Search Equilibrium with Unobservable Investment

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Abstract

This article studies a wage posting game in which workers invest in human capital so as to reduce the disutility of labor. If investment were observable, then no workers would invest in skills due to the holdup problem, and all employers would offer the monopsony wage as in the Diamond paradox. With unobservable investment, however, the equilibrium wage and skill distributions are nondegenerate, despite agents being ex ante identical. An asymptotic efficiency result is obtained in which investment converges to the efficient level as the arrival rate of job offers tends to infinity, with firms receiving all the surplus in the market. The model generates distinctive comparative statics predictions, and the theory can rationalize any wage distribution that satisfies a few regularity conditions. The analysis is robust to the inclusion of some forms of direct search costs.

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1 Introduction

A classic problem in search theory is the Diamond paradox, in which search frictions, even if infinitesimally small, may induce all employers to offer the monopsony wage. If workers are homogenous, then employers can hire any unemployed worker by offering a wage equal to the common reservation wage, potentially causing the market wage to fall until it equals the flow utility while unemployed. The result is a degenerate wage distribution that gives workers no incentive to search. Consequently, a number of authors have relaxed the assumptions of Diamond (1971) so as to generate wage dispersion by, for example, introducing worker heterogeneity as in Albrecht & Axell (1984) or on-the-job search as in Burdett & Mortensen (1998).

In contract theory, an important topic is the holdup problem, which arises when one agent makes a sunk investment in a relationship but must split the concomitant gains with another agent. As discussed by Williamson (1979), opportunistic behavior by the latter party may lead to an inefficient outcome in which the former party underinvests. This issue has motivated a large literature proposing various solutions, such as the allocation of property rights (Grossman & Hart, 1986), the contractual design of a renegotiation game (Aghion *et al.* , 1994), price competition with directed search (Acemoglu & Shimer, 1999), bargaining under incomplete information (Gul, 2001), and a dynamic process of investment and bargaining (Che & Sákovics, 2004).

The current paper examines a setting that features the holdup problem as well as the Diamond paradox and demonstrates how unobservable investment helps to resolve both issues simultaneously. We consider a continuous-time model of random search in the labor market (section 2). The arrival rate of job offers and the destruction rate of filled jobs are constant and exogenous. All workers and firms are ex ante homogenous, and there is no on-the-job search. Employers are assumed to post flat-wage contracts that maximize their steady-state profit flow, and job seekers follow an optimal reservation wage policy in equilibrium. We add to this basic setup an investment stage prior to labor market entry in which workers can take a costly action that reduces the disutility of labor. Individuals are assumed to invest so as to maximize their expected payoff, which is the difference between the present value of utility flows expected by a job seeker and the amount invested. We compare and contrast the outcome when investment is unobservable to the case where it is observable (section 3).

A key assumption is that the disutility of labor experienced by a worker may be unobservable to firms. This assumption has some precedent in the literature on search models with asymmetric information; see, for example, Albrecht & Vroman (1992), in which workers differ in their cost of effort while employed, and Guerrieri (2008), in which such heterogeneity is match-specific. In the current article, the disutility of labor is endogenously determined by an investment decision, which can be conceptualized as a form of human capital accumulation that enables an employee to fulfill job requirements with greater facility. Such investments in human capital, which could be related to the amount of effort put into schooling or training, may not be perfectly observed by prospective employers.[1]

This conception of the investment decision as involving skills is not necessarily inconsistent with the large body of literature in labor economics according to which more skilled workers have higher earnings (Card, 1999). Instead of decreasing the disutility of labor, skill investment could be interpreted equivalently as increasing the value that a worker derives from a job, which may include performance-related components of income. For example, more skilled waiters may earn more in tips income, a more skilled professor could earn more in consulting income, and more skilled athletes may earn more in advertising income. Hence, more skilled workers would have higher earnings than less skilled workers when productivity-related income is added to a base wage that does not differ among workers at a firm. This interpretation of the model is empirically relevant as performance pay is becoming increasingly prevalent in the labor market and is of growing importance in generating earnings inequality (Lemieux *et al.* , 2009).[2]

The investment in the model can also be interpreted in terms of human capital broadly defined to include health, fertility, and geography. Individuals may make healthy lifestyle choices in regard to smoking, drinking, diet, and exercise that lower the disutility of labor by reducing disability and disease.[3] Women might invest in their careers by delaying childbearing in order to reduce childcare responsibilities at home that can substantially increase the opportunity cost of work.[4] Workers might pursue economic opportunities by moving closer

[1] In particular, schools may assign grades in an imprecise manner given the preferences of students and teachers (Dubey & Geanakoplos, 2010), and the process of training may require inputs such as diligence and attention that are intangible (Acemoglu & Pischke, 1999).

[2] More generally, the model here is analytically equivalent to one in which firms sell jobs and workers buy jobs. In such a formulation, the worker would be the residual claimant on the revenue from a job, which would naturally be increasing in the skills of the worker. The idea that workers are the residual claimant on the output of a firm is historically important in economic theory (Walker, 1891). Selling the job to the worker is also a solution to the standard moral hazard problem when the worker is risk neutral (Harris & Raviv, 1979).

[3] For example, Reichert (2015) studies a weight loss intervention that increased employment rates among women.

[4] For example, Miller (2011) shows that delaying motherhood results in higher labor supply among women.

to employment centers, which would lower the transportation cost of commuting to work.[5]

The literature in contract theory distinguishes between self-investment, which benefits the investor as when a seller invests to lower the cost of supplying a product, and cooperative investment, which benefits a partner as when a seller invests to raise the value of a product to a buyer (Che & Hausch, 1999).[6] The current paper focuses on a form of self-investment by workers that decreases the cost of providing labor. While the model does not allow for cooperative investment that increases an employer's revenue from a job, it does allow for self-investment that increases a worker's valuation for a job, which may include productivity-based earnings. In addition, the current article considers the "gap" case from the bargaining literature in which positive gains from trade exist even for the least skilled workers. While there is also a "no-gap" case in which gains from trade arise only if human capital investment is sufficiently high, it may not be empirically relevant since a labor market for unskilled workers exists in the real world, albeit with higher levels of unemployment.

As a benchmark, we consider the equilibrium of the model when investment is observable. Under this assumption, workers do not invest in human capital, and employers all offer the monopsony wage. Due to wage posting by employers, a holdup problem occurs because if a worker invests in lowering the disutility of labor, then firms reduce wages by the amount of this decrease. Hence, workers have no incentive to invest as they do not receive any of the associated returns. In addition, the Diamond paradox arises because employers offer workers the reservation wage, which is always accepted by workers when offered. Consequently, the market wage declines until the flow utility is the same while employed and unemployed. If job seekers were to face direct costs of search, then they would leave the labor force, causing the market to collapse.

The main case of interest is where investment is unobservable. In this situation, the equilibrium distributions of wage offers and human capital are nondegenerate. The wage distribution has a convex support and is continuous except possibly at the supremum of its support. The skill distribution is continuous except at the supremum of its support, which is the interval between zero and the efficient investment level. Intuitively, there cannot be

[5] Highly educated workers, who are extremely mobile geographically (Molloy *et al.* , 2011), may relocate after finding a job. However, according to the spatial mismatch hypothesis in urban economics (Gobillon *et al.* , 2007), residents of disadvantaged neighborhoods where jobs are scarce face high costs of finding and holding a job outside their neighborhood. Hence, such individuals may be unable to find work unless they incur a substantial upfront moving cost, which has motivated important policy interventions like Moving to Opportunity that facilitate migration out of depressed areas.

[6] Gul (2001), Lau (2008), and Nguyen & Tan (2019) consider self-investment while investment has cooperative elements in Hermalin (2013), Kawai (2014), and Dilmé (2019).

an equilibrium in which no workers invest because otherwise a worker could profitability deviate by unobservably investing with no effect on the wages that employers offer. In particular, an individual could make a small investment at relatively low cost and enjoy all of the benefits resulting from a decrease in the disutility of labor. Furthermore, there cannot be an equilibrium in which all workers invest the same positive amount because otherwise employers would lower wages by the amount that this investment reduces the disutility of labor. If so, workers would have an incentive to deviate by not investing as all the benefits of investment would be appropriated by firms.

Despite its parsimony, the model in this case is consistent with a number of stylized facts about the labor market. One is the apparent negative duration dependence of unemployment (van den Berg & van Ours, 1996), which occurs here because the arrival rate of acceptable job offers is directly related to the unobserved human capital of a worker. Another is the positive employer size-wage differential (Brown & Medoff, 1989), which exists here because firms each face an upward sloping labor supply curve due to the heterogeneous reservation wages of workers. In addition, the model is compatible with a large set of wage distributions, including those that are positively skewed and have a long right tail as in reality (Neal & Rosen, 2000). The atom at the supremum of the support of the equilibrium investment distribution may be plausible insofar as it represents a group of workers like in Eckstein & Wolpin (1990) that would accept every available job offer. Like in Bontemps *et al.* (2000), the equilibrium wage distribution is such that the full measure of employed workers is paid strictly more than their reservation wage.

The remainder of this section explains the contribution of the paper in the context of existing work. Thereafter, the paper is organized as follows. Section 2 presents a search model with unobservable investment and defines an equilibrium in this environment. The model is solved in section 3. As a point of reference, the outcome when investment is efficient or observable is also considered. In section 4, we derive comparative statics for the wage and skill distributions with respect to the parameters of the model. In section 5, we demonstrate that the model is capable of fitting an arbitrary distribution of wages, and we apply the model to explain differences between real-world wage distributions. Section 6 examines the solution to the model in the limit as the arrival rate of job offers becomes infinite. The conclusion is in section 7, which reviews the findings and mentions potential applications and extensions.

1.1 Relation to Literature

The present article contributes to two major lines of work in economics: an active literature in contract theory on the holdup problem with asymmetric information and an extensive literature in search theory on wage dispersion. The solution to the model has two main implications in relation to these areas of research. The first is an identification result according to which any wage offer distribution satisfying a few regularity conditions can be rationalized by a unique investment cost function. The second is an asymptotic efficiency result in which investment converges to the efficient level as the arrival rate of job offers becomes infinite.

A seminal paper on the holdup problem with asymmetric information is Gul (2001), which analyzes a bargaining game with one-sided offers by a seller where a buyer invests so as to raise its own valuation of a good. The respective outcomes when investment is observable and unobservable are considered, and so are both the cases of one-shot and repeated offers. A key result in Gul (2001) is that unobservable investment by the buyer converges to the efficient level as the time between offers approaches zero, with trade happening instantly in the limit and the seller receiving the entire surplus. Lau (2008) and Nguyen & Tan (2019) examine the optimal information structure in such an environment, demonstrating the role of asymmetric information in creating an ex ante incentive to invest as well as an ex post inefficiency in trade. A related problem of information design, in which a seller observes the investment distribution chosen by a buyer but not the realized value of the investment variable, is presented by Condorelli & Szentes (2020). In addition, Hermalin (2013) studies a setting in which a seller makes a private investment that enhances the value of an object to a buyer, and Dilmé (2019) considers a situation where the seller also values this investment. A similar problem is analyzed by Kawai (2014) in a dynamic setting.[7]

The current paper extends this literature by embedding a holdup problem into a search model. While existing research largely focuses on interactions between a single buyer and seller, we consider a market with a continuum of agents on each side. In the setting here, if a seller rejects the offer made by a buyer, then the seller has the option to search for another buyer. Moreover, the distribution of types among the pool of available sellers is determined not only by the investment decisions of sellers, but also by the search behavior of each type of seller. In section 6, we extend the asymptotic efficiency result in Gul (2001)

[7]Other relevant papers include: González (2004), who presents a screening model in which an agent makes a capital investment that is unobservable to a principal; Goldlücke & Schmitz (2014), who construct a signaling game in which investment by a seller reveals private information about its outside option to a buyer; and Rao (2015), who examines a common value auction in which one bidder invests in an item while the other bidder is uninformed about the amount invested.

to a search theoretic environment, showing that investment approaches the efficient level as the expected time between offers becomes arbitrarily small. In the limit, the expected delay in obtaining employment converges to zero, and employers capture all of the ex ante surplus in the labor market. Despite these parallels, the setup here, where buyers each make take-it-or-leave-it offers and sellers have a nontrivial outside option, differs from that in Gul (2001), in which there is just one seller that makes repeated offers to a buyer. Furthermore, by situating the analysis in a labor search context, we obtain a number of implications regarding the properties of the wage distribution and its comparative statics as well as the Diamond paradox and the incentive to search. Importantly, we find that a form of the Diamond paradox survives as the welfare of workers is the same as under monopsony. Questions involving firm size and unemployment durations can also be examined.

Turning to the literature in search theory on wage dispersion, one strand of this research examines the role of heterogeneity among workers. Albrecht & Vroman (1992) show that an equilibrium in which all employers post the same wage fails to exist in an environment where workers differ in their disutility of labor. In the wage posting model of Albrecht & Axell (1984), there are two types of individuals who differ in their opportunity cost of employment, and there may be a two-point distribution of wages in equilibrium. The present article endogenizes the distribution of worker heterogeneity. We demonstrate the existence, uniqueness, and properties of the equilibrium, which features a continuum of worker types and wage offers when investment is unobservable.

