Data-driven optimization of railway maintenance for track geometry

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ABSTRACT

Railway big data technologies are transforming the existing track inspection and maintenance policy deployed for railroads in North America. This paper develops a data-driven condition-based policy for the inspection and maintenance of track geometry. Both preventive maintenance and spot corrective maintenance are taken into account in the investigation of a 33-month inspection dataset that contains a variety of geometry measurements for every foot of track. First, this study separates the data based on the time interval of the inspection run, calculates the aggregate track quality index (TQI) for each track section, and predicts the track spot geo-defect occurrence probability using random forests. Then, a Markov chain is built to model aggregated track deterioration, and the spot geo-defects are modeled by a Bernoulli process. Finally, a Markov decision process (MDP) is developed for track maintenance decision making, and it is optimized by using a value iteration algorithm. Compared with the existing maintenance policy using Markov chain Monte Carlo (MCMC) simulation, the maintenance policy developed in this paper results in an approximately 10% savings in the total maintenance costs for every 1 mile of track.

1. Introduction

Rail across the world is experiencing an increase in demand that is driven by increased global trade (Li et al., 2014). In the United States, railways are one of the major modes of freight transportation. In the 2007 Commodity Flow Survey, rail accounted for 46% of total national ton-miles (Bureau of Transportation Statistics, 2014). Rail is also used, to a fair extent, by people to commute between two places. Therefore, there is increasing pressure on railways to maintain a high service level at all times. Railway tracks are essential components in the rail industry. As typical mechanical systems, tracks are prone to faults and failures with time and usage. In 2009, out of 1890 train accidents, 658 were due to track defects (Peng et al., 2011). Major failures of railway tracks can cause heavy economic losses, lawsuits, huge delays in recovery operations and, in extreme cases, fatalities. The severe consequences due to track defects increase the pressure to maintain rail tracks in a good state of repair. In addition, with the advance of new communication and sensing tools, big data is becoming increasingly emerging in railway transportation (Nunez and Attoh-Okine, 2014).

The main objective of this paper is to develop a data-driven track preventive maintenance strategy, which also takes spot corrective maintenance into account, to maintain the best service level of railway track with minimal costs. Preventive maintenance helps to prevent major failures from occurring. The primary objective of preventive maintenance is to preserve system functions in a...
cost-effective manner (Tsang, 1995). Preventive maintenance can be further classified as condition-based or interval-based (time interval or tonnage interval) (Yang, 2003). In interval-based preventive maintenance, maintenance activities occur after a certain period of time, and the system is restored to its initial state. In condition-based preventive maintenance, maintenance actions are taken depending on the current state of the system after each inspection (Su et al., 2016). The focus of this research is related to condition-based preventive maintenance at discrete time intervals. After an inspection, this paper considers three maintenance actions: no action taken on the system, minor maintenance work to restore the system back to the previous working state, or major maintenance work to greatly restore the system to much better conditions (Chen and Trivedi, 2005). In this study, minor maintenance refers to preventive tamping, while major maintenance refers to corrective tamping (Khouy et al., 2014).

This paper adopts the track quality index (TQI) as an overall track-state indicator for decision making in preventive maintenance. TQI is a numeric representation of the ability of railroad track to perform its design function, or, more precisely, to support the required train movements (Fazio and Corbin, 1986). In short, TQI indicates whether the track is in a good state or a bad state. If the track is in a bad state, the railroad performs appropriate maintenance activities to improve its condition and restore it to the good state. The railroad can plan the actions depending on the TQI value. This makes it easier to develop a state using a range of TQI values. Assuming the future state of the track only depends on the current state, one can regard the track as a Markovian system. Additionally, there are more than one actions that can be taken for a state so that the problem can be formulated as a Markov decision process (MDP). This paper aims to assist the railway industry in maintaining a state of good repair for existing track systems. The proposed maintenance strategy will help in reducing the cost expenditures by railroads as well as preventing failures that may lead to derailments and accidents.

In contrast to preventive maintenance, corrective maintenance aims to recover the track into a state in which it can perform a required function after fault recognition. In this paper, we refer corrective maintenance as rectifications of spot defects reported by daily manual inspections or scheduled track inspections. Over time, the spot conditions of railway track can degrade from a good state to an unusable state, either gradually or abruptly. This can occur due to cumulative tonnage, defective wheels, and the impulsive force on tracks.

Railway track spot defects can be classified into two different types, namely, track structural defects (also known as rail defects) and track geometry defects. Track structural defects occur when the structure and support system of the railway tracks, comprising sleepers, joints, fasteners, ballast and other underlying structures, fail. Track geometry defects arise due to irregularities in the various track geometry measurements (Zarembski, Einbinder and Attoh-Okine, 2016). In practice, railroads collect massive raw inspection data on several dozens of track geometry parameters. However, due to data limitations, this paper focuses on track geometry defects that exceed the threshold. The majority of track geometry defects fall into a few types of geometry measurements. Without loss of generality, this paper only investigates the following five prevailing track geometry measurements: (1) Cant: the amount of vertical deviation (in radians) between two flat rails from their designed value; (2) Cross-level: the difference in elevation between the top surfaces of the rails at a single point in a tangent track segment; (3) Gage: the distance between the heads of the inner surface of the rails; (4) Surface: the uniformity of the rail surface measured in short distances along the tread of the rails; (5) Twist: the difference between two cross-level measurements a certain distance apart. Fig. 1 illustrated these five defects. The dashed lines in Fig. 1 indicate the deviation from the normal state. Therefore, one can tell how each type of defects deviates from the normal state. One can refer to He et al. (2015) for more detailed explanations of track geometry measurements.

A flow chart of this study is presented in Fig. 2. In this paper, Policy is a course of action or decision proposed by the model for track maintenance. Starting from raw track related data, this study examines both optimal policy and existing policy for track maintenance. Regarding optimal policy, this paper considers both corrective maintenance and preventive maintenance. First, a random forest is employed to forecast the occurrence of geo-defects that have to be rectified after inspections. Second, we use an equation to measure TQI model by aggregating raw geometry measurements and build a Markov model based on field observations. The occurrence of geo-defects is then modeled as a Bernoulli process. Then, we define actions in track maintenance and derive an MDP model which incorporates both maintenance costs and geo-defect repair costs. We use a value iteration algorithm to solve the MDP model and determine the optimal policy. In contrast, the existing policy is derived directly from the raw track data. Finally, we employ Markov chain Monte Carlo (MCMC) simulation to calculate and compare the total cost of different policies. This paper makes the following three contributions: (1) as a first attempt, it builds a prediction model for the occurrence of geo-defects with massive foot-by-foot track geometry data, traffic in million gross tonnage (MGT), track speed limit, and historical maintenance activities. The relationships between the values of TQI and the arrival probability of geo-defects are quantified; (2) it establishes a Markov chain to model the track deterioration process and calibrate the transition probability with the real-world data; (3) it develops a Markov decision process (MDP) for track maintenance decision making, incorporating both preventive maintenance based on TQI and spot corrective maintenance based on geo-defects.

