Production of Quark-Antiquark Pairs

Figure 46.6: World data on the total cross section of $e^+e^- \rightarrow \text{hadrons}$ and the ratio $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}, s)/\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)$. $\sigma(e^+e^- \rightarrow \text{hadrons}, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \rightarrow \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see “Quantum Chromodynamics” section of this Review, Eq. (9.7) or, for more details, K. G. Chetyrkin et al., Nucl. Phys. B586, 56 (2000) [Erratum *ibid.* B634, 413 (2002)]. Breit-Wigner parameterizations of $J/\psi$, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the $R$ ratio extraction from them can be found in [arXiv:hep-ph/0312114]. Corresponding computer-readable data files are available at http://pdg.lbl.gov/current/xsect/. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010.)
Particle Data Group: cross section plots \text{Cross Section Plots}.

International Linear Collider \text{ILC Parameters Document} “The maximum centre-of-mass energy should be 500 GeV, allowing for operation at any energy in the range between 200 GeV and 500 GeV.”

\section*{Theoretical Calculation}

Total cross section $e^+e^- \rightarrow \text{hadrons}$

$$\sigma = \frac{4\pi\alpha}{3E_{cm}^2} \left( \sum_f Q_f^2 \right) \left[ 1 + \frac{\alpha_s}{\pi} + K \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right]$$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$= 3 \left( \sum_f Q_f^2 \right) \left[ 1 + \frac{\alpha_s}{\pi} + K \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right]$$


$$K_{\overline{\text{MS}}} = \frac{365}{24} - \frac{11}{12}n_f - 2\beta_0\zeta(3) \simeq 1.986 - 0.115n_f$$
FIG. 1. Graphs contributing to $R$ through order $g^4$

![Diagram](image)

**FIG. 4.** The “Mercedes-Benz” diagram C1 showing a “planar” choice of momentum routings. The external momentum $q$ is held fixed. It is easily seen that each propagator involves at most two momenta.


**Figure 1:** Examples of non-singlet diagrams (a), (b), where the two $Z$ vertices are connected by a fermion line, and of singlet diagrams (c),(d), where the diagram can be split by only cutting gluon lines. The imaginary part of the non-singlet diagrams gives $R^{V/A,NS}$, while the imaginary part of the singlet diagrams is denoted by $R^{V/A,S}$. 

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Helicity Amplitudes for Electron-Positron Annihilation


On the General Theory of Collisions for Particles with Spin*

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The general analysis of binary reactions involving particles with arbitrary spin is reformulated in such a way, that it applies equally well to relativistic particles (including photons). This is achieved by using longitudinal spin components ("helicity states") not only in the initial and final states, but also in the angular momentum states which are employed as usual to reduce the $S$-matrix to a simpler form. Expressions for the scattering and reaction-amplitude, intensity and polarization are given. They involve fewer vector-addition coefficients than the customary formulas, and no recoupling coefficients. The application to some examples is sketched, and in the Appendix some formulas are given that may be of use in the applications.
**Helicity Structure of Electron-Positron Annihilation**

The annihilation amplitude is

$$i\mathcal{M} \left( e^-(p)e^+(p') \rightarrow \mu^-(k)\mu^+(k') \right) = \frac{ie^2}{q^2} \left[ \bar{u}(p')\gamma^\mu u(p) \right] \left[ \bar{u}(k)\gamma_\mu v(k') \right].$$

The unpolarized spin averaged and summed squared amplitude is

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4(q^2)^2} \text{Tr} \left[ \gamma^\mu (p + m_e) \gamma^\nu (p' - m_e) \right] \text{Tr} \left[ \gamma_\mu (k' - m_\mu) \gamma_\nu (k + m_\mu) \right].$$

At sufficiently high energies the electron and muon masses can be neglected and the massless annihilation squared amplitude simplifies to

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{4(q^2)^2} \text{Tr} \left[ \gamma^\mu p \gamma^\nu p' \right] \text{Tr} \left[ \gamma_\mu k' \gamma_\nu k \right].$$

**Right-handed:**

![Right-handed diagram]

**Left-handed:**

![Left-handed diagram]

Helicity = Chirality for massless particles
The massless QED Lagrangian density

$$\mathcal{L}_{\text{QED}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{Maxwell}} = \overline{\psi}(x)i\gamma^\mu \partial_\mu \psi(x) - e\overline{\psi}(x)\gamma^\mu \psi(x)A_\mu(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$ 

is invariant under global chiral transformation of the electron field

$$\psi(x) \rightarrow e^{i\theta \gamma^5} \psi(x), \quad \theta = \text{constant}$$

and the photon-electron interaction conserves chirality.

---

**Figure 5.4.** Conservation of angular momentum requires that if the z-component of angular momentum is measured, it must have the same value as initially.
**Helicity Amplitudes using Trace Technology**

Define the left-handed and right-handed projection matrices

\[
\gamma^L \equiv \frac{1 - \gamma^5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \gamma^R \equiv \frac{1 + \gamma^5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad 1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

Consider the polarized channel process \( e^- e^+ \rightarrow \mu^- \mu^+ \). Trace technology gives

\[
\sum_{\text{spins}} \bar{u}(p') \gamma^\mu \gamma^R u(p) \bar{u}(p) \gamma^\nu \gamma^R v(p') = \text{Tr} \left[ p'^\mu p^\nu - g^{\mu\nu} p' \cdot p - i\epsilon^{\alpha\mu\beta\nu} p'_\alpha p_\beta \right] = 2 \left( p'^\mu p^\nu + p^\nu p'^\mu - g^{\mu\nu} p' \cdot p - i\epsilon^{\alpha\mu\beta\nu} p'_\alpha p_\beta \right).
\]

The corresponding muon contribution is

\[
\sum_{\text{spins}} \bar{u}(k) \gamma^\mu \gamma^R v(k') \bar{v}(k') \gamma^\nu \gamma^R u(k) = 2 \left( k_\mu k'_\nu + k_\nu k'_\mu - g^{\mu\nu} k' \cdot k' - i\epsilon_{\rho\mu\sigma\nu} k^\rho k'^\sigma \right).
\]

and the product of electron and muon contributions gives

\[
|\mathcal{M}|^2 = e^4 (1 + \cos \theta)^2.
\]
The four non-vanishing polarized cross sections are

\[
\frac{d\sigma}{d\Omega} \left( e_R^+ e_L^- \rightarrow \mu_R^- \mu_L^+ \right) = \frac{\alpha^2}{4E_{\text{cm}}^2} (1 + \cos\theta)^2 ,
\]

\[
\frac{d\sigma}{d\Omega} \left( e_R^+ e_L^- \rightarrow \mu_L^- \mu_R^+ \right) = \frac{\alpha^2}{4E_{\text{cm}}^2} (1 - \cos\theta)^2 ,
\]

\[
\frac{d\sigma}{d\Omega} \left( e_L^+ e_R^- \rightarrow \mu_R^- \mu_L^+ \right) = \frac{\alpha^2}{4E_{\text{cm}}^2} (1 - \cos\theta)^2 ,
\]

\[
\frac{d\sigma}{d\Omega} \left( e_L^+ e_R^- \rightarrow \mu_L^- \mu_R^+ \right) = \frac{\alpha^2}{4E_{\text{cm}}^2} (1 + \cos\theta)^2 .
\]