Ex 1.1

CHAPTER 1

Convert 2 km/h to m/s. How many ft/s is this?

1 km = 1000 m
1 h = 60 minutes = 60 \times 60 \text{ seconds}

\[
\frac{2 \text{ km}}{\text{h}} = \frac{2 \times (1000 \text{ m})}{60 \times 60 \text{ seconds}} = 0.556 \text{ m/s}
\]

1 ft = 0.3048 m \Rightarrow 1 \text{ m} = \frac{1}{0.3048} \text{ ft}

\[
0.556 \text{ m/s} = 0.556 \left( \frac{1}{0.3048} \text{ ft/s} \right)/\text{s}.
\]

= 1.82 \text{ ft/s}

Ex 1.2

Convert the quantities 300 lb·s and 52 slugs/ft³ to appropriate S.I. units.

Solution

1 lb = 4.4482 N

300 lb·s = 300 \times (4.4482 \text{ N}) \cdot \text{s}

= 1334.5 \text{ N·s} = 1.33 \text{ kN·s}
52 slug/ft$^3$

$1 \text{ slug} = 14.5938 \text{ kg}$

$1 \text{ ft} = 0.3048 \text{ m}$

$$\frac{52 \times (14.5938 \text{ kg})}{(0.3048 \text{ m})^3} = \frac{26.8 \times 10^3 \text{ kg/m}^3}{\text{S.I.}}$$
VECTORS

Vector is a quantity having direction as well as magnitude.

Scalar: a physical quantity that is completely described by a real number (e.g., Time, Mass)

Vector representation: by bold face or an arrow on top (e.g., $\vec{u}$, $\vec{v}$, $\vec{w}$)

VECTOR OPERATIONS

1) VECTOR ADDITION

\[ \vec{u} + \vec{v} = \vec{w} \]

2) TRIANGLE LAW OF VECTOR ADDITION

Consider two vectors $\vec{u}$, $\vec{v}$. If we place them head to tail, their sum is defined to be vector from tail of $\vec{u}$ to head of $\vec{v}$. 
b) Parallelogram Rule

All vector quantities obey the parallelogram law of addition.

\[ \vec{a} + \vec{b} = \vec{c} \]  

\[ \text{(demonstrates parallelogram rule).} \]

2. Product of Scalar and a Vector

The product of scalar \( a \) and vector \( \vec{u} \)

\[ a \vec{u} \]

\[ \frac{1}{2} \vec{u} \]
3) Vector Subtraction
\[ \overrightarrow{u} - \overrightarrow{v} = \overrightarrow{u} + (-1)(\overrightarrow{v}) \]

\[ (-\overrightarrow{v}) \]
\[ \overrightarrow{u} - \overrightarrow{v} \]
\[ \overrightarrow{u} \]

*) Vector addition is commutative.
\[ \overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u} \]

*) Vector addition is associative.
\[ (\overrightarrow{u} + \overrightarrow{v}) + \overrightarrow{w} = \overrightarrow{u} + (\overrightarrow{v} + \overrightarrow{w}) \]

*) Product is associative with respect to scalar multiplication
\[ a (b \overrightarrow{u}) = (ab) (\overrightarrow{u}) \]

*) Product is distributive with respect to scalar addition
\[ (a+b)\overrightarrow{u} = a\overrightarrow{u} + b\overrightarrow{u} \]

*) Product is distributive with respect to vector addition
\[ a(\overrightarrow{u} + \overrightarrow{v}) = a\overrightarrow{u} + b\overrightarrow{v} \]
Unit Vectors

A unit vector is simply a vector whose magnitude is 1. If a vector \( \vec{v} \), and \( \frac{\vec{v}}{|\vec{v}|} \) has same direction, then,

\[
\hat{v} = \frac{\vec{v}}{|\vec{v}|}
\]

\[
\hat{v} = \frac{\vec{v}}{101}
\]

EXAMPLES

(1)

A structure supported by cables AB & AC. \( |F_{AB}| = 100 \text{ kN} \) \( |F_{AC}| = 60 \text{ kN} \). Determine the magnitude of sum of forces exerted on the structure.
\[ \alpha + 30 = 180 \implies \alpha = 150^\circ \]

Using the Law of Cosines:

\[
(F_{AC} + F_{AB})^2 = |F_{AB}|^2 + |F_{AC}|^2 - 2 |F_{AB}| |F_{AC}| \cos \alpha
\]

\[
= (100)^2 + (60)^2 - 2(100)(60)\cos 150^\circ
\]

\[ = 155 \text{ kN} \]

To find direction
\[ \frac{|\text{F}_{\text{AB}} + \text{F}_{\text{AC}}|}{\sin \alpha} = \frac{|\text{F}_{\text{AB}}|}{\sin \beta} \]

\[ \sin \beta = \frac{|\text{F}_{\text{AB}}| \sin \alpha}{|\text{F}_{\text{AB}} + \text{F}_{\text{AC}}|} = \frac{100 \sin(150^\circ)}{155} = 0.8 \]

\[ \beta = \sin^{-1} \left( \frac{100 \sin(150^\circ)}{155} \right) = 18.2^\circ \]
Cartesian Components

Components in Two dimensions

\[ \mathbf{u} = \mathbf{u}_x + \mathbf{u}_y \quad \text{(Triangle Law)} \]

Let \( \mathbf{i} \) be unit vector in \( x \) direction and \( \mathbf{j} \) be unit vector in \( y \) direction.

\[ \mathbf{u} = u_x \mathbf{i} + u_y \mathbf{j} \]

\( u_y, u_x \) = scalar components of \( \mathbf{u} \).