As mentioned by Rogerson *et al.* (2005) and discussed further by Gaumont *et al.* (2006), a potential problem with theories of wage dispersion based on worker heterogeneity is the sensitivity of the equilibrium to the presence of direct costs of search. In an environment like Albrecht & Axell (1984), for example, where there is a finite number of types of workers, workers having the highest reservation wage do not obtain any gains from search and would withdraw from the labor force in response to even an infinitesimal cost of search. In the current setting, however, where a continuum of worker types arises in equilibrium when investment is unobservable, the full measure of workers has a strict incentive to search once investments have been sunk. Consequently, the equilibrium of the model is robust to the inclusion of some forms of direct search costs, as detailed in the online appendix, which states and proves an equivalence result relating the equilibrium of the baseline model to the equilibria of an extended model with direct costs of search.

A primary objective of empirical work on job search is to explain the observed distribution of wages. A pioneering attempt at estimating an equilibrium search model is Eckstein

& Wolpin (1990), who develop a framework based on Albrecht & Axell (1984), in which there is heterogeneity in the characteristics of workers, particularly in the value of leisure. However, as noted in the survey by Mortensen & Pissarides (1999), this approach, which accommodates only a small number of discrete types due to computational limitations, cannot adequately replicate the empirical wage distribution, so that variation in wages is attributed predominantly to measurement error.

Accordingly, subsequent research emphasizes the role of on-the-job search in generating wage dispersion, with several studies estimating versions of the model in Burdett & Mortensen (1998).[8] Examples include: van den Berg & Ridder (1998), who construct a segmented labor market in which agents are homogeneous within but not across segments; Bowlus *et al.* (1995), who categorize firms into a finite number of different levels of productivity; and Bontemps *et al.* (2000), who allow for a continuous distribution of productive heterogeneity across employers. Another approach involves sequential auctions as in Postel-Vinay & Robin (2002b), where employed workers may receive outside offers, and incumbent employers can react with counteroffers, resulting in Bertrand competition.[9]

Although the basic model of on-the-job search in Burdett & Mortensen (1998) counterfactually predicts a wage distribution with an increasing density when employers are homogenous, the observed cross-sectional distribution of wages can be perfectly reproduced by introducing heterogeneity among firms as in the framework of Bontemps *et al.* (2000). Nonetheless, Bontemps *et al.* (2000) demonstrate that the setup in Burdett & Mortensen (1998) imposes some restrictions on the potential shape of the wage distribution even in the case where there are productivity differentials across employers. In particular, the rate at which the density can increase is bounded above. Relatedly, the model of sequential auctions with heterogenous firms in Postel-Vinay & Robin (2002a) is also slightly restrictive insofar as it implies that the wage distribution admits a density that is equal to zero and continuous at the supremum of its support, in which case the wage distribution has a thin upper tail.

Whereas prior studies like Eckstein & Wolpin (1990) suggest that worker heterogeneity may play little role in explaining wage dispersion in a search context, section 5 of the current

[8]This model is extended by Carrillo-Tudela & Kaas (2015) to investigate adverse selection and employer learning and by Piyapromdee (2018) to incorporate moral hazard and efficiency wages. Relatedly, Garrett *et al.* (2019) and Lester *et al.* (2019) analyze screening problems in a simultaneous search environment based on Burdett & Judd (1983). The former studies a setting with private values, while common values are involved in the latter.

[9]Other theories of wage dispersion include those of Galenianos & Kircher (2009), who examine a portfolio choice problem in a directed search context where workers can apply to multiple jobs, and Menzio & Trachter (2015), who construct a sequential search model in which there is a large seller and a fringe of small sellers.

article shows that worker heterogeneity when endogenized through unobservable investment can generate a wide variety of wage distributions. Given any values of the search and technology parameters, the disutility of labor can be specified so as to match any equilibrium wage distribution that has a bounded and convex support and is continuous except possibly at the supremum of its support. Thus, the current framework has perhaps an even greater capacity to produce wage dispersion than standard models of on-the-job search with heterogeneous firms. In addition, its comparative statics properties are somewhat distinctive as discussed in section 4.

In the present environment, wages are influenced primarily by supply-side factors associated with the cost of working instead of demand-side factors related to the output from a worker. Provided that it is socially optimal for every type of worker to be employed, the equilibrium wage distribution does not depend on the revenue flow produced by an employed worker, whereas an increase in the flow utility enjoyed by an unemployed worker results in a wage offer distribution that first-order stochastically dominates the original one. This property of the model reflects the fact that the expected payoff of a worker remains at the monopsony level regardless of whether investment is observable or unobservable. In the case where investment is unobservable, a lower arrival rate of job offers or a higher destruction rate of filled jobs increases wages in the sense of first-order stochastic dominance because workers must be compensated for the loss of regular work if their expected payoff is to remain unchanged. These comparative statics are diametrically opposed to the predictions of the model of on-the-job search in Burdett & Mortensen (1998), in which such changes in the parameters decrease wages by reducing competition among employers.

There has also been some work linking wage dispersion to variation in investment. In the search model of Acemoglu & Shimer (2000), firms offering higher wages attract more job applicants and invest more in physical capital. Likewise, Mortensen (1998) and Quercioli (2005) solve for search equilibria in which there is a positive relationship between wages and investment in firm-specific human capital. General human capital investment in the presence of search frictions and wage dispersion is studied by Fu (2011). In those papers, investment decisions are made by firms, whereas here workers are the ones to invest. Moreover, those authors do not consider the role of asymmetric information about investment decisions, which is the focus here. The direction of causation between wage dispersion and variation in investment also differs. The former causes the latter in those papers, but here the latter causes the former. In particular, those authors extend the models of Burdett & Judd (1983) and Burdett & Mortensen (1998) in which wage dispersion arises even in the absence of investment by workers or firms, but the wage distribution here would be degenerate in line with Diamond (1971) if agents did not invest.

2 Model

This section introduces unobservable skill investment into a model of labor market search. Before starting job search, workers invest in skill, which lowers the disutility of labor. Firms post wages so as to maximize their profits, and workers optimally choose reservation wages. The framework is similar to the wage posting game in Mortensen & Pissarides (1999) with respect to the search dynamics but adds a pre-market investment stage that generates unobserved heterogeneity among workers.

2.1 Basics

There is a continuum of firms of measure 1 and a continuum of workers of measure n. Before labor market entry, each worker chooses an amount $h \geq 0$ to invest in human capital, which is assumed to be unobservable to employers. Let K denote the distribution of human capital across workers. Upon entering the labor market, a worker is unemployed. The arrival of job offers to unemployed workers follows a Poisson process with rate parameter $\lambda > 0$. There is no on-the-job search; that is, employed workers do not search for a job. Matches between workers and firms are destroyed at Poisson rate $\delta > 0$. Labor force participants have the discount rate ρ.

While unemployed, a worker receives the flow utility b, which may represent unemployment benefits as well as the value of leisure and home production. While employed, a worker receives the flow utility $w - c(h)$, where w denotes the wage rate, and $c(h)$ represents the cost of working. Let F denote the distribution of wage offers across firms. An employed worker generates a revenue flow $p > b + c(0)$. The cost function c is assumed to satisfy the following regularity conditions.

Assumption 1 *The disutility of labor* $c : \mathbb{R}_+ \to \mathbb{R}$ *is a differentiable convex function with* $c'(0) < -\rho(\delta + \lambda + \rho)/\lambda$ *and* $\lim_{h\uparrow\infty} c'(h) = 0$.

In words, the disutility of labor is required to be smooth and decreasing in skills, but the rate of decrease declines towards zero as skills increase. The condition $c'(0) < -\rho(\delta+\lambda+\rho)/\lambda$ is equivalent to assuming that the socially efficient investment level is positive.[10]

[10]The analysis of the model also accommodates the case in which c is not differentiable at 0 but is instead

We now derive the payoff functions of workers and firms, so that an equilibrium can be defined.

2.2 Workers

The expected payoff of a worker is determined as follows. Let $U(h)$ denote the expected present value of lifetime utility flows for an unemployed worker with skill h. Let $V(w,h)$ be the expected present value of lifetime utility flows for a worker with skill h who is employed at a job paying the wage w.

The values $U(h)$ and $V(w,h)$ can be calculated using standard dynamic programming techniques. It is helpful to begin with a discrete-time approximation. Let Δt represent a small unit of time. The Bellman equation for an employed worker with skill h at a job paying wage w is given by:

$$V(w,h) = [w - c(h)]\Delta t + (1+\rho\Delta t)^{-1}[(1-\delta\Delta t)V(w,h) + \delta\Delta t\, U(h)], \tag{1}$$

where the left-hand side is the expected present value of employment, and the right-hand side is the sum of the current flow payoff while employed and the discounted continuation value, which takes into account the probabilities of continued employment and unemployment and the respective values of these states. The Bellman equation for an unemployed worker with skill h is given by:

$$U(h) = b\Delta t + (1+\rho\Delta t)^{-1}\left((1-\lambda\Delta t)U(h) + \lambda\Delta t \int_{-\infty}^{\infty} \max[V(w,h), U(h)]\, dF(w)\right), \tag{2}$$

where the left-hand side is the expected present value of unemployment, and the right-hand side is the sum of the current flow payoff while unemployed and the discounted continuation value, taking into account the probabilities and resulting payoffs from not receiving and receiving a job offer, which a worker can accept or reject depending on the specified wage.

Noting that $(\Delta t)^2$ becomes vanishingly small relative to Δt in the limit as Δt approaches zero, the continuous-time Bellman equations can be obtained by multiplying each side of equations (1) and (2) by $(1+\rho\Delta t)$ and retaining terms of order Δt and eliminating terms of order $(\Delta t)^2$. After some rearrangement it follows that the Bellman equation for employment

continuous at 0 with $\lim_{h\downarrow 0} c^{'}(h) = -\infty$, which implies that $c^{'+}(0) = -\infty$. As shown in section 3.3, allowing for this case enables the model to generate a wage distribution that is continuous everywhere.

is:

$$\rho V(w,h) = [w - c(h)] + \delta[U(h) - V(w,h)], \tag{3}$$

and the Bellman equation for unemployment is:

$$\rho U(h) = b + \lambda \int_{-\infty}^{\infty} \max[V(w,h) - U(h), 0]\, dF(w). \tag{4}$$

The optimal strategy of a job seeker can be characterized by a reservation wage such that a worker accepts a wage offer if and only if it is no less than the reservation wage. The Bellman equation (3) for employment implies that $V(w,h)$ is continuously increasing in w with $\lim_{w\downarrow-\infty} V(w,h) = -\infty$ and $\lim_{w\uparrow\infty} V(w,h) = \infty$. Moreover, the value of unemployment $U(h)$ is independent of the wage w. Hence, there exists a unique reservation wage $R(h)$ such that $V(w,h) < U(h)$ for $w < R(h)$, $V(w,h) > U(h)$ for $w > R(h)$, and $V[R(h),h] = U(h)$.

The Bellman equation (4) for unemployment can now be expressed as:

$$\rho U(h) = b + \lambda \int_{R(h)}^{\infty} V(w,h) - U(h)\, dF(w) = b + \frac{\lambda}{\delta + \rho} \int_{R(h)}^{\infty} w - c(h) - \rho U(h)\, dF(w), \tag{5}$$

where the second equality follows from using equation (3) to substitute for $V(w,h)$. Equation (3) reduces to $\rho U(h) = R(h) - c(h)$ for $w = R(h)$, so that equation (5) yields the following condition for the reservation wage:

$$R(h) - c(h) = b + \frac{\lambda}{\delta + \rho} \int_{R(h)}^{\infty} w - R(h)\, dF(w). \tag{6}$$

The reservation wage equation (6) implies that $R(h)$ is continuous and decreasing in the skill level h since $c(h)$ is continuous and decreasing in h. The expected payoff of a worker investing the amount h in human capital is specified as:

$$Y(h) = U(h) - h = R(h)/\rho - c(h)/\rho - h, \tag{7}$$

where the second equality proceeds from equation (3) in the case where $w = R(h)$. In words, the expected payoff of a worker is the value of unemployment net of investment costs. Note that a worker is assumed to be unemployed upon labor market entry. Although discounting between the time when the worker invests in skill and the time when the worker enters the labor market is not explicitly parameterized, a higher discount rate between these two times can be conceptualized as a proportionate decrease in the magnitude of b, p, and c.

2.3 Firms

The profit flow of an employer in steady state is determined as follows. First, the unemployment rate of each type of worker is calculated based on labor flows. Letting $u(h)$ denote the steady state unemployment rate among workers of skill h, the proportion of type h workers flowing from employment to unemployment per unit of time is $\delta[1-u(h)]$, and the proportion of type h workers flowing from unemployment to employment per unit of time is $\lambda\{1-F[R(h)^{-}]\}u(h)$, where the notation $G(\hat{z}^{-}) = \lim_{z\uparrow\hat{z}} G(z)$ is used. Equating the flows into and out of employment, the steady state unemployment rate among workers with skill level h is given by:

$$u(h) = \delta / \big(\delta + \lambda\{1 - F[R(h)^{-}]\}\big). \tag{8}$$

Based on the unemployment rate of each type of worker and the distribution of types, the distribution of worker types conditional on unemployment can be computed. The measure of workers who are unemployed and have skill no greater than $h \geq 0$ can be expressed as:

$$m(h) = n\bigg(\int_0^h u(z)\, dK(z) + u(0)K(0)\bigg). \tag{9}$$

Let $m(\infty) = \lim_{h\uparrow\infty} m(h)$ represent the measure of all workers who are unemployed. The conditional distribution of human capital given that a worker is unemployed is denoted by $J(h) = m(h)/m(\infty)$. In words, $J(h)$ is the probability that an unemployed worker has a skill level no greater than $h \geq 0$.