2. Literature review

2.1. Track quality index

The track quality index (TQI) is one of the most widely used indices to represent the track state. Traditionally, TQI is derived from foot-by-foot track geometry measurements, which reflect how well the track structure is performing. An overall track maintenance planning model can be developed easily if the geometry TQI data are supplemented with additional data pertaining to the structural data (Fazio and Corbin, 1986). TQI also helps in maintaining a track deteriorating record (El-Sibaie and Zhang, 2004).

The track geometry is measured for each foot by a track geometry car, an automated track inspection vehicle on a rail transport
system used to test several geometric parameters without obstructing normal railroad operations. These measurements are later aggregated at 0.1 mile-segment level to calculate the TQI (Schlake et al., 2011).

The method of calculating TQI varies by country. A one-mile segment of railway track is divided into smaller 0.1 mile-segments, and various geometry parameters are measured. These geometry statistics are then summed to obtain the TQI value of the section (Berawi et al., 2010). In China, the TQI is calculated as the sum of the standard deviation of seven track geometry measurements (Bai et al., 2015). In the United States, the TQI is calculated as the ratio of the traced space curve length to the track segment length (El-Sibaie and Zhang, 2004). In Europe, the J synthetic coefficient is used as an indicator of the track quality based on the standard deviation in Polish Railways (Madejski and Grabczyk 2002). In addition, the rail track geometry on the sample segment is also assessed according to European Standard EN 13848-5 (Berawi et al., 2010). In India, a formula, called the track geometry index (TGI), has been developed by Indian Railways to represent the quality of the track. This model is based on the standard deviation of different geometry parameters over a 200 m segment (Podofillini et al., 2006).

However, as the TQI is calculated in an aggregated manner, it is possible for the TQI to miss severe spot failures, such as geo-defects. In this paper, we first calculate the TQI as an aggregate model. Later, we incorporate the stochastic arrival of geo-defects, as an individual external factor, to assess the track conditions in a better manner.

2.2. Track preventive maintenance

Track preventive maintenance refers to the procedure where tracks have a set of maintenance schedule to prevent the track failure during use. Track preventive maintenance can be time-based, where the tracks are monitored and maintenance activity occurs from time to time, or condition-based, where the maintenance activities depend on the current condition of the track. Condition-based
planning and management are a significantly more efficient methods of managing the rail asset than the traditional rules-based approach because they account for the local differences in behavior and performance, which affect the degradation of the rails (Zarembski, 2010).

Track preventive maintenance involves many complex costs, such as inspection cost, different types of maintenance cost, track downtime costs, labor costs, and material costs. Substantial research has been conducted in the field of effectively scheduling preventive maintenance to improve the cost incurred by a company. When the cost incurred by device failure is larger than the cost of preventive maintenance, it is worthwhile to perform preventive maintenance (Chen and Trivedi, 2005). In addition, studies have been conducted to determine the relationships between renewal and maintenance activities (Grimes and Barkan, 2006).

Preventive maintenance is expensive when performed too early or too late. (Peng, 2011) used models to significantly reduce travel and penalty costs. Time-space network models have been created to address the rising issue of maintenance cost (Peng and Ouyang, 2012). Optimization models were built to minimize both maintenance and renewal costs, as well as delays related to operational services (Andrade, 2014). Mathematical programs have been suggested to schedule routine maintenance activities and unique maintenance activities (Budai et al., 2006). Another formulation was proposed for both track maintenance activities and crew optimization, which was solved by Tabu search (Higgins, 1998)). Decision rules models were developed to provide planning/scheduling solutions by following a set of rules used in maintenance scheduling (Santos et al., 2015). Sometimes it is possible for a system to continue to operate in a degraded way, even after failure. The optimal time to perform repairs to maximize the reward was calculated using mathematical equations (Castro and Sanjuán, 2008).

Implementation of condition-based preventive maintenance requires accurate failure identification and predictions (Gibert et al., 2017). Multivariate statistical models have been developed to improve the ability to predict the probability of broken rails and other types of failure (Dick et al., 2003). Survival analysis was used to estimate the derailment risk due to geo-defects (He et al., 2015). Methods have been developed to create a system for reliable fault diagnosis and to identify trends of equipment failures using neural networks (Yam et al., 2011). Machine vision techniques have been used to recognize and detect defects in track components (Molina et al. 2011). Big Data techniques have also been applied to facilitate maintenance decision making (Núñez et al., 2014). In addition, Jamshidi et al. (2017) proposed an image processing based approach to estimate the probability of failure risk of a railway. This
probability was obtained based on the information of two defect-related variables, including visual length and crack growth. In this paper, the rail system’s deterioration is modeled by a Markov process, and the spot geo-defects are predicted by a random forest. This modeling can help in taking better maintenance actions.

2.3. Markov decision process (MDP)

MDP is a sequential decision model where the set of available actions, rewards, and transition probabilities depends on the current state and not the states occupied or actions chosen in the past (Puterman, 2005). MDP has been successfully implemented in the optimization of maintenance schedules and procedures for deteriorating systems. Studies have been conducted to optimize the maintenance policy of circuit breakers (Ge et al., 2007), corroded structures (Papakonstantinou and Shinozuka, 2014), water distribution system (Kim et al., 2015), infrastructure networks (Smilowitz and Madanat, 2000), and wind turbines (Byon and Ding, 2010). MDP has provided optimal effective maintenance decisions based on the conditions revealed at a point of time (Amari et al., 2006). When we have partial information about a deteriorating system, MDP can still be implemented to gain knowledge about condition-based maintenance (Hontelez et al., 1996).

In the transportation domain, MDP is wildly used in decision making about maintenance activity to be performed for the condition of road pavement (Ben-Akiva et al., 1993; Feighan et al., 1988). Similar to pavement maintenance, (Ferreira and Murray, 1997) demonstrated the potential to implement the MDP model in rail track maintenance. Moreover, MDP does not only provide an optimal effective maintenance decision based on the conditions revealed at each point of time (Amari et al., 2006), it also works when there is only partial information about the deterioration system in condition-based maintenance (Hontelez et al., 1996). Therefore, MDP is an appropriate approach to planning transportation maintenance.

There are several implementations of Markov modeling in railway track inspection and maintenance. A previous study developed a Markov model where the TQI was calculated in a range of 0–100 based on the unevenness, twist, alignment and Gage measurements (Shafahi and Hakhamaneshi, 2009). Five states were used to represent the 100 unit TQI range in the Markov model. According to (Shafahi and Hakhamaneshi, 2009), several factors affect the track degradation process, including axle load, traffic speed, weather, and geometry. However, after carefully processing the data, they found that track will have different deterioration rates despite having some of the same factors. Therefore, track deterioration process is a complex stochastic process which comes from the complex interactions among trains, tracks, environment-weather conditions, and others. Given this sense, there are some other studies that leveraged Markov process to describe such random process. (Podofillini et al., 2006) developed a Markov model to predict rail breakage probability and to calculate the risks and costs associated with an inspection strategy. The proposed maintenance model was eventually optimized by a genetic algorithm. A recent study analyzed track degradation data from the UK rail network to generate degradation distributions that were used to define transition rates within the Markov model (Prescott and Andrews, 2013). Another Markov model with 50 states was adopted to model the variation of twist over time, each state representing the twist on a section of track in the range of 1–50 mm (Lyngby et al., 2008). Most of the previous work focuses on track degradation modeling using Markov models. However, no prior studies have examined the benefits of implementing MDP model with spot defect repair. There remains a research gap in using MDP for track preventive maintenance with the consideration of spot corrective maintenance.