From Pythagorean theorem, its magnitude is

\[ |\mathbf{u}| = \sqrt{u_x^2 + u_y^2} \]

The component can be written as

\[ u_x = u \cos \theta \]
\[ u_y = u \sin \theta \]

These form a right triangle.

\[ \tan \theta = \frac{u_y}{u_x} \Rightarrow \theta = \tan^{-1} \left( \frac{u_y}{u_x} \right) \]
Force acting on sailplane in the figure are its weight \( \vec{W} = -600 \hat{j} \) (b) the drag \( \vec{D} = -200 \hat{i} + 100 \hat{j} \) and the lift \( \vec{L} \).

a) If sum of the forces on the sailplane is zero, what are the components of \( \vec{L} \)?

b) If the lift \( \vec{L} \) has components determined in (a) drag increases by factor of 2, what is the magnitude of the sum of the forces on the sailplane?

**Solution**

\[ \vec{F} = \vec{W} + \vec{D} + \vec{L} = 0 \]

\[ -600 \hat{j} + (-200 \hat{i} + 100 \hat{j}) + \vec{L} = 0 \]

\[ \vec{L} = 200 \hat{i} + 500 \hat{j} \]

b) By drag is twice

\[ \vec{F}_{\text{sum}} = \vec{W} + 2\vec{D} + \vec{L} \]

\[ = -600 \hat{j} + 2(-200 \hat{i} + 100 \hat{j}) + 200 \hat{i} + 500 \hat{j} \]
\[ \vec{F} = -200 \hat{i} + 100 \hat{j} \]

\[ |\vec{F}| = |\vec{F}_1 + \vec{F}_2 + \vec{F}_3| = \sqrt{(200)^2 + (100)^2} = 224.1 \text{ lb} \]

3. Compute \( x \) and \( y \) components of \( \vec{F}_1 \) and \( \vec{F}_2 \)

\[ F_{1x} = F_1 \cos 30^\circ = -F_1 \sin 30^\circ \]

\[ F_{1x} = -200 \left( \frac{1}{2} \right) = -100 N \]

\[ F_{1y} = F_1 \sin 60^\circ \]

\[ F_{1y} = +200 \left( \frac{\sqrt{3}}{2} \right) = 173 N \]

For \( \vec{F}_2 \), using proportional parts of similar triangle

\[ \frac{F_{2x}}{F_2} = \frac{12}{13} \quad \frac{F_{2y}}{F_2} = \frac{5}{13} \]

\[ F_{2x} = 260 \left( \frac{12}{13} \right) = 240 N \]

\[ F_{2y} = 260 \left( \frac{5}{13} \right) = 100 N \]
The link is subjected to $F_1$ and $F_2$. Determine the magnitude and direction of resultant force.

Solution:

$F_x = F_1 \cos \alpha - F_2 \sin \beta$

$= 600 \cos 30 - 400 \sin 45$

$= 268.8 \text{ N (→)}$

$= 236.8 \text{ N}$
\[ F_y = F_1 \sin \alpha + F_2 \cos \beta \]
\[ = 600 \sin 30 + 400 \cos 45 \]
\[ = 582.8 \text{ N (up)} \]

Resultant Force:
\[ R = \sqrt{(F_x)^2 + (F_y)^2} \]
\[ = \sqrt{(263.8)^2 + (582.8)^2} \]
\[ = 629 \text{ N} \]

Direction of resultant force:
\[ \tan \theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1} \left( \frac{582.8}{263.8} \right) = 67.9^\circ \]

Components in three dimensions:
\[ \vec{u} = \vec{u}_x + \vec{u}_y + \vec{u}_z \]
\[ = u_x \hat{i} + u_y \hat{j} + u_z \hat{k} \]
\[ |\vec{u}| = \sqrt{u_x^2 + u_y^2 + u_z^2} \]

To determine direction, use direction cosines. (Please see the book for the 3D figure)

These are also known as coordinate direction angles.
Express the force $F$ as a Cartesian vector.

Solution

$F' = F_x + F_y$

From parallelogram law: $F = F' + F_z$
Magnitude of components

\[ F_z = F \sin 60^\circ = 100 \sin 60^\circ = 86.6 \text{ lb} \]

\[ F' = F \cos 60^\circ = 100 \cos 60^\circ = 50 \text{ lb} \]

\[ F_x = F' \cos 45^\circ = 50 \cos 45^\circ = 35.4 \text{ lb} \]

\[ F_y = F' \sin 45^\circ = 50 \sin 45^\circ = 35.4 \text{ lb} \]

\[ \vec{F} = (35.4 \hat{i} - 35.4 \hat{j} + 86.6 \hat{k}) \text{ lb} \]

\[ F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(35.4)^2 + (35.4)^2 + (86.6)^2} = 100 \text{ lb} \]

The coordinate angles of \( \vec{F} \) can be found from the components of unit vector acting in the direction of \( \vec{F} \).

\[ \vec{u} = \frac{\vec{F}}{F} = \frac{F_x}{F} \hat{i} + \frac{F_y}{F} \hat{j} + \frac{F_z}{F} \hat{k} \]

\[ = \frac{35.4}{100} \hat{i} - \frac{35.4}{100} \hat{j} + \frac{86.6}{100} \hat{k} \]

\[ = 0.354 \hat{i} - 0.354 \hat{j} + 0.866 \hat{k} \]
\[
\begin{align*}
\cos \alpha &= 0.354 \\
\cos \beta &= -0.354 \\
\cos \gamma &= 0.866 \\
\alpha &= \cos^{-1}(0.354) = 69.3^\circ \\
\beta &= \cos^{-1}(-0.354) = 111^\circ \\
\gamma &= \cos^{-1}(0.866) = 30^\circ
\end{align*}
\]