Next, the employment of a firm given the wage it offers is calculated based on hires and separations. The flow of workers recruited by an employer offering wage w per unit of time is given by $\lambda m(\infty)\{1 - J[R^{-1}(w)^{-}]\}$ for $w < R(0)$ and by $\lambda m(\infty)$ for $w \geq R(0)$. Letting $\ell(w)$ denote the employment of a firm offering wage w in steady state, the flow of workers separating from such an employer is $\delta\ell(w)$ per unit of time. Equating the flow of workers joining and leaving the firm, the steady state employment of a firm offering wage w is given by:

$$\ell(w) = \begin{cases} \lambda m(\infty)\{1 - J[R^{-1}(w)^{-}]\}/\delta, & \text{for } w < R(0) \\ \lambda m(\infty)/\delta, & \text{for } w \geq R(0) \end{cases}. \tag{10}$$

The profit flow in steady state of an employer offering wage w is specified as:

$$\pi(w) = (p - w)\ell(w). \tag{11}$$

2.4 Equilibrium

An equilibrium of the model is formally defined below. First, the search behavior of workers is optimal, so that the reservation wage satisfies equation (6). Second, workers choose an investment level h so as to maximize the expected payoff $Y(h)$ given by equation (7). Third, firms choose a wage w so as to maximize the profit flow $\pi(w)$ given by equation (11).

Definition 1 *A wage posting and skill investment equilibrium is a triple* (R, K, F) *such that the following hold:*

a) $R(h)$ *satisfies equation (6) for all* $h \in \mathbb{R}_+$.

b) $h' \in \operatorname{argmax}_{h \in \mathbb{R}_+} Y(h)$ *for all* h' *on the support of* K.

c) $w' \in \operatorname{argmax}_{w \in \mathbb{R}} \pi(w)$ *for all* w' *on the support of* F.

It is instructive to relate the definition above to the Nash equilibrium of the static version of the two-player game in Gul (2001). First, since the holdup problem in that paper is now embedded in a search model, an additional condition is required here to represent optimal search behavior by the investing party. Second, as in that paper, the investing party chooses its investment so as to maximize its expected payoff, which in the current environment takes into account optimal search behavior. Third, as in that paper, an agent making an offer acts to maximize its expected payoff, but in the current environment where multiple agents make offers, the expected payoff depends on the offers made by other agents, which influence the distribution of reservation values among searchers. This reflects the dynamic multi-agent nature of the present setting, in which the composition of the pool of searchers is determined by investment decisions in conjunction with the dynamics of the matching process that is generated by optimal search behavior along with the distribution of offers. In the solution below the details of the search model are relevant largely insofar as they matter for the specification of the payoff functions in the definition of an equilibrium.

3 Solution

This section solves for the wage and skill distributions that arise in search equilibrium. It begins by presenting two simple benchmarks before proceeding to the primary case of interest. First, the efficient investment level, which maximizes the expected present value of payoff flows in the labor market net of investment costs, is characterized. Second, the situation where investment is observable to employers is considered, which results in an equilibrium with zero investment and a degenerate wage distribution. Thereafter, the model is analyzed under the assumption that investment is unobservable, which produces wage and skill dispersion in equilibrium.

3.1 Efficient Outcome

The efficient amount to invest in human capital before entering the labor market is identified as follows. Given the assumption that $p > b + c(0)$, payoff flows in the labor market are maximized by a policy in which workers accept every job offer received. A formal statement and proof are given in the online appendix, along with precise specifications of the relevant action spaces and payoff functions.[11]

Assuming that every job offer is accepted, let $\hat{U}(h)$ and $\hat{V}(h)$ respectively denote the expected present values of the payoff flows generated by an unemployed and employed worker with skill level $h \geq 0$. The values $\hat{U}(h)$ and $\hat{V}(h)$ satisfy the following system of Bellman equations:

$$\rho\hat{U}(h) = b + \lambda[\hat{V}(h) - \hat{U}(h)] \text{ and } \rho\hat{V}(h) = [p - c(h)] + \delta[\hat{U}(h) - \hat{V}(h)], \tag{12}$$

which has the solution:

$$\hat{U}(h) = \frac{\lambda[p - c(h)] + (\delta + \rho)b}{\rho(\delta + \lambda + \rho)} \text{ and } \hat{V}(h) = \frac{\delta b + (\lambda + \rho)[p - c(h)]}{\rho(\delta + \lambda + \rho)}. \tag{13}$$

An efficient investment level maximizes $\hat{U}(h) - h$, which represents the expected present value of the payoff flows generated by a worker net of the investment costs, where new entrants to the labor market are initially unemployed. The result below characterizes the socially optimal investment level.

Proposition 1 *The efficient investment level is given by* $h^e = c^{'-1}[-\rho(\delta + \lambda + \rho)/\lambda]$.

The efficient investment level is such that an additional dollar of human capital investment results in a one dollar reduction in the present value of the expected disutility flow from labor. Since $c^{'}$ is a continuously increasing function, the efficient investment level in this case is decreasing in the discount rate ρ and the job destruction rate δ but increasing in the job arrival rate λ.

[11] These details are omitted from the main text because the formalization is straightforward but requires the introduction of a large amount of notation that is not used elsewhere in the paper.

3.2 Observable Investment

We next identify the outcome of the model when the amount invested by a worker is observable to employers. The setup is the same as in section 2, except that the wage offer distribution F depends on the skill level h.

An equilibrium when investment is observable satisfies the three conditions in definition 1 where F now depends on h. Workers follow an optimal reservation wage rule. They invest in skill so as to maximize their expected payoff. Firms offer wages that maximize their profits given the skill level of each worker.

The following result characterizes the equilibrium in this case. There is no investment in human capital, and the wage distribution is degenerate, with each worker being offered the common reservation wage.

Proposition 2 *An equilibrium exists and is unique when investment is observable. The equilibrium is as follows:*

a) The reservation wage is given by $R(h) = b + c(h)$ for all $h \in \mathbb{R}_+$.

b) The skill distribution is $K(h) = 1$ for all $h \in \mathbb{R}_+$.

c) For all $h \in \mathbb{R}_+$, the wage offer distribution is given by $F(w) = 0$ for $w < b + c(h)$ and by $F(w) = 1$ for $w \geq b + c(h)$.

In equilibrium, the expected payoff of each worker is $Y(0) = b/\rho$, and the profit flow of each employer is $\pi[b + c(0)] = n[p - b - c(0)]\lambda/(\delta + \lambda)$.

When investment is observable, wage posting leads to the Diamond paradox. Since employers know the reservation wage of each worker, every employer offers a worker the reservation wage. Accordingly, a worker has no incentive to reject an offer and search for another, so the wage is at the monopsony level for each type of worker, equal to the sum of the flow payoff while unemployed and the disutility of labor while employed. If there were a cost to search for a job instead of remaining out of the labor force, then workers would withdraw from the labor market instead of searching.

The Diamond paradox, which arises when investment is observable, results in the holdup problem. Ordinarily, the holdup problem applies to situations in which an investment is relationship-specific: if there were instead an outside market for the investment, the holdup problem would disappear. In the current setting, there is an outside market for the investment made by a worker because a worker can reject the offer made by one firm and seek employment in another firm. However, other employers offer the monopsony wage due to the Diamond paradox, so that a worker does not receive any returns on investment from the outside market. This is what gives rise to the holdup problem, in which workers have no incentive to invest. If there were no discounting or search frictions, an equilibrium without the Diamond paradox could be supported with the wage being at the competitive level, in which case the holdup problem would vanish.

3.3 Unobservable Investment

The model is now solved in the case where investment in skills is unobservable. The analysis is conducted in a number of steps. First, some key properties of an equilibrium are characterized. Thereafter, the unique human capital distribution, wage offer distribution, and reservation wage function satisfying these properties is determined.

Let w^u denote the supremum of the support of F. The result below shows that the supremum of the support of the equilibrium wage distribution is less than the revenue flow that a worker generates. This implies that the equilibrium profit flow is positive, given that employment is positive in equilibrium.

Lemma 1 *In equilibrium,* $w^u < p$.

The logic behind the preceding result is as follows. Note that an employer can secure a profit flow of zero by offering the wage p. First, suppose to the contrary that there exists an equilibrium in which $w^u > p$. Then a firm offering w^u must have zero employment so that profits are nonnegative. Since no jobs are being accepted, workers are continually receiving a flow payoff of b, in which case they would be better off accepting a job paying w^u. This results in the contradiction that a firm offering w^u has positive employment. Next, suppose to the contrary that there exists an equilibrium in which $w^u = p$. Since wage offers are no greater than p, reservation wages must be less than p as search is time consuming. Hence, an employer could have positive employment while offering a wage less than p, which results in positive profits. This violates the equilibrium condition that w^u is a profit maximizing wage offer.

Let h^u denote the supremum of the support of K. Let w^l denote the infimum of the support of F. In essence, the next result shows that the reservation wage of the most highly skilled workers is no greater than the lowest wage offered in equilibrium. This implies that those workers who have the smallest disutility of labor in equilibrium accept every job offered.

Lemma 2 *In equilibrium,* $R(h^u) \leq w^l$.

The proof of the lemma above is basically as follows. Suppose first that $R(h^u) > w^u$, which implies that workers are not accepting any wage offers. Then firms have zero profits, and workers constantly have the utility flow b. Hence, workers would be willing to accept a wage offer of $b + c(0) < p$, in which case firms can earn positive profits by offering the wage $b + c(0)$. Since firms would have an incentive to deviate, there is no equilibrium in which $R(h^u) > w^u$. Now suppose that $R(h^u) > w^l$. Then firms offering a wage less than $R(h^u)$ do not hire any workers and get zero profits. However, a firm could employ some workers by offering a wage higher than $R(h^u)$. Because $R(h^u) \leq w^u$ and $w^u < p$ in equilibrium, a firm could earn positive profits by offering a wage that is slightly higher than $R(h^u)$. Since some firms would have an incentive to deviate, there is no equilibrium in which $R(h^u) > w^l$.

Let h^l denote the infimum of the support of K. The lemma below essentially states that the reservation wage of the least highly skilled workers is no less than the highest wage offered in equilibrium. The implication of this is that those workers who have the highest disutility of labor in equilibrium do not enjoy any gains from search.

Lemma 3 *In equilibrium,* $R(h^l) \geq w^u$.

The proof is simple. Suppose that $R(h^l) < w^u$ in equilibrium. Then there exists a wage $\hat{w}$ slightly less than w^u that every worker would accept. Hence, a firm could increase its profits by offering the wage $\hat{w}$ instead of the wage w^u, which contradicts the equilibrium condition that the wage offer w^u is profit maximizing.

The preceding result is used to show that 0 is the infimum of the support of the equilibrium skill distribution and that the expected payoff of a worker in equilibrium is b/ρ. Consequently, the expected payoff of workers is no different from the monopsony level when investment is observable. In equilibrium, workers who invest in skills receive gains from search, but these gains are exactly offset by the cost of investment.

Lemma 4 *In equilibrium,* $h^l = 0$ *and* $\max_{h \in \mathbb{R}_+} Y(h) = b/\rho$.

The intuition behind the lemma above is straightforward. Since $R(h^l) \geq w^u$, workers with skills no greater than h^l do not gain anything from search, meaning that their flow payoff in the labor market is constant at b. Since investment is costly and investments no greater than h^l do not provide any returns in the labor market, the only way that investment level h^l could be payoff maximizing as required in equilibrium is if $h^l = 0$, and investment level h^l generates an expected payoff of b/ρ.

The result below states that the distribution function K is increasing between 0 and h^u in equilibrium. That is, the skill distribution has a connected support.

Lemma 5 *In equilibrium,* $K(h') < K(h'')$ *for all* $h', h'' \in [0, h^u]$ *with* $h' < h''$*.*

The proof is by contradiction, and the following is a sketch. Suppose to the contrary that there exists an equilibrium in which skill levels h^a and h^b with $h^a < h^b$ are on the support of K and the support of K has a gap between h^a and h^b. Then employers would receive lower profits by offering a wage strictly between $R(h^b)$ and $R(h^a)$ than by offering the wage $R(h^b)$ because they would have the same employment while paying each worker more. Hence, profit maximizing firms will not offer a wage strictly between $R(h^b)$ and $R(h^a)$. Moreover, workers with skill level h^b are willing to reject a wage offer equal to their reservation wage $R(h^b)$. This implies that workers with skill levels h^a and h^b effectively have the same job search behavior. Since investment levels h^a and h^b must both maximize the expected payoffs of workers, they must both maximize the reduction in the disutility of labor given job search behavior minus the cost of investment. However, given the convexity of the disutility of labor, this objective function cannot have two distinct maximizers, holding job search behavior constant.

We now proceed to solve for the equilibrium of the model. The lemmata above imply that any equilibrium has the following properties: First, the reservation wage $R(h^u)$ of a worker with skill level h^u is no greater than the infimum of the support of F. Second, the support of K is the interval $[0, h^u]$. Third, the maximum value of the expected payoff $Y(h)$ to a worker over all possible levels of skill $h \in \mathbb{R}_+$ is b/ρ. We search for a triple (R, K, F) having these properties that satisfies the equilibrium conditions in definition 1. The theorem below states that the model has a unique equilibrium and provides expressions for the reservation wage equation, human capital distribution, and wage offer distribution in equilibrium.

