STATICALLY INDETERMINATE OBJECT

To solve for unknown forces:
\[ \Sigma F = 0 \]
\[ \Sigma M = 0 \]

Is it possible to have more number of unknown forces or couples than number of independent equilibrium equations?

For example - you can write only 3 equations for a two dimensional problem. What if there are more than 3 unknowns?

This can happen in 2 situations:

(i) When an object has more support than minimum number necessary to maintain it in equilibrium. Such an object is said to have REDUNDANT SUPPORT [usually for safety reasons].

(ii) When support is improperly designed such that they cannot maintain equilibrium under the loads acting on it. The object is said to have IMPROPER SUPPORT.

In either situation - the object is said to be STATICALLY INDETERMINATE.
\[ \Sigma F_x = A_x = 0 \]

\[ \Sigma F_y = A_y - F = 0 \]

\[ \Sigma M_A = M_A - \left( \frac{L}{2} \right) F = 0 \]

\text{Known: } F \quad \text{unknown: } A_x, A_y, M_A
\[ \sum F_x = A_x = 0 \]
\[ \sum F_y = A_y - F + B = 0 \]
\[ \sum M_A = M_A - \left( \frac{1}{2} \right) F + LB = 0 \]

If \( F \) is known, \( \text{unknowns: } A_x, A_y, B, M_A \).

3 equations but 4 unknowns.
ii) Is the beam statically indeterminate?

iii) Determine as many reactions as possible

Solution

\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]
\[ \sum M_A = 0 \]

Apply Equilibrium Equations:

\[ A_x + B_x = 0 \]
\[ A_y + B_y - 2 = 0 \]

Unknown:

\[ A_x, B_x, A_y, B_y \]

Plug in (iii)\( \Rightarrow \)

\[ |Ay| = 0.8 \text{ kN} \]

\[ |By| = 1.2 \text{ kN} \]
IMPROPER SUPPORTS

An object has improper supports if it will not remain in equilibrium under the action of loads exerted on it.

In 2D problems, it occurs in two ways:
(i) Support can exert only parallel forces

\[ F \]

No horizontal force to balance.
2. The supports can exert only concurrent forces.

If the load exert a moment about the point where line of action of the support forces intersect - the object is not in equilibrium.

A & B exert no moment about point P, but the load F does.
2) Determine if the bar is properly supported or improperly supported.

\[ \text{Line of action due to roller support intersect at } P. \]
\[ \Rightarrow F \text{ exerts moment about } P. \text{ Thus this bar is improperly supported.} \]
Three Dimensional Applications

\[ \Sigma F = 0 \]
\[ \Sigma M = 0 \]

3 equations

The three components of sum of forces = 0

The three components of sum of moments about any point = 0

Supports in 3D:

(i) Ball & Socket Support

The socket permits the ball to rotate freely but prevents it from translating in any direction.
The **Roller Support**

A roller support can exert only a force normal to the supporting surface.
If you imagine a bar attached, you can rotate the bar about hinge axis (the Z axis).
- The bar cannot be translated in any direction.
- Hinge results rotation about x & y axis. Thus can exert couples about those axis to result motion.
Z axis is aligned with the axis of the shaft.

Built-in support.
The bar AC is 4ft long and is supported by a hinge at A and cable BD. The hinge axis is along the Z axis. The centerline of bar lies in the XY plane, and cable attachment point B is the mid point of the bar. Determine the tension in the cable and reactions exerted on the bar by the hinge.
Solution

Forces: Ax, Ay, Tx, Ty

\[ \overline{T} = \overline{T_{EBD}} \]

\[ \overline{e_{BD}} = \frac{\overline{T_{EBD}}}{|\overline{T_{EBD}}|} \]

\[ \overline{\lambda_{RD}} = \overline{\lambda_{BD}} - \overline{B} = (2 - 2(\cos 30))^i + [2 - (-2\sin 30)]^j + (-1-0)^k \]

\[ = 0.268^i + 3^j - k \]

\[ \overline{e_{BD}} = \frac{\overline{T_{EBD}}}{|\overline{T_{EBD}}|} = 0.084^i + 0.945^j - 0.315^k \]

\[ \overline{e_{BD}} = \overline{T_{EBD}} = T(0.084^i + 0.945^j - 0.315^k) \]
\[ \Sigma F_x = 0 \Rightarrow Ax + 0.084T = 0 \]
\[ \Sigma F_y = 0 \Rightarrow Ay + 0.945T - 100 = 0 \]
\[ \Sigma F_z = 0 \Rightarrow Az - 0.315T = 0 \]

\[ \Sigma M_A = M_{Ax} \hat{i} + M_{Ay} \hat{j} + \left[ \overline{\text{MA}} \times (T \overline{\text{BD}}) \right] + \left[ \overline{\text{MA}} \times (-100 \hat{j}) \right] = 0 \]

\[ \overline{\text{MA}} = 2 \cos 30° \hat{i} - 2 \sin 30° \hat{j} \]
\[ \overline{\text{MA}} = 4 \cos 30° \hat{i} - 4 \sin 30° \hat{j} \]
\[ T \overline{\text{BD}} = T (0.084 \hat{i} + 0.945 \hat{j} - 0.315 \hat{k}) \]

\[ \Sigma M_A = M_{Ax} \hat{i} + M_{Ay} \hat{j} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.732 & -1 & 0 \\ 0.084T & 0.945T & -0.315T \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3.464 & -2 & 0 \\ 0 & -100 & 0 \end{vmatrix} \]
\[
\Rightarrow (M_{Ax} + 0.315T) \hat{i} + (M_{Ay} + 0.546T) \hat{j}
\]
\[
+ (1.72T - 346) \hat{k} = 0
\]

\[
\Sigma M_x = 0 \Rightarrow M_{Ax} + 0.315T = 0
\]
\[
\Sigma M_y = 0 \Rightarrow M_{Ay} + 0.546T = 0
\]
\[
\Sigma M_z = 0 \Rightarrow 1.72T - 346 = 0
\]

**All equations:**

**Forces:**

\[
F_x = A_x + 0.084T = 0
\]
\[
F_y = A_y + 0.945T - 100 = 0
\]
\[
F_z = A_z - 0.315T
\]

**Unknowns:** \( A_x, A_y, A_z, T, \theta \)

\( M_{Ax}, M_{Ay} \)

**Equations (6):**

Solve to obtain:

\( T = 201 \text{ lb} \)

\( M_{Ax} = -63.4 \text{ ft-lb} \)

\( M_{Ay} = -109.8 \text{ ft-lb} \)

\( A_x = -17 \text{ lb} \)

\( A_y = -90.2 \text{ lb} \)

\( A_z = 63.4 \text{ lb} \)