrium, compared with the systematical optimization, the Model 3 is able to ensure the equity of each intersection in the simulation, but the loss of the total delay of the system is relatively high.
Table 6. Comparisons of the Systematic Approach and the Game Theoretic Approach of Model 2

<table>
<thead>
<tr>
<th></th>
<th>Total Delay</th>
<th>Delay Max Difference</th>
<th>VAR of Delay Max Difference</th>
<th>VAR of Delay 1, 2 and 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>4946.344</td>
<td>1213.313</td>
<td>109703.448</td>
<td>393910.056</td>
</tr>
<tr>
<td>Epsilon-Equilibrium</td>
<td>6056.375</td>
<td>827.031</td>
<td>30270.418</td>
<td>184813.930</td>
</tr>
<tr>
<td>Difference</td>
<td>1110.031</td>
<td>-386.281</td>
<td>-79433.029</td>
<td>-209096.126</td>
</tr>
<tr>
<td>Change (%)</td>
<td>22.4%</td>
<td>-31.8%</td>
<td>-72.4%</td>
<td>-53.1%</td>
</tr>
</tbody>
</table>

4.1.3 Simulation of Model 3

The model 3 has four intersections. The layout of the Model 3 is shown in Figure 8. The traffic flow of westbound and eastbound is 400 veh/h. The traffic flow of northbound is 200 veh/h to each intersection.

The average results of Model 3 are calculated from running this model 20 times. Delay 1, Delay 2, Delay 3 and Delay 4 represent the delay of intersection 1, intersection 2, intersection 3 and intersection 4, respectively. The total delay is the summation of the delays of the four intersections.

Based on the two different methods, the results are summarized in the Table 7 and Figure 8.

Table 7. Average Delay Results of Model 3

<table>
<thead>
<tr>
<th></th>
<th>Delay 1 (s)</th>
<th>Delay 2 (s)</th>
<th>Delay 3 (s)</th>
<th>Delay 4 (s)</th>
<th>Total Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>1103.96</td>
<td>965.92</td>
<td>2139.48</td>
<td>2053.2</td>
<td>6262.56</td>
</tr>
<tr>
<td>Epsilon Algorithm</td>
<td>1527.92</td>
<td>1539.72</td>
<td>1809.84</td>
<td>1772.2</td>
<td>6649.68</td>
</tr>
</tbody>
</table>
Table 8. Comparisons of the Systematic Approach and the Game Theoretic Approach of Model 3

<table>
<thead>
<tr>
<th></th>
<th>Total Delay</th>
<th>Delay Max Difference</th>
<th>VAR of Delay Max Difference</th>
<th>VAR of Delay 1, 2 and 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>6262.56</td>
<td>1173.560</td>
<td>116036.143</td>
<td>523995.827</td>
</tr>
<tr>
<td>Epsilon-Equilibrium</td>
<td>6649.68</td>
<td>281.920</td>
<td>46321.327</td>
<td>36461.813</td>
</tr>
<tr>
<td>Difference</td>
<td>387.120</td>
<td>-891.640</td>
<td>-69714.817</td>
<td>-487534.013</td>
</tr>
<tr>
<td>Change (%)</td>
<td>6.2%</td>
<td>-76.0%</td>
<td>-60.1%</td>
<td>-93.0%</td>
</tr>
</tbody>
</table>

From the Table 8, we can find that compared with the systematical optimization, the epsilon algorithm only increases the total delay of the model by 6.2%. However, the approach of epsilon-equilibrium decreases the variance of the delay max difference by 76.0%. And the variance of the average delay 1, delay 2, delay 3 and delay 4 improves significantly by 93.0%, which means the delays of each intersection in Model 3 become much more stable to use epsilon-equilibrium than the systematical optimization. Compared with the systematic optimal, under the control of the epsilon-equilibrium, the Model 3 results in an impressively low level of loss of the total delay. Furthermore, the stability of the delay of each intersection could be improved excellently.
4.1.4 Simulation of Model 4

The Model 4 has five intersections. The layout of the Model 4 is shown in Figure 10. This model is more complicated than the before ones. Different with the previous corridor traffic network, the layout of Model 4 is more representative which can be a typical component of a general big traffic network. For the main roads, the traffic flow of westbound, eastbound, southbound and northbound is 400 veh/h. For the side road, the traffic flow of westbound and northbound is 200 veh/h.