Theorem 1 *A wage posting and skill investment equilibrium exists and is unique. The equilibrium is as follows:*

a) The reservation wage is given by:

$$R(h) = \begin{cases} b + \rho h + c(h), & \text{if } h \in [h^l, h^u] \\ b + \rho h^u + \dfrac{\lambda c(h^u) + (\delta + \rho) c(h)}{\lambda + \delta + \rho}, & \text{if } h > h^u \end{cases}, \tag{14}$$

where $h^l = 0$ *and* $h^u = c'^{-1}[-\rho(\delta + \lambda + \rho)/\lambda]$*.*

b) The skill distribution is given by:

$$K(h) = \begin{cases} \dfrac{[p - R(h^u)]S(h)}{[p - R(h^u)]S(h^u) + [p - R(h^l)](\delta + \lambda)}, & \textit{if } h \in [h^l, h^u) \\ 1, & \textit{if } h \geq h^u \end{cases}, \tag{15}$$

where $S(h) = [p - R(h^l)] \int_0^h \frac{\rho^2 - \delta c'(z)}{[p - R(z)]^2} dz$ *for* $h \in \mathbb{R}_+$.

c) The wage offer distribution is given by:

$$F(w) = \begin{cases} 0, & \textit{for } w \leq w^l \\ 1 + \dfrac{\rho(\delta + \rho)}{\lambda\{c'[R^{-1}(w)] + \rho\}}, & \textit{for } w \in (w^l, w^u) \\ 1, & \textit{for } w \geq w^u \end{cases}, \tag{16}$$

where $w^l = b + \rho h^u + c(h^u)$ *and* $w^u = b + c(0)$.

The proof has three key steps. First, the equilibrium condition that workers make investment decisions so as to maximize their expected payoff is imposed. Since the maximum expected payoff is b/ρ in equilibrium and the support of the skill distribution is the interval $[0, h^u]$, this requirement implies that any investment level in the interval $[0, h^u]$ must generate an expected payoff of b/ρ. For skill levels no greater than h^u, an expression for the reservation wage is thereby obtained.[12] Second, the equilibrium condition that workers search optimally in accordance with the reservation wage equation is applied. This requirement is used to determine the wage offer distribution that is consistent with the reservation wage obtained earlier. Third, the equilibrium condition that firms make wage offers so as to maximize profits is imposed. Given the reservation wage and wage distribution identified previously, a human capital distribution can be derived such that all wage offers on the support of the wage offer distribution result in the same maximal level of profits.

The skill distribution that arises endogenously in equilibrium is continuous except at the supremum of its support. This helps to explain why previous work like Albrecht & Axell (1984) and Albrecht & Jovanovic (1986) in which the distribution of worker or match charac-

[12]For workers whose skill levels are higher than h^u, the reservation wage equation along with the property that such workers accept any wage offer in equilibrium is used. The value of h^u follows from the need for the investment level h^u to be payoff maximizing.

teristics is exogenous is more limited than the current model in generating wage dispersion.[13] Those studies restrict attention to forms of heterogeneity that are either continuous or discrete and do not consider a distribution of types that is a mixture of both a continuous and a discrete component as is the case here.

In the current setting, the equilibrium wage distribution has a convex support like the skill distribution and is continuous except possibly at the supremum of its support. The online appendix elaborates on the properties of the equilibrium and discusses the existence of a density and its characteristics. This helps to clarify the conditions under which the model generates a realistic wage distribution, whose density is first increasing and then decreasing. The online appendix also contains a numerical example that illustrates the attributes of the equilibrium.

Finally, we consider the welfare of the market participants. According to lemma 4, the expected payoff of a worker in the equilibrium with unobservable investment in theorem 1 is b/ρ, which is at the monopsony level stated in proposition 2 for the case where investment is observable. The following corollary of theorem 1 calculates the steady state profit flow of an employer when investment in unobservable. Profits are greater in the equilibrium in theorem 1 than the level identified in proposition 2 for the case where investment is observable. While workers are evenly compensated for their investments in skills, any surplus generated by such investments that is not dissipated by the inefficiencies arising from asymmetric information is absorbed by employers.

Corollary 1 *In the equilibrium with unobservable investment in theorem 1, the profit flow of an employer is given by:*

$$\max_{w\in\mathbb{R}} \pi(w) = \frac{n(p-w^l)[p-b-c(0)]\lambda}{(p-w^l)S(h^u)+(p-w^u)(\delta+\lambda)} > \bar{\pi}, \tag{17}$$

where w^l, w^u, h^u, and S are as specified in the statement of theorem 1, and

$$\bar{\pi} = \frac{n[p-b-c(0)]\lambda}{\delta+\lambda} \tag{18}$$

is the profit flow of an employer in the equilibrium with observable investment in proposition 2.

[13] In Albrecht & Axell (1984), there are two types of workers, and at most two distinct wages are offered in equilibrium. In Albrecht & Jovanovic (1986), the distribution of match types admits a density, and there is a single wage in equilibrium.

There is a simple logic behind why firms have higher profits when investment is unobservable than when it is observable. Consider the labor market in equilibrium. The wage offer $b + c(0)$, which is the profit maximizing wage offer when investment is observable, is also an optimal wage offer when investment is unobservable. Moreover, unemployment is higher when investment is unobservable and workers have private information about their reservation wages because some wage offers are rejected in this case and match formation is delayed. Nonetheless, the wage offer $b + c(0)$ is always accepted as it meets or exceeds the reservation wage of every worker, regardless of whether investment is observable or not. Since the number of job searchers is greater when investment is unobservable, it follows that a firm offering $b + c(0)$ recruits more workers and has higher profits in the equilibrium with unobservable investment in theorem 1 than in the equilibrium with observable investment in proposition 2.

Together lemma 4 and corollary 1 imply that social welfare in equilibrium is higher when investment is unobservable than when it is observable. Although the expected payoff of workers is the same regardless of the observability of investment, the profit flow of firms is higher when investment is unobservable. Thus, the efficiency gain from enhanced investment outweighs the efficiency loss from delayed matching when investment is unobservable, so that the holdup problem is partly resolved.[14] This result is a dynamic search-theoretic counterpart to the finding of Gul (2001) that when bargaining is repeated, the unobservability of investment raises social welfare as the gains from the reduction in underinvestment exceed the losses from the delay in trade.[15] Although the wage offer distribution is nondegenerate when investment is unobservable, the Diamond paradox is not entirely absent because the expected payoff of workers remains at the monopsony level.

In equilibrium, only a measure zero of workers invest nothing, and workers who make a positive investment enjoy a higher expected payoff from search than from remaining out of the labor force and receiving a flow utility equal to the value of leisure. This mitigates a common problem with models of wage dispersion based on heterogeneity across workers in reservation wages. In Albrecht & Axell (1984), where there are two types of workers, a nondegenerate equilibrium exists only if a positive measure of workers, particularly those with the higher reservation wage, can search costlessly. Since such workers do not receive a surplus from

[14] In section 6, it is demonstrated that as the arrival rate of wage offers becomes arbitrarily large, skill investment converges to the efficient level with unemployment approaching zero, so that the holdup problem disappears in the limit.

[15] By contrast, in a one-shot static bargaining game, Gul (2001) finds that social welfare is the same when investment is fully unobservable as when it is fully observable, although as Lau (2008) and Nguyen & Tan (2019) show, partial information about investment may increase social welfare.

participating in the labor market, they would withdraw from the labor force if there is even an infinitesimal search cost. In the current setting, however, after the investment stage the set of workers receiving a positive ex post surplus from labor market participation has full measure, so that essentially all workers have a strict incentive to remain in the labor force. This makes the equilibrium here comparatively robust to the presence of direct search costs. In the online appendix, we formally derive an equivalence result between the solutions to the baseline model and an extended model in which all workers effectively incur out-of-pocket search costs in equilibrium.

4 Comparative Statics

This section presents comparative statics for the effect of the market parameters on the wage and skill distributions. We consider the model with unobservable investment. Distributions are compared based on whether one first-order stochastically dominates (FOSD) the other.

We begin by examining skill levels. The result below characterizes the effect of the parameters on K in terms of first-order stochastic dominance.

Proposition 3 *In a wage posting and skill investment equilibrium, the distribution K improves in the sense of FOSD if p or λ is higher or if b or δ is lower. The effects of changes in ρ on K cannot in general be ranked in terms of FOSD.*

These comparative statics can be understood in terms of two properties of K in equilibrium. First, the supremum of the support of K is the efficient investment level in proposition 1, which is increasing in λ, decreasing in δ and ρ, and independent of p and b.

Second, K is such that firms offering a wage higher on the support of F receive the same profits as firms offering a wage lower on the support of F. If p rises or b falls, then firms that offer higher wages and therefore have lower profit margins experience a greater percentage increase in profits. If ρ rises, then the present value of future returns to investment by workers declines, so that firms offering lower wages must pay more to rationalize investment by workers in equilibrium, which causes a decrease in the profits of firms paying lower wages. When λ rises or δ falls, the effect on profits is similar because unemployment decreases more among higher skilled workers with lower reservation wages, which reduces recruitment comparatively more among firms offering lower wages. In order to keep profits equal among firms, these changes in parameters must be offset by an increase in the acceptance of wage offers that is greater among firms paying lower wages, which is achieved through a decrease in reservation wages due to greater skill investment by workers.

Combining both sorts of effects, higher values of p and λ and lower values of b and δ result in a distribution of human capital that FOSD the original one. In the case of ρ, these two effects work in opposite directions at different parts of the distribution, so such a comparison is not possible.

We now analyze the determinants of wages. The following proposition describes how changes in the parameters of the model affect F as specified in theorem 1.

Proposition 4 *In a wage posting and skill investment equilibrium, the distribution F improves in the sense of FOSD if b, δ, or ρ is higher or if λ is lower. The parameter p has no effect on F.*

These comparative statics demonstrate that the equilibrium wage distribution is determined by supply-side factors involving the disutility of work as opposed to demand-side factors involving the revenue product of labor. In particular, since p does not influence the expected payoff of a worker, the distribution function F is independent of p, provided that $p > b + c(0)$. By contrast, the other parameters of the model directly affect the expected payoff function of a worker and can produce changes in the equilibrium wage offer distribution.

The intuition behind the results on first-order stochastic dominance is as follows. First, consider b. A higher value of b makes employment less advantageous to workers relative to unemployment, so employers must offer higher wages in order to induce workers to accept a job. Thus, the wage offer distribution following such changes in b FOSD the original one.

Next, consider λ, δ, and ρ. If λ rises or δ falls, then because of the greater number or length of employment opportunities, wages can simultaneously decline without changing the expected payoff of a worker. At a lower value of ρ, a future gain in utility is less devalued relative to the current cost of investment, so workers do not require as much compensation for a given investment in human capital. In addition, when λ increases or δ or ρ decreases, some workers invest more in human capital, which lowers the disutility of labor, making them willing to accept lower wages. Therefore, the original wage offer distribution FOSD the one that results from such changes in λ, δ, and ρ.

It is instructive to relate the comparative statics above to those in the existing literature on equilibrium search models. In particular, consider the framework in Burdett & Mortensen

(1998), in which wage dispersion across homogenous workers arises due to on-the-job search.[16] In their framework, a higher arrival rate of jobs while unemployed or a lower destruction rate of matches effectively makes the labor market more competitive by improving the outside option of a worker that rejects an offer. Correspondingly, their model predicts that an increase in λ or a decrease in δ results in a wage offer distribution that FOSD the original one, which runs contrary to the effects that λ and δ have in the environment here. Another property of their framework is that p affects the equilibrium wage distribution while ρ does not. Intuitively, the wage distribution in their model is determined by an isoprofit condition for firms, where the instantaneous profit flow of firms is affected by p but not ρ. In the current setting, where the wage distribution is determined by an indifference condition for workers, the opposite holds with the equilibrium wage distribution depending on ρ but not p because ρ but not p affects the present value of utility flows to workers.

The effect of the parameter b on the wage offer distribution is qualitatively the same in the current model as in Burdett & Mortensen (1998). The arrival rate of job offers while employed is zero here but positive there.

5 Wage Distribution

This section examines the ability of the model with unobservable investment to generate realistic wage distributions. First, we present comparative statics results to illustrate how changes in the cost function affect the equilibrium wage distribution. Second, we derive an identification result demonstrating that arbitrary wage distributions can be rationalized by appropriately specifying the cost function. Third, the model is applied to explain regional differences in wage distributions based on the costs and benefits of investment.

[16] The equilibrium wage offer distribution in their model is given by:

$$F(w) = \begin{cases} 0, & \text{if } w \leq R \\ (\alpha+\delta)[1-\sqrt{(p-w)/(p-R)}]/\alpha, & \text{if } R < w < p-(p-R)[\delta/(\alpha+\delta)]^2 \\ 1, & \text{if } w \geq p-(p-R)[\delta/(\alpha+\delta)]^2 \end{cases}, \tag{19}$$

with the reservation wage R being equal to:

$$R = [b(\alpha+\delta)^2 + (\lambda-\alpha)\alpha p]/[(\alpha+\delta)^2 + (\lambda-\alpha)\alpha], \tag{20}$$

where $\alpha > 0$ is the Poisson arrival rate of job offers while employed, and the other notation is the same as in the current model.