3. Methodology

The following reasonable assumptions are made in this paper:

- Without loss of generality, the maintenance activities are classified as minor and major maintenance. One can treat minor maintenance as preventive tamping and major as corrective tamping, respectively (Khoye et al., 2014).
- Only one maintenance action is allowed to be taken after each inspection.
- Minor maintenance action only takes the Markov chain from its current state to the previous state.
- Major maintenance action improves the track condition by two states or more. The probability mass function of the final state is observed based on historical data.
- This study does not consider the downtime cost and derailment cost, which can be estimated accordingly (He et al., 2015). In addition, the total costs for maintenance activities already include labor costs.
- We only consider the red-tag or critical geo-defect that exceeds the FRA thresholds. There is maximal one geo-defect that can be found per segment after each inspection. Once identified, the geo-defect has to be repaired before the next inspection run.
- To ensure safety, a track segment that is found in the worst state i.e. state 5 in our case study below, will always request some maintenance action to improve the state.

Note that the effects of minor and major maintenance may not align with the above assumptions. Later, a sensitivity analysis has been conducted to address this issue.

3.1. TQI measurement

The track geometry data for each foot is first aggregated into a 0.1 mile-segment, and each 0.1 mile-segment is $L_0$ in length. The TQI is then calculated for each type of track geometry measurement individually using the following formula (El-Sibaie and Zhang,
\[
TQI = \left( \frac{L^2}{L_o} - 1 \right) \times 10^6, \tag{1}
\]

where

\[TQI = \text{track quality index};\]
\[L_s = \text{traced length of space curve (ft.)};\]
\[L_o = \text{fixed length of track segment (ft.)}.\]

The value of \(L_o\) is fixed at 528 ft. \(L_s\), the traced space curve length, is calculated by summing the distances between any two points within the track segment.

\[
L_s = \sum_{i=1}^{n} \sqrt{(\Delta y_i^2 + \Delta x_i^2)} = \sum_{i=1}^{n} \sqrt{(\Delta y_i)^2 + (\Delta x_i)^2}, \tag{2}
\]

where

\[\Delta y_i = \text{difference in two adjacent measurements (ft.)};\]
\[\Delta x_i = \text{sampling interval along the track (=1 ft.)};\]
\[i = \text{sequential number}.\]

In the presence of separate track geometry data for the left track and right track, as in the case of surface and cant, we always choose the measurement (error) with higher absolute value.

### 3.2. Data mining methods for geo-defect forecasting

Given the assumption of maximal one geo-defect that can be found per segment after each inspection, we model the arrival of geo-defects as a Bernoulli process with probability \(p_g\). Further, we apply three data mining algorithms, random forest (Breiman, 2001) support vector machine (SVM) (Cortes and Vapnik, 1995) and logistic regression (Jr et al., 2013), to model the relationship between the explanatory variables and the dependent variable, which is the occurrence of geo-defects. The purpose of applying these algorithms is to determine \(p_g\) with which geo-defects occur with a particular TQI. Random forest is an ensemble learning method for classification and regression that constructs a multitude of decision trees during training. Random forest can correct the problem of overfitting in the decision tree algorithm to its training set. Li and He (2015) has successfully implemented random forest for forecasting railcar remaining useful life. SVM is another popular supervised learning model for classification and regression models (Schuldt et al., 2004; Gibert et al., 2015; Tong and Koller, 2001). It separates data from different classes by mapping data into a high dimensional space and then dividing them with decision boundaries that are as wide as possible.

Support vector machines (SVM) has been implemented to different problems in railway. Cárdenas-Gallo et al., (2017) developed an ensemble methodology to forecast degradation of track geometry. Gilbert et al. (2015) devised a combination of linear SVM classifiers to inspect ties for missing or defective rail fastener problems. Park et al. (2008) applied a two-step SVM classifier for monitoring railroad track. Moreover, an SVM has also been developed to inspect rail corrugation (Li et al., 2017), detect and diagnose misalignment faults of electrical railway point machine (Asada, et al., 2013), predict hot box detector failures (Li et al., 2014). Logistic regression measures the odds or probability of a categorically dependent variable based on one or more independent variables using a logit function. It provides a probabilistic explanation for class separation. Logistic regression works well as long as the features are roughly linear and the classes are linearly separable. It is a prevalent method in banking, including bank failure prediction (Zaghdoudi, 2013).

### 3.3. Markov decision process model

#### 3.3.1. Development of the Markov decision process model

The methodology starts creating a discrete-time Markov chain. A Markov chain is a process in which events remain in the same state or move from one state to another. As the future state is dependent only on current state and not on any previous state, it is therefore memoryless in nature. When we describe the process over discrete periods of time, the Markov chain is known as a discrete-time Markov chain or DTMC. The Markov property will be proved later by historical datasets.

Decision epochs are the time points when the maintenance actions are carried out. This study assumes that decisions are made right after each inspection which is scheduled periodically. The downtimes of maintenance actions are neglected. The decision horizon is finite and discretized by decision epochs.

The state and defined actions are observed based on past track inspections and measurements. One can easily develop the transitional probability matrix using the following equation,
\[ P(S'|S,A,T) = \frac{N_{SS'AT}}{\sum_{j=1}^{N} N_{SjAT}} \]  

(3)

where

- \( S \) = the current state;
- \( S' \) = the next state;
- \( A \) = the action taken;
- \( T \) = discrete inspection interval;
- \( N \) = total number of states;
- \( N_{SS'AT} \) = the number of times there has been a transition from state \( S \) to state \( S' \) given action \( A \) and inspection interval \( T \);
- \( N_{SjAT} \) = the number of times there has been a transition from State \( S \) to all other states \( (j) \) given action \( A \) and inspection time \( T \).

In Eq. (3), the term \( \sum_{j=1}^{N} N_{SjAT} \) represents the total number of times a transition has occurred from the given state \( S \) to all other states given action \( A \) and inspection interval \( T \). The transition probability matrices are developed using Eq. (3).

A typical MDP problem is defined by the actions taken to maximize the rewards (minimize the total costs) when in current state \( S \). This calculation also considers a discount factor \( \gamma \), where \( 0 \leq \gamma \leq 1 \). Below is the typical mathematical definition of an MDP.

\[ \pi^* = \min \sum_{t=0}^{\infty} \gamma^t C_{\pi_t}(S,A,d), \]  

(4)

where

- \( \pi^* \) = the optimal policy;
- \( \pi_t \) = the current policy to be followed in state \( S \);
- \( \gamma \) = the discount factor representing the difference in future rewards (costs);
- \( C_{\pi_t} \) = the cost function for existing policy to be followed in State \( S \);
- \( d \) = the random variable following Bernoulli distribution. \( d = 1 \) when a geo-defect occurs, otherwise \( d = 0 \).

