Same-Day Delivery with Crowdshipping and Store Fulfillment in Daily Operations

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Abstract

This paper examines the problem of daily operations of same-day delivery with crowdshipping and store fulfillment (SDD-CSF). We aim to close the last-mile delivery gap between local stores and customers. SDD-CSF makes order fulfillment plan from two aspects: order sourcing decision and delivery method selection in order to minimize the cost associated with the order fulfillment plan. We adopt the new concept of last-mile delivery from local stores using crowdsourced shipping, which includes two specific delivery methods based on the distinct characteristics of crowdsourced shippers: Information Sharing Driver (ISDs) and Occasional Drivers (ODs). We devise a dynamic programming model for order fulfillment in a rolling horizon framework, which later is mathematically approximated into a mixed integer linear programming model. The model considers both current received orders and the predicted future demand to make order assignment decision that minimizes the immediate delivery cost plus the resulting future expected cost. It repeatedly solves the model following the timeline in order to construct an optimal fulfillment plan from local stores. With the help of the rolling horizon structure, we also introduce a feedback control system to cope with the inaccurate forecast of future demand. Finally, we prove that under perfect information, the proposed formulation can converge to the global optimal.

Keywords: last-mile delivery, same-day delivery, store fulfillment, crowdsourcing, rolling horizon

Appendix A. Proof – the SDD-CSF model asymptotically converges to the Global-optimal model

From the case study, it can be seen that the objective values of SDD-CSF are always very close to that of the Global-optimal model no matter the size of input orders. In this appendix, we will provide theoretical evidence to suggest that the objective value of SDD-CSF model asymptotically converges to the Global-optimal model. The proof further theoretically validates that SDD-CSF model is a desirable approximation of the dynamic programming model for order fulfillment under relatively accurate customer demand forecasting.

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**Lemma:** When it demonstrates certainty in terms of the forecast demand, crowdshipping availability, and inventory balance, SDD-CSF model is asymptotically optimal with respect to the Global-optimal model. It can be represented by

\[ C^{GO} \leq C^{CSF} \]

where \( C^{GO} \) is the objective value of Global-optimal model while \( C^{CSF} \) is that of SDD-CSF model. When the following three assumptions are met, \( C^{CSF} \) asymptotically converges to \( C^{GO} \).

- The forecast demand is the exact daily customer orders will be realized.
- The sequence of arriving orders from future demand at the same period is negligible for inventory checking and order assignment.
- The crowdshipping delivery performs with certainty.

The outline for the proof is as follows. First, we show that the Global-optimal model is able to be reformulated in form of the cost function with forecast orders \( C^F \). We prove that \( C^F \) is essentially a form of relaxation problem of the Global-optimal model and always has the same optimal value as original one under certain assumptions. Second, we tweak the input schemes of the customer demand in order to combine the formulation of the Global-optimal model and \( C^F \) into one new MIP model, which essentially is the SDD-CSF model without considering the OD pickup probability. By solving the model in form of the rolling horizon, we present that the objective value of Global-optimal model is the lower bound of the consolidation of order assignment result at each period through the theorem of Jensen’s inequality. Finally, we add the OD pickup probability back to the model as the original SDD-CSF model, and shows that the order assignment result from the SDD-CSF model approaches the Global-optimal model when crowdshipping deliveries are done with certainty.

**Step 1.** Reformulate the Global-optimal model in the form of the cost function with forecast orders.

The proof starts from the Global-optimal model. Section 3.2.1 shows that the order indexes \( o \in I \) or \( J \) implies not only the order identification but also the arrival time \( \tau \in t_0, ..., T \) and shipping destination \( z \in Z \) of the order. Therefore, we may transform the order sourcing decision variables from binary \( a_{ikt}, b_{ikt}, e_{ikh} \) into \( a'_{zikt}, b'_{zikt}, e'_{ztkh} \), which physically mean that the sourcing decision variables now are changed to be the group orders placed at time \( \tau \) for destination \( z \) rather than just one order \( i \). Note that \( a'_{zikt}, b'_{zikt}, e'_{ztkh} \) are no longer binary variables. Same transformation is applied to consolidating the quantity of customer orders from the individual order \( d_i \) to the group of orders \( d'_{zt} \).