5.1 Cost Function

This section provides comparative statics relating the equilibrium wage and skill distributions to the disutility function. The specification $c(h) = \chi + \kappa\hat{c}(h)$ is used without loss of generality, where $\kappa > 0$, and $\hat{c}$ satisfies $\hat{c}(0) = 0$ in addition to having the properties of c in section 2.1. As in section 4, the concept of FOSD is used to rank distributions.

We begin by examining the effects of χ and κ on K. Lower values of χ and κ tend to raise the profits of firms paying higher wages relative to those paying lower wages. For profits to remain equal among firms offering different wages, as is required in equilibrium, the acceptance of wage offers must increase among firms paying lower wages relative to those paying higher wages. This happens through greater skill investment, which causes reservation wages to decline. In addition, the supremum of K, which is the same as the efficient investment level, is independent of χ and increasing in κ. Hence, as the proposition below states, a reduction in χ raises skill investment, whereas the effect of κ is ambiguous because there are two opposing mechanisms.

Proposition 5 *In a wage posting and skill investment equilibrium, the distribution K improves in the sense of FOSD if χ is lower. The effects of changes in κ on K cannot in general be ranked in terms of FOSD.*

Next we turn to the effects of χ and κ on F, which are summarized in the proposition below. A higher value of χ makes employment less favorable to workers, so firms must offer higher wages to compensate. A higher value of κ reduces the disutility of labor among skilled individuals and induces some workers to invest more in human capital, causing their reservation wages to fall, which firms exploit by paying less.

Proposition 6 *In a wage posting and skill investment equilibrium, the distribution F improves in the sense of FOSD if χ is higher or if κ is lower.*

These results are compatible with existing empirical evidence. The parameter χ can be interpreted as a disamenity from working such as physical danger, an undesirable location, or an inflexible schedule. As in the literature on compensating wage differentials (Rosen, 1986), an increase in the disamenity is associated with higher wages, and there is a tendency for less skilled workers to be employed in jobs having a greater disamenity, with χ exerting a negative effect on the human capital distribution. A higher value of κ can be interpreted as a greater rate of return to skill investment, which is associated with lower wages in the current setting. This is consistent with the stylized facts in Psacharopoulos (1985), who finds that the returns to education are greater in lower-income countries.

5.2 Identification Result

This section characterizes the set of wage offer distributions that can arise in equilibrium. An identification result is obtained relating the wage distribution to the disutility function. We fix the market parameters p, b, δ, λ, and ρ at arbitrary values. We consider any wage offer distribution satisfying a few regularity conditions, namely that it has a convex and compact support on the real line and is continuous except possibly at the supremum of its support. We then show that there exists a disutility function c consistent with the assumptions in section 2.1 that generates this wage offer distribution as part of an equilibrium.

Moreover, this disutility function is uniquely determined on an interval that includes zero, which represents the support of the human capital distribution that corresponds to the given wage offer distribution. The theorem below specifies a one-to-one mapping that enables the disutility function on the relevant interval to be inferred from the wage offer distribution, and vice versa. Once the disutility function has been recovered in this way, the skill distribution associated with the given wage distribution is uniquely determined as described in theorem 1, and so is the reservation wage function on the relevant interval.

Theorem 2 *Choose any values of the parameters p, b, δ, λ, and ρ. Let F be any wage offer distribution with the properties:*

a) F is supported on a proper interval $[w^l, w^u] \subset \mathbb{R}$.

b) F is continuous for all $w \neq w^u$.

Then the following hold for the disutility function c:

a) There exists c such that one can find R and K for which (R, K, F) is a wage posting and skill investment equilibrium.

b) Any such c is given by the equation below for all $h \in [0, h^u]$:

$$c(h) = A^{-1}(h) - \rho h - b, \tag{21}$$

where $A(w) = \lambda \int_w^{w^u} 1 - F(z)\, dz / [\rho(\delta + \rho)]$ for all $w \in [w^l, w^u]$ and $h^u = A(w^l)$.

The foregoing theorem is obtained as follows. The expression for F in theorem 1 provides a differential equation for c, with the expression for the supremum of the support of F supplying a boundary condition. This boundary value problem is solved to obtain an explicit solution for c, with the expression for the infimum of the support of F helping to determine the domain over which this solution holds.[17] Moreover, c can be defined beyond this domain so as to be sufficiently well behaved. It is then verified that the function c determined in this way satisfies the assumptions in section 2.1.

The result above is pertinent to the empirical literature on the estimation of equilibrium search models of the labor market. One approach to generate wage dispersion, as in Albrecht & Axell (1984), is to allow for heterogeneity in worker characteristics, such as the value of leisure. Nonetheless, Eckstein & Wolpin (1990), who allow for only a small finite number of worker types, find that this approach provides a poor fit to the wage data, with the vast majority of the observed variation in wages being attributed to measurement error. Consequently, much of the literature, including van den Berg & Ridder (1998) for instance, pursues on-the-job search as another explanation for wage dispersion. When firms are homogenous, the on-the-job search model in Burdett & Mortensen (1998) implies that the wage distribution has an increasing density, which is inconsistent with the data. However, when heterogeneity across firms in productivity is introduced, this framework can produce a realistic wage distribution, as in Bontemps *et al.* (2000).

In the current paper, worker heterogeneity and wage dispersion arise endogenously through what are effectively mixed strategies over continuous action spaces. The resulting skill and wage distributions are nondegenerate for an open set of parameter values and have a convex support. In a certain sense, this approach to introducing worker heterogeneity can accommodate even more forms of wage dispersion than models of on-the-job search with firm heterogeneity. Specifically, Bontemps *et al.* (2000) show that their model imposes a restriction on the shape of the equilibrium wage distribution, namely that the density of the wage offer distribution does not increase too rapidly. By contrast, the potential shape of the wage offer distribution is largely unrestricted in the current setting except for a few regularity conditions noted above. As the next section illustrates, the flexibility of the model in this regard facilitates its empirical application and testing.

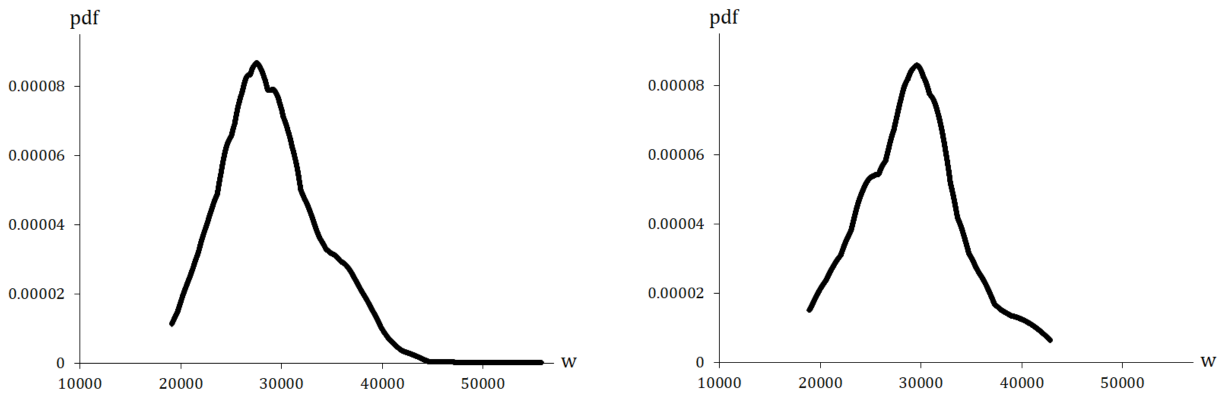
5.3 Empirical Application

In order to illustrate how the disutility function can be specified to match real-world wage distributions, the model is fitted to data from the Employment Opportunities Pilot Project (EOPP). These data, which were collected in 1980, have been widely used in research on job search, including recent papers by Faberman & Menzio (2018) and Wolthoff (2018). An

[17] Note also that A^{-1} in theorem 2 represents the reservation wage on this domain.

Figure 1: Wage Density Functions in Two Labor Markets

(a) Baton Rouge, Louisiana (b) Outagamie, Wisconsin



Note: Probability density functions are fitted to wage data from the EOPP employer survey using adaptive kernel density estimation with an Epanechnikov kernel function. Wages are annualized and expressed in current dollars as detailed in the main appendix.

important component of the EOPP is a survey of employers that contains unusually detailed information about the recruitment and separation of employees as well as wages and sales.

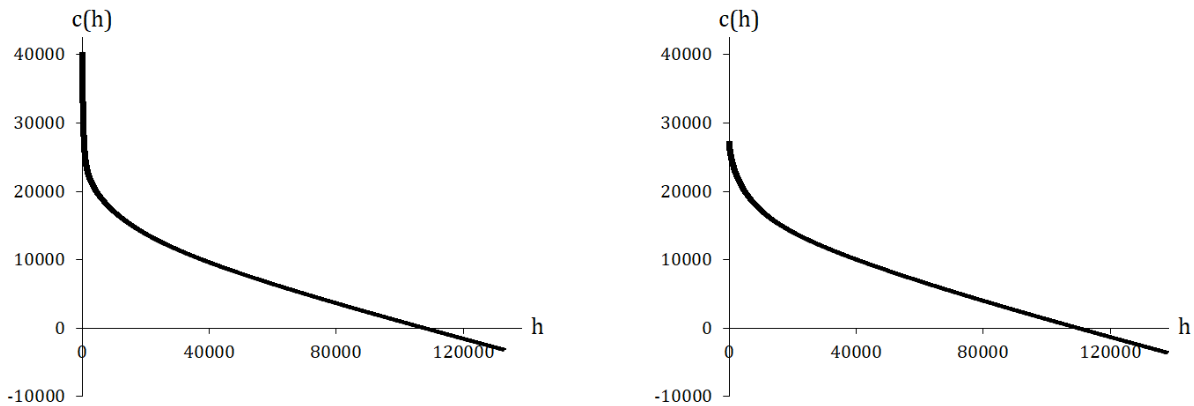
The parameters are calibrated using these data in conjunction with standard government sources as explained in detail in the main appendix. Labor productivity is set at $p = \$62,597$ based on the gross annual sales receipts per worker, and the value of leisure is set at $b = \$15,615$ based on the average annual level of unemployment benefits. The job destruction rate is set at $\delta = 0.6443$ per year based on job separation data, and the job arrival rate is set at $\lambda = 0.4652$ per year based on job recruitment data and labor force statistics. The annual discount rate ρ is chosen to be 0.05 in keeping with standard values in the literature. All figures are adjusted for inflation to current dollars.

The EOPP surveyed employers in about thirty sites throughout the US, with locations in the South and Midwest of the country being overrepresented. For illustrative purposes, we select two sites with conspicuously different wage distributions: Outagamie, Wisconsin and Baton Rouge, Louisiana. According to estimates of the Gini coefficient during the last quarter of the twentieth century (Levernier *et al.* , 1995), Louisiana was the most unequal state in the United States while Wisconsin was among the top handful in terms of equality.

The data from the EOPP are used to calculate the annual earnings of the typical worker in each firm as described in the main appendix. A probability density function is fitted to the

Figure 2: Implied Disutility Functions in Two Labor Markets

(a) Baton Rouge, Louisiana (b) Outagamie, Wisconsin



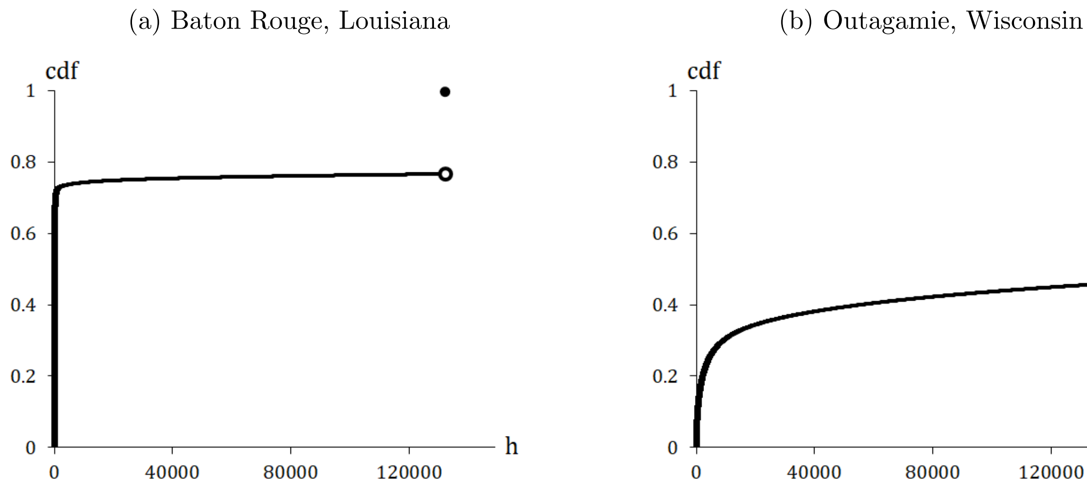
Note: Disutility functions are computed from the estimated wage distributions based on theorem 2. The parameter values are set at $p = \$62,597$, $b = \$15,615$, $\lambda = 0.4652$, $\delta = 0.6443$, and $\rho = 0.05$ as detailed in the main appendix.

wage data in each location using adaptive kernel density estimation with an Epanechnikov kernel function. Figure 1 depicts the resulting wage density for each site. The disutility function for each site is then computed from the wage distribution using equation (21) in theorem 2 and is displayed in figure 2. Thereafter, the skill distribution is calculated based on equation (15) in theorem 1 and is displayed in figure 3. All values are expressed in current dollars.