In this study, the cost shall reflect the impacts of taking a maintenance action, including inspection costs, maintenance costs, and repair costs of spot defects. The cost function \( C(S,A) \) in each state \( S \) and action \( A \) can be written as,

\[
C(S,A) = \begin{cases} 
  c_i & A = 0, d = 0 \\
  c_i + c_m & A = 1, d = 0 \\
  c_i + c_M & A = 2, d = 0 \\
  c_i + c_R & A = 0, d = 1 \\
  c_i + c_m + c_R & A = 1, d = 1 \\
  c_i + c_M + c_R & A = 2, d = 1 
\end{cases}
\]  

(5)

where

- \( c_i \) is the inspection cost;
- \( c_m \) is the cost of minor maintenance;
- \( c_M \) is the cost of major maintenance;
- \( c_R \) is the repair cost of a geo-defect.

The MDP problem can be solved optimally using various techniques, such as value iteration, policy iteration, and linear programming. In this paper, we use value iteration to solve the MDP problem. In addition, one may argue the proposed model is not exactly the same as the practice since the number of states and actions are limited in MDP. However, railroads can learn from the differences between the existing policy and optimal policy. The obtained optimal policy will provide a general guidance on how to design the practical maintenance strategy.

3.3.2. Solution algorithm for the Markov decision process

We formulated an MDP in the previous sections. In this section, we aim to optimize the MDP. Our main aim is to determine an optimal policy \( \pi^* \) that describes the optimal action to be taken in each state.

The most widely used techniques to solve an MDP problem are the value iteration algorithm, policy iteration algorithm and linear programming method. In this paper, we use the value iteration algorithm, which converges exponentially fast. The value iteration method has been developed to solve different types of problems. For example, it can be used to solve dynamic programming problems (Bertsekas, 1998).

The value iteration algorithm (Bellman, 1957) is a method to compute the optimal value of an MDP. Value iteration works well if the state space is cyclic. The idea behind value iteration is to maximize the rewards (minimize the costs) collected over a period of
time. However, when moving from one state to another in an MDP, we are concerned with only the immediate reward (cost); we do not know whether the path will lead us to a state with a high reward (low cost). Thus, value iteration searches for the true value of the state and follows the path where it can obtain the minimal costs

\[
Q_{i+1}(S,A) = \sum_{S'} P(S'|S,A,T) (C(S,A,d) + \gamma V_i(S'))
\]

(6)

\[
V_{i+1}(S) = \min_{A \in A}(Q_{i+1}(S,A))
\]

(7)

Initially, we define two functions, \(Q_i\), which represents the Q-function with \(i\) stages remaining, and \(V_i\), which represents the value function with \(i\) stages to go. The value iteration algorithm works recursively. An optimal solution is found when the values of \(Q^*\) and \(V^*\) converge. While applying this algorithm, we start at the end and move backward, updating the values of \(Q^*\) and \(V^*\). \(\gamma\) in Eq. (2) represents a discount in rewards (costs) over a period of time. In this paper, it is assumed that \(\gamma = 1\).

We start with an initial value \(V_0 = 0\). We calculate \(Q_i\) for the current value of \(V_0\) and proceed to calculate the value of \(V_i\). We continue iterating the value of \(V_i\) at each step for all states until the values converge and we obtain an optimal policy.

3.4. Markov chain Monte Carlo simulation

Monte Carlo methods are procedures where samples are repeatedly drawn from random distributions to obtain numerical results that are close to the actual results. When this method is applied to Markov chains, the process is known as Markov chain Monte Carlo (MCMC) simulation (Gilks, 2005). MCMC has been applied in other research domains such as healthcare (Li et al., 2016) and marine transportation (Faghih-Roohi et al., 2014). However, to the best of our knowledge, there only exist a few papers about the applications of MCMC in railway maintenance. Mokhtarian et al., (2013) developed a Bayesian nonparametric method using MCMC algorithm to estimate the lifetime of railway system components. Andrade and Teixeira (2013a,b) applied a hierarchical Bayesian model to predict rail track geometry degradation for maintenance purposes. A similar approach was proposed to model the main two quality indicators of rail track geometry, including the standard deviation of longitudinal level defects and the standard deviation of horizontal alignment defects (Andrade and Teixeira, 2015). Elberinka et al. (2013) presented a method for detecting and modeling rail defects. An MCMC algorithm is used to obtain an estimation by sampling the joint probability distribution of the orientation parameters. Wellalage et al. (2014) developed a Metropolis-Hasting algorithm (MHA)-based Markov chain Monte Carlo (MCMC) simulation technique to predict the future condition of railway bridge elements.

MCMC simulation is adopted to assess the total cost of different maintenance policies given the previously built Markov model. The entire dataset is randomly shuffled and then the mileposts are split into the training dataset and the test dataset. The optimal policy and existing policy are derived from the training dataset only, whereas test dataset is used to generate test transition matrices and to be implemented in the MCMC simulation. We begin the MCMC simulations by defining the end state from the data as the current state for each 0.1 mile-segment of the track. The information about the current state is used in the simulation, which continues until the end of the simulation cycle.

At each step, for a given state and time period, we generate action and derive the next state according to the transition probability matrices. During the MCMC simulation, when a transition occurs from the current state to the next state, a geo-defect is generated, given the TQI values in the current state and other required predictors in Table 1. When the next step is reached, we repeat the above procedure and move forward. During this procedure, we keep a count of each action taken to move from the current state to the next state, as well as the number of defects generated and repaired.

4. Case studies

4.1. Data collection

The data for this study were collected from a Class I railroad, which has a minimum carrier operating revenue of $467 million or more in 2013 (Association of American Railroads, 2014), during March 2009 to December 2011. 50 miles data was selected for the analysis. The entire dataset contains 9,673,453 rows and 35 columns, in total 1.54 GB. The data consist of various foot-by-foot geometry measurements, including gage, cross-level, surface, twist, warp, dip, and cant, along with the geo-location details, such as milepost, track number, and line segment, giving each section a unique identity.

Table 1

<table>
<thead>
<tr>
<th>Variables</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>TQI</td>
<td>Track quality index in the current inspection run</td>
</tr>
<tr>
<td>TONNAGE</td>
<td>Total cumulative tonnage (MGT) from the last inspection run to the current inspection run</td>
</tr>
<tr>
<td>FSpd</td>
<td>Freight car speed limit (mph)</td>
</tr>
<tr>
<td>T</td>
<td>Number of days between the last inspection run and the current inspection run</td>
</tr>
<tr>
<td>Def_1</td>
<td>Indicator of whether the last inspection had any geo-defects in the segment. If Def = 1, there was at least one geo-defect; otherwise, Def = 0</td>
</tr>
<tr>
<td>Def</td>
<td>Indicator of whether the current inspection had any geo-defects in the segment. If Def = 1, there was at least one geo-defect; otherwise, Def = 0</td>
</tr>
</tbody>
</table>
For all the 50 miles-segments considered in the case study, there are 13 inspections within the study period. As shown in Fig. 3, the timeline of the inspection runs is determined based on the data between March 2009 and December 2011. The numbers represent number of days from the previous inspection. Because we have no data before the first inspection, we consider the number of days from the previous inspection to be zero.