![Figure 10. Network 4 (Five intersections)](image)
The average results of Model 4 are calculated from running this model 20 times. Delay 1, Delay 2, Delay 3, Delay 4 and Delay 5 represent the delay of intersection 1, intersection 2, intersection 3, intersection 4 and intersection 5 respectively. The total delay is the summation of the delays of the five intersections. Based on the two different methods, the results are summarized in the Table 9 and Figure 11.
Table 9. Average Delay Results of Model 4

<table>
<thead>
<tr>
<th></th>
<th>Delay 1 (s)</th>
<th>Delay 2 (s)</th>
<th>Delay 3 (s)</th>
<th>Delay 4 (s)</th>
<th>Delay 5 (s)</th>
<th>Total Delay (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>1857.773</td>
<td>2133.318</td>
<td>2159.455</td>
<td>1917.045</td>
<td>1946.273</td>
<td>10013.864</td>
</tr>
<tr>
<td>Epsilon Algorithm</td>
<td>2163.227</td>
<td>2240.545</td>
<td>2263.955</td>
<td>2177.182</td>
<td>2192.409</td>
<td>11037.318</td>
</tr>
</tbody>
</table>

Figure 11. Results of Model 4
Table 10. Comparisons of the Systematic Approach & the Game Theoretic Approach of Model 4

<table>
<thead>
<tr>
<th></th>
<th>Total Delay</th>
<th>Delay Max Difference</th>
<th>VAR of Delay Max Difference</th>
<th>VAR of Delay 1, 2, 3, 4, 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Optimal</td>
<td>10013.864</td>
<td>301.682</td>
<td>136540.738</td>
<td>18289.431</td>
</tr>
<tr>
<td>Epsilon Equilibrium</td>
<td>11037.318</td>
<td>100.727</td>
<td>42217.760</td>
<td>1846.528</td>
</tr>
<tr>
<td>Difference</td>
<td>1023.455</td>
<td>-200.955</td>
<td>-94322.978</td>
<td>-16442.902</td>
</tr>
<tr>
<td>Change (%)</td>
<td>10.2%</td>
<td>-66.6%</td>
<td>-69.1%</td>
<td>-89.9%</td>
</tr>
</tbody>
</table>

From the Table 10, we can find that compared with the systematical optimization, the epsilon algorithm only increases the total delay of the model by 10.2%. However, the approach of epsilon-equilibrium decreases the variance of the delay max difference by 66.6%. And the variance of the average delay 1, delay 2, delay 3, delay 4 and delay 5 improves significantly by 89.9%, which means the delays of each intersection in Model 4 become much more stable to use epsilon-equilibrium than the systematical optimization. Compared with the systematic optimal, under the control of the epsilon-equilibrium, the Model 4 results in an impressively low level of loss of the total delay. Furthermore, the stability of the delay of each intersection could be improved excellently.

Table 11. Summary of Running Time

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running Duration (s)</td>
<td>8</td>
<td>8</td>
<td>112.5</td>
<td>52</td>
</tr>
</tbody>
</table>
4.2 Results and Analysis of CAVs

Assuming the penetration rate of automated vehicles is 100%. The CAVs are modeled by the method from section 3.4. Only Model 3 and Model 4 are simulated, because these two networks are more representative. For Model 3, CAVs can reduce the total delay by 41.8% compared with HVs through the approach of systematical optimization. For Model 4, CAVs can reduce the total delay by 46.2%.

Table 12. Comparison of HVs and CAVs

<table>
<thead>
<tr>
<th>Total Delay (s)</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>HVs</td>
<td>6262.56</td>
<td>10013.86</td>
</tr>
<tr>
<td>CAVs</td>
<td>3647.83</td>
<td>5383.24</td>
</tr>
<tr>
<td>Change</td>
<td>-41.8%</td>
<td>-46.2%</td>
</tr>
</tbody>
</table>
CHAPTER 5: CONCLUSION AND FUTURE WORK

The epsilon equilibrium can obviously reduce the variance of the delay of each intersection to make the traffic system more stable. Unlike the approach of systematical optimization, no intersection would sacrifice its own benefit too much after using the game theoretical approach. The total delays of the traffic network will increase under the control of epsilon-equilibrium, but the loss is acceptable, especially for the model with four intersections whose performance is the best one. The variance of the average delay 1, delay 2, delay 3 and delay 4 improves significantly by 93.0% with only 6.2% penalty of the total delay of the system. The total delays of automated vehicles are much less than the total delay of human driven vehicles.

In the proposed game theoretic algorithm, only offsets are available strategies for intersections. Other traffic signal parameters like cycle length and green splits are fixed. More complex strategies contain several parameters can be studied. Also, to make this algorithm more practical, bigger networks should be considered. Due to the fact of computational limitation resulted by increasing number of intersections, how to divide a big network into several groups to utilize game theoretic approach and how these groups coordinate can be studied.
BIBLIOGRAPHY