A key difference between the wage distributions in figure 1 is that the right tail is much longer in Baton Rouge than in Outagamie. When the wage distribution has a longer tail, workers must lower their asking wages by more in order to effectuate a given increase in their employment prospects. Since investment is less beneficial for increasing employment in this case, it must be more effective in decreasing the disutility of labor for workers to be willing to invest. This explains why the disutility function near zero investment has a steeper slope in Baton Rouge than in Outagamie.

In order for the wage distribution to have a longer right tail, firms must be willing to raise wages by more, which would be the case if raising wages increases employment by more, necessitating a greater concentration of workers that would respond to higher pay. This explains why the investment distribution is relatively polarized with a higher density close to zero in Baton Rouge as compared to Outagamie. These patterns are broadly consistent with the findings of Glaeser *et al.* (2009), according to which more unequal locations have greater returns to skill and heterogeneity in skill levels.

Figure 3: Implied Skill Distributions in Two Labor Markets



Note: Cumulative distribution functions for skills are computed from the implied disutility functions based on theorem 1. The parameter values are set at $p = \$62,597$, $b = \$15,615$, $\lambda = 0.4652$, $\delta = 0.6443$, and $\rho = 0.05$ as detailed in the main appendix.

6 Asymptotic Efficiency

This section analyzes the model in the limit as the arrival rate λ of job offers goes to infinity. The result below shows that as λ becomes arbitrarily large, the human capital distribution given in theorem 1 converges to an atom at the limiting value of the efficient investment level given by proposition 1. The wage offer distribution converges to an atom at the amount that just covers the cost of this investment given the reduction in the disutility of labor due to human capital acquisition.

In both situations, the expected payoff of a worker remains the same as under monopsony. Consequently, firms absorb the entire surplus in the labor market, receiving in the limit a profit flow per worker that is the difference between the revenue flow of an employed worker and the sum of the flow utility of an unemployed worker, the amortized cost of any investment, and the disutility of labor. For all $\lambda \in \mathbb{R}_{++}$ as well as in the limit $\lambda \to \infty$, the reservation wage of a worker who invests no more than the efficient level is the least wage offer that adequately compensates the worker for the cost of investment. When λ is real-valued, the reservation wage decreases as a worker invests beyond what is efficient, but this does not hold in the limit. Regardless, a worker has no incentive to invest such large amounts.

Theorem 3 *Letting* $h^* = c'^{-1}(-\rho)$*, the following hold in a wage posting and skill investment equilibrium as* λ *tends to* ∞*:*

a) Given any set $E \subset \mathbb{R}_+$ *such that* $\inf\{c(h) : h \in E\} > -\infty$*, the reservation wage function* R *converges uniformly on* E *to* $R^*(h) = \max[b + \rho h + c(h), b + \rho h^* + c(h^*)]$*.*

b) Skills distributed according to K *converge in probability to the constant* h^**.*

c) Wage offers distributed according to F *converge in probability to the constant* $w^* = b + \rho h^* + c(h^*)$*.*

d) The profit flow $\max_{w\in\mathbb{R}} \pi(w)$ *of an employer converges to* $n(p - w^*)$*.*

e) The expected payoff $\max_{h\in\mathbb{R}_+} Y(h)$ *of a worker is* b/ρ*.*

Gul (2001) obtains a similar resolution to the holdup problem in a model in which a buyer makes an unobservable investment in its valuation of an item and a seller makes repeated unilateral offers to the buyer. Gul (2001) finds that as the time between offers tends to zero, the investment by the buyer converges to the efficient level, with the seller receiving all of the surplus. Analogously, in the current environment, as the arrival rate of job offers becomes infinite, investment by workers approaches the efficient level, and employers receive all of the surplus. Note that the roles of the buyer and the seller are reversed here in relation to Gul (2001). The sellers of labor instead of the buyer make an investment, and the buyers of labor instead of the seller make unilateral offers.

In the present setting like in Gul (2001), investment is efficient in the limit because the expected delay in transacting approaches zero as the expected time between offers becomes infinitesimally small. In Gul (2001), this follows from the Coase conjecture, which applies when the distribution of valuations among buyers is exogenous as in Gul *et al.* (1986). In the search model here, however, it is not obvious ex ante that the expected delay would tend to zero because there is no existing analogue to the Coase conjecture that applies when the distribution of disutilities among workers is exogenous.[18] Nonetheless, the model that we

[18] Although Albrecht & Vroman (1992) consider a pertinent model with exogenously distributed disutilities of labor, they do not prove existence of an equilibrium or characterize its limiting properties.

study has the property that the expected delay approaches zero as the arrival rate becomes infinite because the wage converges to a fixed level and workers can find a job instantly at this wage. In the limit, a firm cannot profit by unilaterally offering a wage below this constant level because a worker can immediately find another employer paying the prevailing wage.

In the present setting like in Gul (2001), the party making offers has no incentive to offer anything more favorable than the reservation price of an individual making the smallest investment. It follows that the support of the equilibrium investment distribution must include zero, with all of the surplus being extracted from the investing party by the party making offers. Otherwise, if the smallest investment level were positive, then the payoff to the investing party would be negative, which is not consistent with payoff maximization in equilibrium. In the current environment, this means that as λ becomes arbitrarily large, h^l does not approach the efficient investment level h^*, even though K converges in probability to h^*.

The resolution of the holdup problem in the limit is not a consequence of the Diamond paradox disappearing when the arrival rate is infinite. The main points in the preceding argument are that the expected delay in job finding approaches zero and the payoff to workers remains at the monopsony level. However, these properties are not necessary to solve the Diamond paradox. For example, Albrecht & Axell (1984) present a solution involving heterogeneity in the value of leisure in which there is an expected delay in job finding, and Burdett & Mortensen (1998) present a solution involving on-the-job search in which workers obtain an expected payoff above the monopsony level.

When investment is observable, the Diamond paradox can be solved in the case where $\lambda = 0$ but as shown in section 3.2 not for any $\lambda > 0$. That is, the limit of the equilibrium as λ goes to 0 is different from the set of equilibria when $\lambda = 0$. In the model with unobservable investment, there is still a sense in which the Diamond paradox persists in the limit as λ goes to 0. Although the wage is above the monopsony level in this case, the expected payoff of workers remains at the monopsony level. Moreover, the limiting wage distribution is still degenerate despite the wage being above the monopsony level.

7 Conclusion

This paper has examined a form of human capital investment in the context of an equilibrium search model. With observable investment, the equilibrium would be characterized by the holdup problem, resulting in zero skill investment, and the Diamond paradox, resulting in a degenerate wage distribution. These issues are mitigated when investment is unobservable: the unique equilibrium of the model features wage and skill dispersion among ex ante identical workers and firms. Investment is asymptotically efficient as the expected time between job offers approaches zero, but the welfare of workers is the same as under monopsony like in the Diamond paradox. The model has comparative statics properties that differentiate it from standard theories and is potentially consistent with any wage offer distribution that satisfies a few boundedness, connectedness, and continuity requirements. While some models of wage dispersion require there to be no direct search costs, this assumption can be weakened here.

There are some ways in which the analysis could perhaps be extended in future work. Instead of assuming that firms post wages to maximize their profit flow in steady state, the objective function of employers might be specified as the value of posting a vacancy, which would be driven to zero by free entry. Moreover, the wage setting behavior of firms outside of a steady state and the transitional dynamics of the model towards a steady state remain open questions.[19] Finally, while employers make take-it-or-leave-it offers in the current setup, other wage determination mechanisms involving alternating offers or collective bargaining are possible.

The framework in this paper can potentially be applied to various policy issues. One question is the effect of wage controls, which could be modeled as a restriction on the action space of employers. In the current setting, if a minimum wage were imposed at the competitive level equal to the revenue product of labor, then the holdup problem would be resolved and skill investment would be efficient as there would be no scope for ex post bargaining. Another question involves the optimal design of unemployment insurance and tax systems. In the current environment, the duration of unemployment and the wage upon reemployment may serve as signals to policymakers of unobservable human capital. Since all workers receive the same expected payoff, which is at the monopsony level in equilibrium, taxation of labor income may not have much of a redistributive role despite the heterogeneity in wages and skills, although there may be room for redistribution from employers to workers.

The model is consistent with a number of stylized facts about the labor market and can replicate a remarkably large class of wage offer distributions. Accordingly, we demonstrated how the model can be calibrated using standard data to match real-world wage distributions. A potential direction for further research would be to conduct a more comprehensive empirical analysis that allows for other sources of wage dispersion. This would help to de-

[19] Coles (2001) extends the model of on-the-job search in Burdett & Mortensen (1998) to relax the assumption that firms commit to a wage that is constant over time.

termine the quantitative importance of unobservable investment as modeled in this paper in comparison with mechanisms involving on-the-job search, productivity dispersion among employers, and exogenous differences in worker characteristics.

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Appendix

Section I contains proofs of the results in the main text. Section II provides details on the calibration of the model.

I Proofs

I.1 Proof of Proposition 1

The efficient investment level h^e solves $\max_{h\in\mathbb{R}_+} \hat{U}(h) - h$. The first-order condition is $-\lambda/[\rho(\delta+\lambda+\rho)]c^{'}(h) - 1 \leq 0$, with equality if $h^e > 0$. Recall that $c^{'}$ is a continuous increasing function with $\lim_{h\uparrow\infty} c^{'}(h) = 0$. Hence, the global maximizer is as specified in the statement of the proposition. ∎

I.2 Proof of Proposition 2

For each $h \in \mathbb{R}_+$, the expected profits to a firm from making a wage offer w are decreasing in w for all $w \geq R(h)$. It follows from profit maximization that $F[R(h)] = 1$ in equilibrium. Substituting $F[R(h)] = 1$, the reservation wage equation reduces to $R(h) - c(h) = b$ for all $h \in \mathbb{R}_+$, so that the reservation wage in equilibrium is as specified in the statement of the proposition.

For the equilibrium value of $R(h)$, the profit maximization problem of firms has the solution $w = b + c(h)$ for all $h \in \mathbb{R}_+$, so that the wage offer distribution in equilibrium is as specified in the statement of the proposition.

For the equilibrium value of $R(h)$, the value of unemployment satisfies $U(h) = b/\rho$ for all $h \in \mathbb{R}_+$. Given this value of $U(h)$, the utility maximization problem of workers has the solution $h = 0$. Hence, the skill distribution in equilibrium is as specified in the statement of the proposition.

The expected payoff of a worker in equilibrium is given by $Y(0) = U(0) - 0 = b/\rho$.

In this case, the measure of all workers who are unemployed is given by $m(\infty) = n\delta/(\delta+\lambda)$, and the employment of each firm is given by $\ell[b+c(0)] = n\lambda/(\delta+\lambda)$. The profit flow of an employer in equilibrium is given by $\pi[b+c(0)] = [p-b-c(0)]\ell[b+c(0)] = n[p-b-c(0)]\lambda/(\delta+\lambda)$.
∎

I.3 Proof of Lemma 1

First, we show that $w^u \leq p$. Suppose to the contrary that there exists an equilibrium in which $w^u > p$. Noting that $\pi(p) = (p-p)\ell(p) = 0$, it must be that $\ell(w^u) = 0$, for otherwise $\pi(w^u) = (p - w^u)\ell(w^u) < 0$, contradicting the equilibrium condition that $w' \in \operatorname{argmax}_{w \in \mathbb{R}} \pi(w)$ for all w' on the support of F. This implies that $R(h^u) \geq w^u$ and that $R(h^u) > w^u$ if K has an atom at h^u. However, the reservation wage equation in this case implies the following:

$$R(h^u) - c(h^u) = b + \frac{\lambda}{\rho + \delta} \int_{R(h^u)}^{\infty} w - R(h^u)\, dF(w) = b, \tag{22}$$

so that $R(h^u) = b + c(h^u) < p$, given the assumptions that $c(h)$ is decreasing on $\mathbb{R}_+$ and $p > b + c(0)$. This contradicts the fact that $R(h^u) \geq w^u > p$.

Second, we further show that $w^u < p$. Suppose to the contrary that there exists an equilibrium in which $w^u = p$. Note that $\pi(w^u) = (p-p)\ell(p) = 0$. Since $b + c(0) < p = w^u$, we have for all $h \geq 0$:

$$p - c(0) > b = b + \frac{\lambda}{\rho + \delta} \int_{p}^{\infty} w - p\, dF(w). \tag{23}$$

Noting that the left-hand side of the reservation wage equation (6) is increasing in $R(h)$ and the right-hand side is nonincreasing in $R(h)$, it follows from the preceding inequality that $R(0) < p$ and, consequently, $R(h) < p$ for all $h \in \mathbb{R}_+$. Therefore, $\ell(w) = \lambda m(\infty)/\delta > 0$ and so $\pi(w) = (p - w)\ell(w) > 0$ whenever $R(0) < w < p$. This contradicts the equilibrium condition $w' \in \operatorname{argmax}_{w \in \mathbb{R}} \pi(w)$ for all w' on the support of F. ∎

I.4 Proof of Lemma 2

We first show that $R(h^u) \leq w^u$. Suppose to the contrary that there exists an equilibrium in which $R(h^u) > w^u$. Then it must be that $\ell(w) = 0$ and so $\pi(w) = 0$ for all w on the support of F. Moreover, the reservation wage equation in this case implies the following for all h on the support of K:

$$R(h) - c(h) = b + \frac{\lambda}{\rho + \delta} \int_{R(h)}^{\infty} w - R(h)\, dF(w) = b, \tag{24}$$

so that $R(h) = b + c(h) < p$, given the assumptions that $c(h)$ is decreasing on $\mathbb{R}_+$ and $p > b + c(0)$. It follows that $\ell[b + c(0)] = \lambda m(\infty)/\delta = \lambda n/\delta > 0$ and so $\pi[b + c(0)] = [p - b -$

$c(0)]\ell[b+c(0)] > 0$. However, this contradicts the equilibrium condition $w' \in \text{argmax}_{w\in\mathbb{R}} \pi(w)$ for all w' on the support of F.