In addition to the geometry measurements, geo-defects, which are extreme geometry measurements over the threshold, were also reported and collected during each inspection run. The geo-defects accounts for sudden changes in track geometry measurements. In this paper, we consider only critical defects or red-tag defects, which violate the FRA pre-defined rules (He et al., 2015). These geo-defects have to be rectified within specific deadlines.

4.2. Data analysis

4.2.1. Calculation of TQI

The data provided to us comprise measurements on each foot of track. Data for 1 mile include data for 5280 ft. We set a threshold of 5000 ft. of data to consider for our analysis. Otherwise, the data measurements in one mile-segment are removed. If the one-mile segment contains less than 5280 ft. of data, the missing data are interpolated by taking the mean value of the given data for that particular one-mile segment and geometry type. We calculate the TQI for gage, twist, surface, cant and cross-level defects separately.

As track irregularities can occur due to a combination of multiple track geometry measurements, we consider multiple track geometry measurements for the TQI.

To normalize the TQI, the 95th percentile of the TQI of each geometry measurement for 0.1 mile-segment is calculated over the inspection period. Then, the original TQI values of each geometry type are divided by the 95th percentile of the TQI. Finally, we take the mean values of the normalized TQI across five geometry types for all 0.1 mile-segment to determine the combined TQI value of each 0.1 mile-segment.

Fig. 4 shows the changes in TQI over time for a variety of one mile where each segment represents 0.1 miles. After each inspection run, the geometry measurements are recorded. Our assumption is that given no maintenance, the track will continue deteriorating (increasing). This is represented by the increasing trend of TQI. However, the decreasing trend of TQI indicates the occurrence of maintenance between the inspections. When an inspection is conducted after maintenance activities, there is a drastic decrease in the TQI. If an inspection is not performed for a long time, the track degrades, and the TQI increases.

4.2.2. The results of geo-defect forecasting

There is a total of 5776 aggregated measurements, of which 204 measurements include red geo-defects. The prediction model forecasts the probability of geo-defect occurrence in the current inspection run. Table 1 defines all the variables and corresponding descriptions. This paper considers six explanatory variables, TQI, TONNAGE, FSpd, T and Def_1, and one dependent variable, Def.

In this paper, we first divide the dataset into a training set, containing 70% of the total observations, and a testing set, with the remaining 30%. Using R,\textsuperscript{1} we obtain three models based on the training set by using three algorithms. Then, we use the test set to assess the model performance in terms of accuracy and precision for each algorithm, as shown in Table 2.

Accuracy = \frac{\text{number of true positives} + \text{number of true negatives}}{\text{number of true positives} + \text{false positives} + \text{false negatives} + \text{true negatives}}

Precision = \frac{\text{number of true positives}}{\text{number of true positives} + \text{number of false positives}}

Random forest reaches the highest accuracy 75.2% and precision 77.4%. A recent study developed an ensemble classifier for geometry defect deterioration classification to determine whether a yellow defect will turn into a red defect after some time (Cárdenas-Gallo et al., 2017). The study reported an average accuracy of 74–82%. Although the prediction problem is different in these two studies, the accuracy and precision appear reasonable in this paper. Therefore, random forest has the best performance and is selected for geo-defect forecasting. To discretize the state, we divide TQI into 5 levels by percentile, 0th-20th percentile, 20th-40th percentile, 40th-60th percentile, 60th-80th percentile, and 80th-100th percentile. These five levels of TQI correspond to the five

\textsuperscript{1} R package “randomForest” was used for random forest. R package “e1071” was used for SVM.
states of the Markov Chain defined in Section 4. In a similar manner, we divide the inspection interval $T$ into 3 levels, 0–58 days, 59–85 days and more than 85 days. Then, we apply the best model (random forest) to make predictions for each TQI level and each level of $T$.

Fig. 5 shows that for the same TQI level, the larger the value of the inspection interval is, the higher the probability of geo-defect occurrence. Fig. 6 shows that for the same inspection interval, the larger the value of TQI is, the higher the probability of geo-defect occurrence. However, there are some exceptions. The probability fluctuates at some mile. For example, the black line, representing the first TQI level, intersects the others at mile segment 303.5. The exception shows that a lower TQI may generate a higher probability of geo-defect occurrence than a higher level TQI. This is reasonable since a geo-defect is a type of spot defect that could be caused by sudden impact from vehicle-track interactions. Therefore, low levels of TQI still suffer from the risk of track failure caused by geo-defects.

4.3. The results of the Markov chain

As the dataset does not have a constant interval between inspections, to create a DTMC, we build a discrete-time model by dividing the data into three levels based on the inspection interval.

$$T = \begin{cases} 
0–58 \text{ days (58th day is the 33rd percentile)} \\
59–85 \text{ days (85th day is the 66th percentile)} \\
86 \text{ days and beyond}
\end{cases}$$

The states of the Markov chain are defined based on the TQI percentiles. Every 20th percentile defines a state, resulting in five states. Fig. 7 illustrates how the TQI values are discretized to build the state of the Markov chain model.

We first demonstrate the Markovian property by a statistical test as follows: We take a portion of the inspection data to obtain both a one-step transition matrix and a two-step transition matrix. The matrices are derived from the 3 inspection runs conducted on 01/28/2010, 04/29/2010 and 08/07/2010. The inspection intervals are 91 days and 100 days, which are similar. The number of

![Fig. 4. TQI plot for 10 segments in 1 mile over time (Milepost 272).](image)
Fig. 4. (continued)

Table 2
Comparison of the three algorithms for geo-defect prediction.

<table>
<thead>
<tr>
<th></th>
<th>Random forest</th>
<th>SVM</th>
<th>Logistic regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>75.2%</td>
<td>67.6%</td>
<td>64.8%</td>
</tr>
<tr>
<td>Precision</td>
<td>77.4%</td>
<td>70.4%</td>
<td>37.8%</td>
</tr>
</tbody>
</table>
state transitions is 234. Each observation contains a measurement for an inspection run. We use the Pearson $\chi^2$ test to determine whether the Markovian property holds in our data.

Let $V$ represent the current state and $U$ the next state, $U,V \in \{1,2,3,4,5\}$

1. We estimate the one-step transition probabilities using the following equation,

Fig. 5. The occurrence probability of geo-defects under different inspection intervals for five TQI levels; (a)-(e) represents TQI levels 1–5, respectively.
and the one-step transition probability matrix is derived as follows,

\[
P_1(x_{i+1} = U | x_i = V) = \frac{\sum_{j} \#x_j = V, x_{i+1} = U}{\sum_{j} \#x_j = V}
\]

and the one-step transition probability matrix is derived as follows,

\[
P_1(x_{i+1} = U | x_i = V) = \begin{bmatrix}
0.63 & 0.21 & 0.11 & 0.01 & 0.04 \\
0 & 0.58 & 0.29 & 0.09 & 0.04 \\
0 & 0 & 0.48 & 0.14 & 0.38 \\
0 & 0 & 0 & 0.50 & 0.50 \\
0 & 0 & 0 & 0 & 1.00
\end{bmatrix}
\]