Comparing to the cost function with forecast orders \( C^F \), it can be found that the decision variables of \( C^F \) adopts the same formation. Furthermore, if we assume the forecast demand in \( C^F \) is the exact daily customer orders that will be realized, we can have

\[ g_{zt} = d'_{zt} \quad \forall z \in Z, t = \tau, .. T \]

and rename the decision variables

- \( w_{zkt} = a'_{zkt} \) \quad \forall z \in Z, t = t_0, .. T, k \in K, t = \tau, .. T \)
- \( x_{zkt} = b'_{zkt} \) \quad \forall z \in Z, t = t_0, .. T, k \in K, t = \tau, .. T \)
- \( v_{ztkh} = e'_{ztkh} \) \quad \forall z \in Z, \tau = t_0, .. T, k \in K, h \in H \)

We introduce an assumption that the sequence of arriving orders from future demand at the same period is negligible for the inventory checking since the order assignment now changes from individual order to one batch of orders placed at time \( \tau \) and zone \( z \). The objective value of Global-optimal model (48)-(57) can be obtained by \( C^F \) with (20)-(30) with \( g_{zt} = d'_{zt} \). Therefore, \( C^F \) is essentially a form of relaxation problem of the Global-optimal model and always has the same optimal value as original one. Mathematically, the convex hull of Global-optimal model can be projected from the order \( I \) to \( s \) higher dimension space which is derived by the combination of the zone set \( Z \) and the order arrival
time set \( t \in 0, ..., T \). When it is with \( g_{zt} = d_{zt} \) and inventory assumption, and the optimal value can be found at the same projected vertex \( C^{GO} = C^F \). For the following steps, these two assumptions hold true unless specified otherwise.

**Step 2.** Integrate the formulation of the Global-optimal model for current received orders and the cost function with forecast orders for future orders.

Next, we divide the quantity of customer orders from \( d_i \) into two parts – \( d_i^{(b)} \) for start point \( t = t_0 \) and \( g_{zt} \) for following hours \( t_0 + 1, ..., T \) to represent the practical scenario. For the purpose of coordinating both \( d_i^{(b)} \) and \( g_{zt} \), we define an new objective function in form of the combination of \( C_\tau \) following hours.

**Step 3.** Incorporate the OD pickup probability back to the model as the original SDD-CSF model.

Finally, we aim to incorporate the OD pickup probability \( p_{zik}^T \) into Equation (A.1). Since ODs perform delivery with uncertainty with \( p_{zik}^T \), the fixed-sourcing orders by set \( J \) and penalty cost would be added to the (A.1) to handle the unfulfilled order by ODs. The objective function of the model changes from Equation (A.1) to Equation (32), which happens to be the exact same objective function of the SDD-CSF model \( C^{CSF} \). Derived from \( C^{CSF} \), we define...
the cost of assignment for existing orders at period $t$ as:

$$\varphi^{CS F}_{t=t_0} = \sum_{ik} \left( \sum_{t=t_0}^{T} c_{ki} \cdot \alpha_{ikt} \right) + \sum_{t=t_0}^{T} \left( p_{tki}^{T} \cdot c_{OD}^{k} \cdot b_{ikt} + (1 - p_{tki}^{T}) \cdot \theta \cdot b_{ikt} \right) + \sum_{h} c_{ki}^{IS} \cdot e_{ikh}$$

$$+ \sum_{jk} \left( \sum_{t=t_0}^{T} c_{kj} \cdot \alpha_{jkt} \right) + \sum_{t=t_0}^{T} \left( p_{tkj}^{T} \cdot c_{OD}^{k} \cdot \beta_{jkt} + (1 - p_{tkj}^{T}) \cdot \theta \cdot \beta_{jkt} \right) + \sum_{h} c_{kj}^{IS} \cdot \gamma_{jkh}$$

$$+ \theta \cdot (I - \sum_{ik} \sum_{t=t_0}^{T} a_{ikt} - \sum_{ik} \sum_{t=t_0}^{T} b_{ikt} - \sum_{ikh} e_{ikh}) + \theta \cdot (J - \sum_{jk} \sum_{t=t_0}^{T} \alpha_{jkt} - \sum_{jk} \sum_{t=t_0}^{T} \beta_{jkt} - \sum_{jkh} \gamma_{jkt})$$

The OD pickup probability $p_{tk}$ differentiates $\varphi^{CS F}$ from $\varphi^{OF}$. Considering there exists a chance that orders can be unfulfilled after the preparation for ODs, we divide the unfulfilled orders once again into two parts, represented by the set $I$ and the set $J$, which are added into $\varphi^{CS F}$.

Since unfulfillment penalty ($\theta$) is much bigger than the rates of order fulfillment, $c_{ki}^{OD} \ll \theta \quad \forall k \in K, i \in I$

then

$$c_{ki}^{OD} \cdot b_{ikt} \leq p_{tki}^{T} \cdot c_{ki}^{OD} \cdot b_{ikt} + (1 - p_{tki}^{T}) \cdot \theta \cdot b_{ikt} \quad \forall t = t_0, ..., T, k \in K, i \in I$$

The comparison between the total cost of SDD-CSF and that of the Global-optimal is

$$C^{GO} \leq C^{OF} = \sum_{t=0}^{T} \varphi^{OF}_{t} \leq C^{CS F} = \sum_{t=0}^{T} \varphi^{CS F}_{t}$$

Thus, we have shown that in some case, the objective function of SDD-CSF model approaches that of the Global-optimal optimization model for the time horizon. It justifies that the SDD-CSF model is a mathematically feasible and desirable approximation of the dynamic programming model for order fulfillment in form of the rolling horizon. □