We now proceed to show that $R(h^u) \leq w^l$. Suppose to the contrary that there exists an equilibrium in which $R(h^u) > w^l$. Then it must be that $\ell(w) = 0$ and so $\pi(w) = 0$ for any $w < R(h^u)$. Since $R(h^u) \leq w^u$ with $R(h)$ being continuous and decreasing in h, we have $J[R^{-1}(w^u + \epsilon)^-] < 1$ and so $\ell(w^u + \epsilon) > 0$ for all $\epsilon > 0$. Noting that $w^u < p$ in equilibrium, it follows that $\pi(w^u + \epsilon) = (p - w^u - \epsilon)\ell(w^u + \epsilon) > 0$ for $\epsilon \in (0, p - w^u)$. This contradicts the equilibrium condition $w' \in \text{argmax}_{w\in\mathbb{R}} \pi(w)$ for all w' on the support of F. ∎

I.5 Proof of Lemma 3

Suppose to the contrary that there exists an equilibrium in which $R(h^l) < w^u$. Then there exists $\hat{w} < w^u$ such that $\ell(\hat{w}) = \ell(w^u) = \lambda m(\infty)/\delta > 0$. It follows that:

$$\pi(w^u) = (p-w^u)\ell(w^u) = (p-w^u)\lambda m(\infty)/\delta < (p-\hat{w})\lambda m(\infty)/\delta = (p-\hat{w})\ell(\hat{w}) = \pi(\hat{w}). \quad (25)$$

However, this contradicts the equilibrium condition $w' \in \text{argmax}_{w\in\mathbb{R}} \pi(w)$ for all w' on the support of F. ∎

I.6 Proof of Lemma 4

For $h \leq h^l$, it follows from $R(h^l) \geq w^u$ that:

$$\int_{R(h)}^{\infty} w - R(h)\, dF(w) = 0, \quad (26)$$

whence the reservation wage equation reduces to $R(h) - c(h) = b$, and the expected payoff of a worker is given by $Y(h) = b/\rho - h$. Since $Y(h)$ is decreasing in h for $h \leq h^l$, it must be that $h^l = 0$, in order to satisfy the equilibrium condition $h' \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ for all h' on the support of K. Because $h^l \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ in equilibrium, we have $\max_{h\in\mathbb{R}_+} Y(h) = Y(h^l) = b/\rho$. ∎

I.7 Proof of Lemma 5

Suppose to the contrary that there exists an equilibrium in which $K(h') = K(h'')$ for some $h', h'' \in [0, h^u]$ with $h' < h''$. Let h^a be the least value of $h \in [0, h^u]$ such that $K(h) = K(h')$. Let h^b be the supremum of the set of values of $h \in [h^a, h^u]$ such that $K(h) = K(h^a)$. It must be that $\lim_{h \uparrow h^b} K(h) < 1$ for otherwise it would follow that $K(h^a) = 1$, contradicting the fact that $h^a < h^u$, where h^u is the supremum of the support of K.

Consequently, employment satisfies $\ell(w) = \ell[R(h^b)] > 0$ for all w such that $R(h^b) \leq w < R(h^a)$. It follows that for all w such that $R(h^b) \leq w < R(h^a)$, profits are given by $\pi(w) = (p - w)\ell[R(h^b)]$, which is positive and decreasing in w. In order to satisfy the equilibrium condition $w' \in \operatorname{argmax}_{w \in \mathbb{R}} \pi(w)$ for all w' on the support of F, it must be that $F(w) = F[R(h^b)]$ for all w such that $R(h^b) \leq w < R(h^a)$. Hence, we have for all $h \in [h^a, h^b]$:

$$\int_{R(h)}^{\infty} w - R(h)\, dF(w) = \lim_{z \downarrow h^a} \int_{R(z)}^{\infty} w - R(h)\, dF(w). \tag{27}$$

Thus, the reservation wage equation can be expressed as follows for all $h \in [h^a, h^b]$:

$$\begin{aligned} R(h) - c(h) &= b + \frac{\lambda}{\rho + \delta} \left(\lim_{z \downarrow h^a} \int_{R(z)}^{\infty} w - R(h)\, dF(w) \right) \\ &= b + \frac{\lambda}{\rho + \delta} \left(\lim_{z \downarrow h^a} \int_{R(z)}^{\infty} w\, dF(w) - R(h)\{1 - F[R(z)]\} \right), \end{aligned} \tag{28}$$

which after some rearrangement yields:

$$R(h) = \left(b + c(h) + \frac{\lambda}{\rho + \delta} \lim_{z \downarrow h^a} \int_{R(z)}^{\infty} w\, dF(w) \right) \Big/ \left(1 + \frac{\lambda}{\rho + \delta} \{1 - \lim_{z \downarrow h^a} F[R(z)]\} \right). \tag{29}$$

For all $h \in [h^a, h^b]$, the expected payoff of workers in equation (7) can now be written as:

$$Y(h) = \frac{\lambda\big(\lim_{z \downarrow h^a} \int_{R(z)}^{\infty} w\, dF(w) - c(h)\{1 - \lim_{z \downarrow h^a} F[R(z)]\}\big) + (\delta + \rho)b}{\rho\big(\lambda\{1 - \lim_{z \downarrow h^a} F[R(z)]\} + (\delta + \rho)\big)} - h. \tag{30}$$

Because h^a is on the support of K and in equilibrium $h' \in \operatorname{argmax}_{h \in \mathbb{R}_+} Y(h)$ for all h' on the support of K, it must be that:

$$Y^{'+}(h^a) = -c^{'}(h^a) \frac{\lambda\{1 - \lim_{z \downarrow h^a} F[R(z)]\}}{\rho\big(\lambda\{1 - \lim_{z \downarrow h^a} F[R(z)]\} + (\delta + \rho)\big)} - 1 \leq 0. \tag{31}$$

Since $c(h)$ is a convex function, we have for all $h \in (h^a, h^b)$:

$$Y^{'}(h) = -c^{'}(h) \frac{\lambda\{1 - \lim_{z \downarrow h^a} F[R(z)]\}}{\rho\big(\lambda\{1 - \lim_{z \downarrow h^a} F[R(z)]\} + (\delta + \rho)\big)} - 1 < 0, \tag{32}$$

and

$$Y^{'-}(h^b) = -c^{'}(h^b)\frac{\lambda\{1 - \lim_{z\downarrow h^a} F[R(z)]\}}{\rho\big(\lambda\{1 - \lim_{z\downarrow h^a} F[R(z)]\} + (\delta + \rho)\big)} - 1 < 0. \tag{33}$$

It follows from the preceding three inequalities that $Y(h^b) < Y(h^a)$. However, because h^b is on the support of K, this contradicts the equilibrium requirement $h' \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ for all h' on the support of K. ∎

I.8 Proof of Theorem 1

The proof is divided into several steps, which are described below.

I.8.1 Finding $R(h)$ for all $h \in [0, h^u]$

From lemma 4, $\max_{h\in\mathbb{R}_+} Y(h) = b/\rho$. In addition, lemmata 4 and 5 imply that the support of K is $[0, h^u]$. Hence, the equilibrium condition $h' \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ for all h' on the support of K is satisfied if and only if $Y(h) = b/\rho$ for all $h \in [0, h^u]$ and $Y(h) \leq b/\rho$ for all $h > h^u$. Since $Y(h) = R(h)/\rho - c(h)/\rho - h$, the condition $Y(h) = b/\rho$ for all $h \in [0, h^u]$ is equivalent to $R(h) = b + \rho h + c(h)$ for all $h \in [0, h^u]$.

I.8.2 Finding $F[R(h)]$ for all $h \in [0, h^u]$

Given the condition $R(h) = b + \rho h + c(h)$ for all $h \in [0, h^u]$, the reservation wage equation (6) can be expressed as follows for $h \in [0, h^u]$:

$$\rho h = \frac{\lambda}{\delta + \rho}\int_{b+\rho h+c(h)}^{\infty} w - [b + \rho h + c(h)]\, dF(w). \tag{34}$$

In the case where $h = 0$, equation (34) reduces to $\int_{b+c(0)}^{\infty} w - [b + c(0)]\, dF(w) = 0$, which implies that $F[b + c(0)] = 1$. Integration by parts yields the following equality for all $h \in [0, h^u]$:

$$\int_{b+\rho h+c(h)}^{b+c(0)} w - [b + \rho h + c(h)]\, dF(w) = [c(0) - c(h) - \rho h] - \int_{b+\rho h+c(h)}^{b+c(0)} F(w)\, dw, \tag{35}$$

so that equation (34) is as below for all $h \in (0, h^u]$:

$$\rho h = \frac{\lambda}{\delta + \rho}\bigg([c(0) - c(h) - \rho h] - \int_{b+\rho h+c(h)}^{b+c(0)} F(w)\, dw\bigg). \tag{36}$$

Taking the left derivative of each side of the preceding equation yields the following equivalent condition for all $h \in (0, h^u]$:

$$\rho = \frac{\lambda}{\delta + \rho}[\rho + c^{'}(h)]\{F[b + \rho h + c(h)] - 1\}. \tag{37}$$

Rearranging the equation above yields the following expression for $F[R(h)]$ where $h \in (0, h^u]$:

$$F[b + \rho h + c(h)] = 1 + \rho(\delta + \rho)/\{\lambda[c^{'}(h) + \rho]\}. \tag{38}$$

When the reservation wage is given by $R(h) = b + \rho h + c(h)$ for all $h \in [0, h^u]$, the reservation wage equation (6) is satisfied for all $h \in [0, h^u]$ if and only if F satisfies equation (38) for all $h \in (0, h^u]$ and $F[b + c(0)] = 1$.

I.8.3 Finding $R(h)$ for $h > h^u$

Noting that $R(h^u) \leq w^l$ in equilibrium from lemma 3, the reservation wage equation (6) is as follows for $h = h^u$:

$$\begin{aligned} R(h^u) - c(h^u) &= b + \frac{\lambda}{\delta + \rho} \int_{R(h^u)}^{\infty} w - R(h^u)\, dF(w) \\ &= b + \frac{\lambda}{\delta + \rho} \left(\int_{R(h^u)}^{\infty} w\, dF(w) - R(h^u) \right) \end{aligned} \tag{39}$$

and as follows for $h \geq h^u$:

$$\begin{aligned} R(h) - c(h) &= b + \frac{\lambda}{\delta + \rho} \int_{R(h^u)}^{\infty} w - R(h)\, dF(w) \\ &= b + \frac{\lambda}{\delta + \rho} \left(\int_{R(h^u)}^{\infty} w\, dF(w) - R(h) \right). \end{aligned} \tag{40}$$

Subtracting equation (39) from equation (40) yields the following for $h \geq h^u$ after some simplification:

$$R(h) = R(h^u) + (\delta + \rho)[c(h) - c(h^u)]/(\delta + \lambda + \rho). \tag{41}$$

Given that the reservation wage equation (6) holds for $R(h^u)$, $R(h)$ satisfies equation (6) for all $h \geq h^u$ if and only if $R(h)$ satisfies equation (41) for all $h \geq h^u$. Substituting the result $R(h^u) = b + \rho h^u + c(h^u)$ into equation (41) yields $R(h) = (b + \rho h^u) + [\lambda c(h^u) + (\delta + \rho)c(h)]/(\lambda + \delta + \rho)$ for $h \geq h^u$.

I.8.4 Finding h^u

When $h^u > 0$, there exists a well defined distribution function F satisfying equation (38) as well as $F[b+c(0)] = 1$ if and only if $F[b+\rho h^u+c(h^u)] \geq 0$ with F given by equation (38). The preceding inequality can be written as $c^{'}(h^u) \leq -\rho(\delta+\lambda+\rho)/\lambda$ after some rearrangement.

Given that $R(h) = b + \rho h + c(h)$ for all $h \in [0, h^u]$, in which case $Y(h) = b/\rho$ for all $h \in [0, h^u]$, the equilibrium condition $h' \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ for all h' on the support of K is satisfied if and only if $Y(h) \leq Y(h^u)$ for all $h \geq h^u$, where Y is related to R through equation (7). For all $h \geq h^u$, using equation (41) to substitute for $R(h)$, the inequality $Y(h) \leq Y(h^u)$ can be expressed as $[c(h) - c(h^u)]/(h - h^u) \geq -\rho(\delta+\lambda+\rho)/\lambda$, following some simplification and rearrangement. Because c is a convex function, the preceding condition holds for all $h \geq h^u$ if and only if $c^{'+}(h^u) \geq -\rho(\delta+\lambda+\rho)/\lambda$.