2. We further obtain the two-step empirical probabilities,
\[
\tilde{p}_{U,V} = \frac{\sum_{i} x_{i+1} = U x_{i+2} = V}{\sum_{i} x_{i} = V}
\]

\[
\tilde{p}_{U,V} = \left( \begin{array}{ccccc}
0.14 & 0.24 & 0.34 & 0.17 & 0.11 \\
0 & 0.05 & 0.33 & 0.46 & 0.16 \\
0 & 0.03 & 0.35 & 0.62 & \\
0 & 0 & 0.10 & 0.90 & \\
0 & 0 & 0 & 1.00 & 
\end{array} \right)
\]

3. Then, we obtain the two-step model probabilities,

\[
\hat{p}_{U,V} = \text{Prob}[x_{i+2} = U|x_{i} = V] = \sum_{w \in \{1,2,3,4,5\}} \text{Prob}[x_{i+2} = U|x_{i+1} = W] \text{Prob}[x_{i+1} = W|x_{i} = V]
\]

\[
\hat{p}_{U,V} = \left( \begin{array}{ccccc}
0.22 & 0.27 & 0.37 & 0.09 & 0.05 \\
0 & 0.15 & 0.36 & 0.33 & 0.16 \\
0 & 0 & 0.20 & 0.61 & 0.19 \\
0 & 0 & 0 & 0.29 & 0.71 \\
0 & 0 & 0 & 0 & 1.00 
\end{array} \right)
\]

4. We form the Pearson test statistic for the goodness-of-fit,

\[
T = \sum_{V} T_{V} = \sum_{V} \frac{(\hat{p}_{U,V} - \tilde{p}_{U,V})^{2}}{\tilde{p}_{U,V}}
\]

\[
T = 1.762
\]

Because \(T_{V} \sim \chi^{2}_{4}\), the total \(T \sim \chi^{2}_{10}\). However, the probability of moving to a better condition state is always 0, so \(T \sim \chi^{2}_{10}\). The critical \(\chi^{2}_{10} = 3.94\) at the 95% confidence level is greater than \(T\). Therefore, \(\hat{p}_{U,V}\) is not significantly different from \(\tilde{p}_{U,V}\). Therefore, we can say that the Markovian property holds for our data.

The entire dataset is split into the training dataset (60%) and the test dataset (40%). Tables 3–5 show the transition probability matrices developed from the training dataset using Eq. (3). We can easily infer how the states degrade when no action is taken, as shown for \(A = 0\). As there are no maintenance actions, the state remains the same or moves from the current state to a worse state. When we apply minimum maintenance action, the states always move from the current state to the previous state, as shown in the matrix under \(A = 1\). Therefore, there is no need to use data to estimate the transition matrix for minor maintenance based on our assumption. When we apply major maintenance action, the states improve by at least 2 states. It is worth noting that when major maintenance is applied to the track in state 2, the consequence is the same as a minimum maintenance action.

Table 3 shows that due to frequent inspections when \(T = 0–58\) days, the state of the track does not degrade much, unless geo-defects occur. When major maintenance activity is performed, the greatest probability of transition is moving from the current state to the next possible improved state. For example, if the current state is 4, then there is a probability of 0.49 that the future state will be state 2. The Table 4 shows the transition probability for state changes when \(T = 59–85\) days. Compared to the smaller inspection interval, the probability of transitioning to a worse state increases substantially. For example, if the current state is 1, there is a probability of 0.06 of moving to state 5 in the next inspection. Table 5 depicts the transition probability for the state changes when \(T > 85\) days. The inspection is conducted after a long period of time; thus, degradation to a worse state is more likely. For example, the probability of moving from state 1 to state 5 is 0.07, and the probability of moving from state 3 to state 5 is 1. Thus, the occurrence of major maintenance work is the highest in this inspection interval. The probability of geo-defect occurrence is also the highest in this period.

Table 3

<table>
<thead>
<tr>
<th>States</th>
<th>(A = 0)</th>
<th>(A = 1)</th>
<th>(A = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>0.72</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.32</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.60</td>
<td>0.31</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>0.77</td>
<td>0.23</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>0.33</td>
</tr>
</tbody>
</table>
This paper considers three maintenance actions ($A$): 0 = no action, 1 = minor, and 2 = major. S and $S'$ denote the current state and the next state, respectively. The action is determined by the following equation,

$$A = \begin{cases} 
0 & \text{if } S' - S \geq 0 \\
1 & \text{if } S' - S = -1 \text{ and } S' \geq 1 \\
2 & \text{if } S' - S < -1 \text{ and } S' \geq 1 
\end{cases}$$

When TQI remains the same or increases over the inspection interval ($S' - S \geq 0$), then no action is taken ($A = 0$). When there is a slight improvement in TQI or the final state is just one before the original state ($S' - S = 1$), then minor maintenance work is performed ($A = 1$). When the transition causes a major drop in TQI or the final state is two or more states less than the initial state ($S' - S < -1$), then major maintenance is performed ($A = 2$).

When a reward is associated with each action, the Markov chain model acts as an MDP. In an MDP, the final states are random and depend on the decision maker.

Fig. 8 presents the diagram of the Markov decision process for $T = 0$–58 days. In this figure, the red arrows indicate the random arrivals of geo-defects. The different actions possible from each state and the probability of moving to the next state following a particular action are shown. Further, the arrival of geo-defects is modeled as a Bernoulli process with probability $p_g$ given by the random forest model developed in Section 3.2.2. When action 0 or no maintenance activity is performed, the state stays in the current state or moves to a worse one, with a higher chance of having a geo-defect, as shown by the red arrows. When inspection and the maintenance activities are performed quite frequently, the states do not degrade substantially, and it is less likely that geo-defects occur.

Fig. 8. The proposed MDP with 5 states and 3 actions for the inspection interval 0–58 days.
In Fig. 8, Action 1 represents minimum maintenance, where the state transitions to the previous operating state. For example, if the current state is 3 and minimum maintenance is conducted, the condition of the track moves to state 2. Therefore, this transition occurs with a probability of 1 for all given states. Similarly, Action 2 represents major maintenance, where the condition of the track improves by at least 2 states. Therefore, major maintenance can be performed for state 3 or worse. When major maintenance is conducted in state 2, it acts as minor maintenance and results in a transition to state 1. When major maintenance activity is performed, the state randomly moves from the current state to the next possible acceptable state. For example, the track condition moves from state 5 to state 3 with a probability of 0.35, to state 2 with a probability of 0.32 and to state 1 with a probability of 0.33 when \( T = 0 - 58 \) days. If major maintenance is applied at state 4, there is a 0.49 probability of moving to state 2 and a 0.51 probability of moving to state 1 when \( T = 0 - 58 \) days.

### 4.4. Occurrence of geo-defects

In this section, the prediction model created in Section 3.2.2 is used to calculate the probability of geo-defects. Given the TQI value from each inspection, the probability of geo-defect occurrence for an individual track segment in the next inspection run can be predicted. In an aggregated way, we can also calculate the mean probability of geo-defect occurrence for a particular state given an inspection interval. Table 6 shows the average probability of geo-defect occurrence at each state given different previous defect status. As one can see, the chance of occurrence increases as the state and inspection interval increase.

