A beam is loaded with a system of forces. Express the resultant force system in Cartesian vector form.

Solution

The system consists of three couples: \( \Sigma M = 0 \).

For \( A \) & \( D \):

\[ M_1 = F_A D \quad \text{(where } D = \text{distance between pin and action)} \]

\[ = (500)(12) = -6000 \text{ in}\cdot\text{lb} \quad (0^\circ, 0^\circ) \]

For \( B \) & \( C \):

\[ F_{Bx} = F_b \cos 60 = 750 \cos 60 = 375 \text{ lb} \]

\[ F_{By} = F_b \sin 60 = 750 \sin 60 = 649.5 \text{ lb} \]

\[ M_2 = F_{Bx} d_2 = (375)(10) = -3750 \text{ in}\cdot\text{lb} \quad (0^\circ, 0^\circ) \]

\[ M_3 = F_{By} d_3 = (649.5)(12) = 7794 \text{ in}\cdot\text{lb} = 7794 \text{ in}\cdot\text{lb} \]

For force \( E \), \( 2F \):

\[ M_4 = F_E d_4 = (800)(12) = 9600 \text{ in}\cdot\text{lb} = -9600 \text{ in}\cdot\text{lb} \]
For any no. of couples in a plane, the resultant couple \( C \) is equal to algebraic sum of individual couples.

\[
C = \Sigma M = -6000 + 7794 - 3750 - 9600
\]

\[
= -11556 \text{ in lb}
\]

\[
= -963 \text{ ft lb}
\]

\[
\overline{C} = -963 \text{ ft lb}
\]

**Equivalent Systems**

A system of forces and moments is simply a particular set of forces and moments of couples.

**Conditions of Equivalence**

Two systems of forces and moments, designated as System 1 \& System 2, to be equivalent if sum of the forces are equal,

\[
(\Sigma F_1) = (\Sigma F_2)
\]

and sum of the moments about a point are equal

\[
(\Sigma M_p)_1 = (\Sigma M_p)_2
\]
First condition for equivalence is:
\[(\Sigma F)_1 = (\Sigma F)_2 .\]
\[F_A + F_B = F_D .\]

Second condition for equivalence is:
\[(\Sigma M_p)_1 = (\Sigma M_p)_2 .\]
\[(\vec{g}_A \times F_A) + (\vec{g}_B \times F_B) + M_C = (\vec{g}_D \times F_D) + M_e + M_f .\]
Three systems of forces and moments act on the beam. Are they equivalent?

(i) Check sum of the forces:

\[(\Sigma F)_1 = 50 \hat{j} \text{ N}\]
\[(\Sigma F)_2 = 50 \hat{j} \text{ N}\]
\[(\Sigma F)_3 = 50 \hat{j} \text{ N}\]

(ii) Sum of the moments about a point:

\[(\Sigma M)_1 = 0\]
\[(\Sigma M)_2 = (50 \text{ N})(0.5 \text{ m}) - (50 \text{ Nm}) = -25 \text{ N} \cdot \text{m}\]
\[(\sum M_0)_3 = (50\text{N})(1\text{m}) - (50\text{N}\text{m}) = 0\]

\[(\sum M_0)_1 = (\sum M_0)_3\]

:. System 1 & 3 are equivalent

2. A 300lb force \( \vec{F}_A \) is applied to a bracket at point A. Replace the force \( \vec{F}_A \) by a force \( \vec{F}_0 \) and a couple \( \vec{C} \) at point O.
Solution

* $\vec{F}_D$ has the same magnitude and sense as $\vec{F}_A$.
* Couple $\vec{C}$ has the magnitude and sense as moment of force $\vec{F}_A$ about point O.

$F_D = F_A = 300 \text{ lb}$.

$\vec{F}_D = \vec{F}_A = F_A \vec{e}_A$

$\vec{e}_A = (-\cos 25^\circ \hat{i} + \sin 25^\circ \hat{j})$.

$\vec{F}_D = \vec{F}_A = (300) \left(-\cos 25^\circ \hat{i} + \sin 25^\circ \hat{j}\right)$

$= -27.2 \hat{i} + 126.8 \hat{j} \text{ lb}$.

$\vec{C} = \mathbf{91}_{DA} \times (-271.89 \hat{i} + 126.8 \hat{j})$

$= 10690 \hat{k} \text{ in-lb}$
Four forces are applied to a rectangular plate. Determine the magnitude and direction of the resultant of the four forces. Compute the moment about \( B \).

Solution

\[
R_x = \sum F_x = -220 + \frac{12}{13} (130) + \left(\frac{4}{3}\right) (125)
\]

\[
= -220 + 220 = 0
\]

\[
R_y = \sum F_y = 25 + \left(\frac{5}{13}\right) (130) - \left(\frac{3}{5}\right) (125)
\]

\[
= 0
\]

\[
M_B = -F_y d_A + F_x d_C
\]

\[
= - (25)(3) + \left(\frac{4}{3}\right) (125)(2) = 125 \text{ ft lb}
\]
Objects in Equilibrium

- Applying Chap 3 & Chap 4 to analyze an equilibrium problem.
- We will look at objects that is acted upon by a system of forces, and moments is in equilibrium if the following conditions are satisfied.

1. Sum of forces is zero
   \[ \Sigma F = 0 \]

2. Sum of the moments about any point is zero
   \[ \Sigma M_{\text{any point}} = 0 \]

Two-Dimensional Application

Reactions: Forces & couples exerted on an object by its support is called "Reactions".

Common Kinds of Support:

* Pin Support:
  - You can rotate the bar about axis of the pin.
* Roller Support

* Built-In Support
The beam above has pin and roller supports and is subjected to a 2 kN force. What are the reactions at the supports?

Solution

\[ A_x = 0.69 \text{ kN} \quad B = 1.39 \text{ kN} \]

\[ A_y = 0.8 \text{ kN} \]

Apply equilibrium condition:

\[ \Sigma F_x = A_x - B \sin 30^\circ = 0 \]

\[ \Sigma F_y = A_y + B \cos 30^\circ - 2 = 0 \]

\[ \Sigma M_A = (5)(B \cos 30^\circ) - (3)(2) = 0 \]
The above object has a built-in support and is subjected to two forces and a couple. What are the reactions at the support?

**Solution**
Apply conditions of equilibrium

\[ \Sigma F_x = (A_x) + 100 \cos 30^\circ = 0 \]
\[ \Sigma F_y = (A_y) - 200 - 100 \sin 30^\circ = 0 \]
\[ \Sigma M_A = (M_A) + 300 - (200)(2) - (100 \cos 30^\circ)(2) + (100 \sin 30^\circ)(4) = 0 \]

Reactions at Support are

\[ A_x = -86.6 \text{ lb} \]
\[ A_y = 150.0 \text{ lb} \]
\[ M_A = 73.2 \text{ ft-lb} \]