### 4.5. Costs associated with the Markov decision process

The annual cost associated with a railway track maintenance process includes the following elements:

\[
\text{Annual cost} = \begin{cases} 
\text{Inspection cost} \\
\text{Minor maintenance cost} \\
\text{Major maintenance cost} \\
\text{Geo-defects repair cost} \\
\text{Other costs, such as materials costs}
\end{cases}
\]

In our cost distribution, we assume that the total cost includes all labor costs. Note that derailment cost is not considered in the scope of this paper. However, it can be easily incorporated into our model using derailment risk modeling (He et al., 2015).

The approximate inspection cost, minor maintenance cost, and major maintenance costs are provided by (Khouy et al., 2014) and geo-defects repair cost is provided by (He et al., 2015). A further sensitivity analysis will follow to examine the sensitivity of several cost items.

- Inspection Cost – $250/mile
- Minor Maintenance Cost – $4000/mile
- Major Maintenance Cost – $10,000/mile
- Geo-defects Repair Cost – $1000/defect.

### 4.6. Analysis of the existing maintenance policy

The policy is defined as the action taken in an MDP in a certain stage. Mathematically, it is represented as \( \pi(A|S) \). The existing policy is that being used in railroad maintenance practice, which can be further derived from the TQI data. First, we count the number of each type of action taken from the current state to reach a future state for various inspection intervals. To obtain the percentage of actions taken in a state, we calculate the ratio of a particular action taken in a state to the sum of all actions taken in that state. The percentage of actions taken for a state in a time interval is shown in Fig. 9(a)–(c).

Fig. 9(a)–(c) show an inconsistent maintenance policy. If we consider the inspection interval \( T = 0 - 58 \) days and refer to Fig. 9(a), we can see that in state 4, no action is performed in 61.1% of the cases. Additionally, in state 5, minimum maintenance is performed in 18.3% of the cases, although in state 5, major maintenance should ideally be executed most of the time.

Similarly, for \( T = 59 - 85 \) days, in state 4, no action is taken 53.5% of the time, which will lead to degradation of the track condition, failure, and accidents. In state 4, 30.5% of the time, minor maintenance action is performed, whereas 16.4% of the times,
major maintenance is conducted. These data confirm the inconsistency in the current maintenance policy. More major maintenance is expected to be conducted in more degraded states. For $T > 85$ days, the inconsistency remains, as 55.7% of the time no action is taken in state 3. Due to the long inspection interval, more maintenance activities should be expected.

4.7. The optimal policy

The optimal policy is determined using the value iteration algorithm. This policy helps to obtain the minimal costs. In our case, the cost is minimized by following this policy. Mathematically, the optimal policy is represented as $\pi^*(A|S)$. In Fig. 10(a)–(c), we plot the percentage of actions taken for a particular state in a particular time interval corresponding to $\pi^*(A|S)$. The computation time for the value iteration algorithm in R is approximately 4.5 s for 40 iterations in a laptop with an i5 CPU and 4 GB memory.

The optimal policy suggests one particular action for a particular state in a particular inspection interval. This helps to maintain uniformity in the maintenance policy. For example, had we followed the existing policy, as shown in Fig. 9(a), for state 4, then no action would be performed 61.1% of the time, minimum maintenance would be performed 25.3% of the time, and major maintenance would be performed 13.6% of the time. Following the optimal policy in Fig. 10(a), when we are in state 4, only major maintenance is suggested. If we perform no action when in state 4, the track might degrade to state 5 or major failure could occur, which might lead to a high frequency of geo-defects and even train accidents. Additionally, the inconsistency increases the cost. If we choose no action in state 4, then there is no benefit to the inspection cost, and we have to pay additional inspection cost later to monitor the new condition to make a decision based on the maintenance or repair policy. The optimal policy also suggests actions to be taken for state 4 in inspection interval $T > 85$ days, which is otherwise unavailable from the data.
4.8. Validation of MCMC simulation of existing policy with actual data

The existing policy is derived from actual data. In this section, we consider 10 miles of railway track and divide them into 5 two-

Fig. 10. Actions in the optimal policy for each state at (a) T = 0–58 days (b) T = 59–85 days (c) T > 85 days.

Fig. 11. The errors in percentage between MCMC Simulation in existing policy and actual data observed for 5 segments with 1000 runs.

4.8. Validation of MCMC simulation of existing policy with actual data

The existing policy is derived from actual data. In this section, we consider 10 miles of railway track and divide them into 5 two-
mile segments. For each segment, we implement MCMC simulation with derived existing policy and compare the results to the actual total maintenance cost within the data collection period. In each simulation run, the actions are selected using a Monte Carlo method after each inspection. For each segment, we run 1000 experiments of MCMC simulation. Fig. 11 reports the percentage error (%) of total costs between simulation and actual data. As one can see, errors only range from –4% to 6% across all 5 segments. This suggests that the existing policy is very close to actual data and can be considered a good approximation of real-world practice.

### 4.9. Comparison of the cost of the optimal policy and the existing policy

Given the optimal policy derived from the training data, we first derive another set of transition matrices from the test dataset. Moreover, we perform an MCMC simulation with 1000 runs to estimate the total maintenance costs over a period of 10 years. If we consider the inspection time interval to be 58–85 days, there are 4 inspections per year or 40 inspections for the length of the simulation. Thus, we have 40 steps of transition. This simulation explores the inconsistency in the actions taken and the inappropriate actions taken by determining the cost incurred in the long term. A sub-optimal policy may cause a heavy loss for a typical Class I railroad.

We implement the optimal policy \( \pi^*(A|S) \) in the MCMC simulation to obtain the total cost associated with maintenance of 1 mile of railway track for 10 years. Table 7 compares the mean costs of the optimal and existing policies under different inspection intervals for 1 mile of track in a period of 10 years. The optimal policy results in approximate 10% cost savings across three scenarios. Further, the inspection interval with \( T = 59–85 \) day will lead to the minimal total cost for both the optimal policy and the existing policy. Such optimal inspection interval achieves the balance between inspection costs and maintenance costs. If one railroad runs 10,000 miles of track with \( T = 59–85 \) days, the optimal policy is anticipated to save more than $83 million in 10 years.

In Table 7, we show the breakdown of total costs. As one can see, inspection cost does not vary between two policies in the same time period because the count of inspection is the same in both the policies. Compared with existing policy, savings by optimal policy come from fewer minor maintenance actions and slightly more major maintenance actions. The cost of major maintenance increased in the optimal policy as number of major maintenance actions increases. This prevents performing repeated minor maintenance on the same section of the track and helps decrease repeated minor maintenance cost. The optimal policy is able to remove unnecessary minor maintenance. As a result, the number of corrective maintenance cost for geo-defect is greatly reduced as well in the optimal policy.

### 4.10. Sensitivity analysis

Sensitivity analysis is conducted to check the dependency of cost on various inputs. We analyze the sensitivity of various types of actions and determine the effects on the overall cost for both the existing policy and the optimal policy. For this purpose, we use a one-factor-at-a-time (OFAT) procedure to individually measure the effect of major maintenance costs and the probability of unexpected maintenance impact. For the unexpected maintenance effect, we consider the effect of major maintenance producing the results of minor maintenance and minor maintenance producing the result of major maintenance on the overall cost simultaneously.

The simulation of each parameter setting is run for a 10-year timeframe and repeated 1000 times. The cost of maintenance actions

### Table 7

| Time interval between inspections (days) | Mean cost estimated using \( \pi(A|S) \) (in USD) | Mean cost estimated using \( \pi^*(A|S) \) (in USD) | Savings |
|----------------------------------------|--------------------------------|--------------------------------|----------|
| T = 0–58 days (33rd percentile)        | 78,924                         | 68,916                          | 12.7%    |
| T = 59–85 days (66th percentile)       | 73,540                         | 65,230                          | 11.3%    |
| T > 85 days                            | 77,341                         | 70,200                          | 9.2%     |

The bold defines the total costs and the savings of the optimal policy compared to the existing policy.
is added to the cost of inspection and the cost of rectifying a geo-defect to generate the total cost.

4.10.1. Major maintenance cost

In this section, we change the cost of major maintenance from $10,000 per 1 mile of track to $8000–$12,000 per 1 mile of track and determine the impact of this change on the optimal value. The count of major maintenance taking places is equal to counts in existing policy and optimal policy.

The total cost comprises the inspection cost, minor maintenance cost, major maintenance cost, and geo-defects repair cost. Fig. 12(a)–(c) shows the sensitivity analysis for the three inspection intervals. In either case, the savings increase with increasing major maintenance cost. Fig. 12(c) shows the major maintenance cost has a huge effect on the savings because the number of major maintenance actions performed in the existing policy is more than the number of major maintenance actions performed in the optimal policy. In the optimal policy for $T > 85$ days, no major maintenance actions are taken in state 3, which reduces the overall costs. Additionally, most of the track is in state 3, where minor maintenance work is optimal. Therefore, one can see that the optimal policy is robust compared with the existing policy when the cost of major maintenance increases.

4.10.2. The probability of unexpected maintenance effect

The observed maintenance results may not reflect what is performed in the field. For example, based on our assumptions, for a certain transition, minor maintenance is observed, while in fact major maintenance is actually performed to result in that transition.
To address this concern, we perform another sensitivity analysis where each maintenance type applied may produce an unexpected maintenance impact with probability $p$. This result in probability $p$ that major maintenance only reduces the TQI state by 1 and that minor maintenance reduces TQI state by more than 1. We vary $p$ from 10% to 50%. The cost of the policy depends on the number of times a particular maintenance action is taken, regardless of the effect. This means that when a major maintenance action is performed, there is a chance that the effect on TQI does not change greatly, i.e., it moves to one better state than the current state. Therefore, depending on the new state, follow-up maintenance may be needed to improve the TQI index, which incurs additional cost. However, when minor maintenance produces the same effect as major maintenance, no additional cost is incurred. This helps to reduce the cost. These changes are implemented in both the existing policy and the optimal policy.

Fig. 13(a)–(c) shows the distribution of actions taken in each time period. For example, in Fig. 13(c), for $T > 85$ days, 66% of the time no action is taken, and 22% of the time, major maintenance is performed under the existing policy. With a higher percentage of major maintenance occurring in the existing policy, there is a greater chance that a major maintenance action will have the effect of a minor maintenance action, which will lead to performing additional major maintenance or minor maintenance, depending on the current state, thus incurring additional cost. However, under the optimal policy, 61% of the time there is no action, 17% of the time minor maintenance is performed and 22% of the time major maintenance is conducted. This policy reduces the risk of a major maintenance activity having the effect of minor maintenance and increases the chances of a minor maintenance activity having the effect of major maintenance. If minor maintenance has the effect of major maintenance, no action may be required during the next inspection run, thereby saving maintenance cost. Thus, under the optimal policy, the savings increase rapidly as the probability of the
maintenance effect increases, as shown in Fig. 14(c). Similar situations occur for the other two inspection intervals, as shown in Fig. 14(a) and (b).

In the current section of sensitivity analysis, we take into account the effect of major maintenance performed instead of minor maintenance and this effect is recognized only during the inspection. However, the distribution of the action depends only on the current state and original policy.

4.10.3. Geo-defect corrective maintenance cost

In this section, we vary the cost of geo-defect maintenance from $0 to $3000 per 1 mile-segment of track and determine the impact of this change on the optimal value. During simulation, the optimal policy is updated under each geo-defect cost in each run.

The total cost comprises the inspection cost, minor maintenance cost, major maintenance cost, and geo-defects repair cost. Fig. 15(a), 15(b) and 15(c) show the sensitivity analysis for the three inspection intervals. In either case, the savings increase with increasing geo-defect maintenance cost. Fig. 15(c) shows the geo-defect maintenance cost has a huge effect on the savings because the number of geo-defect maintenance actions performed in the existing policy is much more than the one in the optimal policy.

5. Conclusions and future research

This research aims to develop a condition-based maintenance policy for the geometry of railway tracks. This paper utilizes 33 months of foot-by-foot inspection data from 50 miles of track of a Class I railroad. Given the geometry measurements of the track, we calculate the TQI for each 0.1-mile segment and develop a random forest model to predict the occurrence of geo-defects. It is found that a lower TQI may still generate a higher probability of geo-defect occurrence than a higher TQI, which is reasonable because a geo-defect is a type of spot defect that can be caused by sudden impact from vehicle-track interactions. Therefore,
Incorporating the Bernoulli model for the occurrence of geo-defect, this study further develops a discrete-time Markov model based on the observed inspection data. Based on the TQI, we consider three maintenance actions (i.e., major maintenance, minor maintenance, and no maintenance) that are conducted under the particular states. After building the Markov decision process (MDP) model, we apply Markov chain Monte Carlo (MCMC) simulation to compare the performance between the optimal policy and the existing policy. The simulation determines the average cost of maintaining 1 mile of track for a period of 10 years.

A value iteration algorithm is applied to obtain the optimal policy using MDP. This optimal policy acts as an input for the MCMC simulation, and we estimate the optimal cost accordingly. The overall cost savings are 12.7% for the inspection interval $T = 0-58$ days, 11.3% for $T = 59-85$ days and 9.2% for $T > 85$ days for 1 mile of track. If one railroad runs 10,000 miles of track with $T = 59-85$ days, the optimal policy is anticipated to save more than $83$ million in 10 years.

Future work could include the following:

- Consider more detailed costs, such as derailment costs and track downtime costs for the policy of preventive maintenance and inspection. Derailment costs can be modeled and estimated using data of historical train derailments caused by geo-defects (He et al., 2015). The downtime cost is a major component of the inspection and maintenance run. If the tracks are not maintained from time to time or a geo-defect occurs, there might be a failure. This failure could cause a derailment, which could impact the overall cost scenario for the railway industry.
- Schedule preventive maintenance activities in a railway network. The cost analysis conducted in this paper can be used to develop a maintenance schedule to decrease the overall cost, including logistics costs and crew labor costs.

Fig. 15. Sensitivity analysis of geo-defect corrective maintenance cost for (a) $T < 58$ days (b) $T = 59-85$ days (c) $T > 85$ days.
The MDP developed in this paper can be extended to a semi-MDP, where the sojourn time in each state is a general continuous random variable.

- Extend the track maintenance to other rail infrastructure, including ballasts, sleepers, turnouts, etc.

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References


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