Combined with the requirement $c^{'}(h^u) \leq -\rho(\delta+\lambda+\rho)/\lambda$ when $h^u > 0$, it follows that $c^{'}(h^u) = -\rho(\delta+\lambda+\rho)/\lambda$ or, equivalently, $h^u = c^{'-1}[-\rho(\delta+\lambda+\rho)/\lambda]$ whenever $c^{'+}(0) < -\rho(\delta+\lambda+\rho)/\lambda$.

I.8.5 Finding w^l, w^u, and $F(w)$

Since $h^u = c^{'-1}[-\rho(\delta+\lambda+\rho)/\lambda] > 0$ in equilibrium, the result that $F[b+\rho h+c(h)]$ satisfies equation (38) for $h \in (0, h^u]$ implies that $w^l = b + \rho h^u + c(h^u)$, and since $F[b+c(0)] = 1$ as well, it must be that $w^u = b + c(0)$. Given that $R(h) = b + \rho h + c(h)$ for $h \in [0, h^u]$ with $R(h) = (b + \rho h^u) + [\lambda c(h^u) + (\delta+\rho)c(h)]/(\lambda+\delta+\rho)$ for $h > h^u$, equation (38) can through a change of variables be expressed as $F(w) = 1 + \rho(\delta+\rho)/\big(\lambda\{c^{'}[R^{-1}(w)] + \rho\}\big)$ for all $w \in [w^l, w^u)$. It follows from the analysis so far that the equilibrium requirements that $R(h)$ satisfies equation (6) for all $h \in \mathbb{R}_+$ and that $h' \in \text{argmax}_{h\in\mathbb{R}_+} Y(h)$ for all h' on the support of K hold if and only if R and F are as specified in the statement of the theorem.

I.8.6 Finding $K(h)$

Assume that R and F are as specified in the statement of the theorem. The steady state employment of a firm offering wage w is given by:

$$\ell(w) = \begin{cases} 0, & \text{for } w < w^l \\ n\frac{\lambda}{\delta}\left(\lim_{h\uparrow R^{-1}(w)} \int_h^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z)\right), & \text{for } w \in [w^l, w^u)\,. \\ n\frac{\lambda}{\delta}\left(\int_0^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z) + \frac{\delta[c^{'+}(0)+\rho]}{\delta c^{'+}(0)-\rho^2} K(0)\right), & \text{for } w \geq w^u \end{cases} \tag{42}$$

Since $\ell(w^u) > 0$ and $p < w^u$, the steady state profit flow satisfies both $\pi(w^u) = (p - w^u)\ell(w^u) > (p-w)\ell(w^u) = \pi(w)$ for all $w > w^u$ and $\pi(w^u) = (p-w^u)\ell(w^u) > 0 = (p-w)0 = \pi(w)$ for $w < w^l$. Consequently, the equilibrium condition $w' \in \operatorname{argmax}_{w\in\mathbb{R}} \pi(w)$ for all w' on the support of F is satisfied if and only if $\pi(w) = (p-w)\ell(w) = (p-w^u)\ell(w^u) = \pi(w^u)$ for all $w \in [w^l, w^u)$.

Using equation (42) to substitute for $\ell(w)$ and $\ell(w^u)$, this condition can be expressed as:

$$\lim_{h\uparrow R^{-1}(w)} \int_h^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2} dK(z) = \frac{p-b-c(0)}{p-w}\left(\int_0^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2} dK(z) + \frac{\delta[c^{'+}(0)+\rho]}{\delta c^{'+}(0)-\rho^2} K(0)\right), \tag{43}$$

where $w \in [w^l, w^u)$. Taking the limit of each side of equation (43) as w approaches w^u from below results in $K(0) = 0$ after some simplification. Setting $w = w^l$ in equation (43) and simplifying yields:

$$\frac{\delta}{\delta+\lambda}[1 - K(h^{u^-})] = \frac{p-b-c(0)}{p-b-\rho h^u - c(h^u)} \int_0^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z). \tag{44}$$

It also follows from equation (43) that for any $w \in (w^l, w^u)$:

$$\lim_{v\uparrow w} \lim_{h\uparrow R^{-1}(v)} \int_h^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z) = \lim_{h\uparrow R^{-1}(w)} \int_h^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z), \tag{45}$$

which implies that $K[R^{-1}(w)] = K[R^{-1}(w)^-]$ whenever $w \in (w^l, w^u)$.

For $w \in (w^l, w^u)$, equation (43) can be written as:

$$\int_{R^{-1}(w)}^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z) = \frac{p-b-c(0)}{p-w} \int_0^\infty \frac{\delta[c^{'}(z)+\rho]}{\delta c^{'}(z)-\rho^2}\, dK(z), \tag{46}$$

which can equivalently be expressed as follows through a change of variables:

$$\int_h^\infty \frac{\delta[c'(z)+\rho]}{\delta c'(z)-\rho^2}\,dK(z) = \frac{p-b-c(0)}{p-b-\rho h-c(h)}\int_0^\infty \frac{\delta[c'(z)+\rho]}{\delta c'(z)-\rho^2}\,dK(z), \tag{47}$$

where $h \in (h^l, h^u)$. Note that the expression on the left-hand side of equation (47) must be differentiable on (h^l, h^u) because the expression on the right-hand side of the equation is differentiable on this interval. Differentiating each side of equation (47) yields the following condition for all $h \in (h^l, h^u)$ after some simplification and rearrangement:

$$K'(h) = \frac{[p-b-c(0)][\rho^2-\delta c'(h)]}{[p-b-\rho h-c(h)]^2}\int_0^\infty \frac{c'(z)+\rho}{\delta c'(z)-\rho^2}\,dK(z), \tag{48}$$

which is equivalent to equation (47) given equation (44). Noting that $K(0)=0$, equation (48) implies that:

$$K(h^{u^-}) = \int_0^{h^u} \frac{[p-b-c(0)][\rho^2-\delta c'(z)]}{[p-b-\rho z-c(z)]^2}\,dz \int_0^\infty \frac{c'(z)+\rho}{\delta c'(z)-\rho^2}\,dK(z). \tag{49}$$

Substituting this expression for $K(h^{u^-})$ into equation (44) and rearranging results in the following:

$$\int_0^\infty \frac{c'(z)+\rho}{\delta c'(z)-\rho^2}\,dK(z) = \frac{p-R(h^u)}{[p-R(h^u)]S(h^u)+[p-R(h^l)](\delta+\lambda)}, \tag{50}$$

where the function S is as defined in the statement of the theorem. The expression for K in the statement of the theorem follows from combining equation (50) with equation (48). Hence, the equilibrium condition $w' \in \operatorname{argmax}_{w\in\mathbb{R}} \pi(w)$ for all w' on the support of F is satisfied if and only if K is as specified in the statement of the theorem. ∎

I.9 Proof of Corollary 1

In equilibrium, $b+c(0)$ is on the support of F, and so $b+c(0) \in \operatorname{argmax}_{w\in\mathbb{R}} \pi(w)$. Hence, the equilibrium profit flow of an employer for any $w \in [w^l, w^u]$ is given by:

$$\pi(w) = \pi[b+c(0)] = [p-b-c(0)]\ell[b+c(0)] = [p-b-c(0)]\lambda m(\infty)/\delta, \tag{51}$$

where unemployment $m(\infty)$ is equal to:

$$m(\infty) = n\left(\int_0^{\infty} \frac{\delta}{\delta + \lambda\{1 - F[R(z)^-]\}}\, dK(z) + \frac{\delta}{\delta + \lambda\{1 - F[R(0)^-]\}} K(0)\right). \tag{52}$$

From theorem 1, we have $F[R(z)^-] > 0$ for all $z \in [0, h^u)$, where $h^u > 0$ and $K(h^{u^-}) > 0$. It follows from equation (52) that $m(\infty) > n\delta/(\delta + \lambda)$, so that the equilibrium profit flow in equation (51) satisfies $\pi(w) > n[p - b - c(0)]\lambda/(\delta + \lambda)$ for any $w \in [w^l, w^u]$.

Substituting the expressions for R, F, and K from theorem 1 into equation (52) results in the following after some simplification:

$$m(\infty) = \frac{n\delta[(p - w^u) - (p - w^l)T(h^u)]}{(p - w^l)S(h^u) + (p - w^u)(\delta + \lambda)}, \tag{53}$$

with $T(h^u)$ being defined as:

$$T(h^u) = (p - w^u)\int_0^{h^u} \frac{c^{'}(z) + \rho}{[p - R(z)]^2}\, dz = (p - w^u)\left(\frac{1}{p - w^l} - \frac{1}{p - w^u}\right), \tag{54}$$

where the second equality follows from applying the second fundamental theorem of calculus. Using equation (53) with $T(h^u)$ given by equation (54) to substitute for $m(\infty)$ in equation (51) results in the profit level $\pi(w) = \{n(p - w^l)(p - w^u)\lambda\}\{(p - w^l)S(h^u) + (p - w^u)(\delta + \lambda)\}$ for all $w \in [w^l, w^u]$. ∎

I.10 Proof of Proposition 3

Consider a wage posting and skill investment equilibrium, in which K is as stated in theorem 1. We begin by introducing some notation. The function m in equation (9) can be expressed as follows for $h \geq 0$ by substituting the values of F and R from theorem 1 into the expression for u in equation (8):

$$m(h) = n\left(\int_0^{h} \frac{\delta[c^{'}(z) + \rho]}{\delta c^{'}(z) - \rho^2}\, dK(z)\right), \tag{55}$$

noting that K as defined in theorem 1 does not have an atom at $h = 0$. Hence, the function J is given by $J(h) = \int_0^h v(z)\, dK(z) / \int_0^{\infty} v(z)\, dK(z)$ for $h \geq 0$, where we denote $v(z) = [c^{'}(z) + \rho]/[\delta c^{'}(z) - \rho^2]$. Substituting the value of K from theorem 1, this yields:

$$J(h) = 1 - \frac{p - b}{p - b - \rho h - c(h)} \tag{56}$$

for $0 \le h < h^u$ with $J(h^u) = 1$.

Let $h^l(\psi)$ and $h^u(\psi)$ respectively denote the infimum and supremum of the support of K as a function of a given parameter ψ of the model. Let $R(h;\psi)$ for $0 \le h \le h^u(\psi)$ represent the reservation wage function and its dependence on the parameter ψ. The distribution function K can be expressed as follows for $0 \le h < h^u(\psi)$ to make its dependence on the parameter ψ explicit:

$$K(h;\psi) = \frac{\{p - R[h^u(\psi);\psi]\}X(h;\psi)}{\{p - R[h^u(\psi);\psi]\}X[h^u(\psi);\psi] + (\delta+\lambda)} \tag{57}$$

with $K[h^u(\psi);\psi] = 1$, where we define $X(h;\psi) = \int_0^h [\rho^2 - \delta c'(z)]/[p - R(z;\psi)]^2\,dz$ for $0 \le h \le h^u(\psi)$.

In addition, we use the notation $J(h;\psi)$ to make explicit the dependence of J on the parameter ψ. Also let $k(h;\psi)$ and $j(h;\psi)$ be the partial probability density functions of $K(h;\psi)$ and $J(h;\psi)$ for $0 \le h \le h^u(\psi)$, where k and j are defined by $k(h) = K'(h)$ and $j(h) = J'(h)$ on $[0, h^u)$. Finally, the notation $v(z;\psi)$ is used to represent the dependence of the function v on the parameter ψ.

I.10.1 Effect of p

We show that $K(h;p)$ improves in the sense of FOSD if p is higher. Partially differentiating the expression for K in equation (57) with respect to p yields the following after some simplification:

$$\frac{\partial K(h;p)}{\partial p} = \frac{(\delta+\lambda)\big(X(h;p) - 2\{p - R[h^u(p);p]\}Y(h;p)\big) + 2\{p - R[h^u(p);p]\}^2 Z(h;p)}{\big(\{p - R[h^u(p);p]\}X[h^u(p);p] + (\delta+\lambda)\big)^2}, \tag{58}$$

where we define $Y(h;p) = \int_0^h [\rho^2 - c'(z)]/[p - R(z;p)]^3\,dz$ and $Z(h;p) = X(h;p)Y[h^u(p);p] - X[h^u(p);p]Y(h;p)$ for $0 \le h \le h^u(p)$. The expression in equation (58) is equal to zero at $h = 0$.

Consider the case where $0 < h < h^u(p)$. First, it follows from the property $p - R[h^u(p);p] > p - R(z;p) > 0$ for $0 \le z < h^u(p)$ that $X(h;p) - 2\{p - R[h^u(p);p]\}Y(h;p) < -X(h;p)$, which is negative.

Second, $Z(h;p)$ has the same sign as $Y[h^u(p);p]/X[h^u(p);p] - Y(h;p)/X(h;p)$, which approaches zero in the limit as h approaches $h^u(p)$. Letting x and y denote the respective derivatives of X and Y with respect to h, it follows from the property $p - R(h;p) > p - R(z;p)$

for $0 \leq z < h$ that $[X(h;p)y(h;p) - Y(h;p)x(h;p)]/[X(h;p)]^2 < 0$, so that $Y(h;p)/X(h;p)$ is decreasing in h for $0 < h < h^u(p)$. Thus, $Y[h^u(p);p]/X[h^u(p);p] - Y(h;p)/X(h;p)$ is negative for $0 < h \leq h^u(p)$.

It follows that the expression in equation (58) is negative for $0 < h < h^u(p)$. Moreover, $h^u(p)$ is independent of p.

I.10.2 Effect of b

We show that $K(h;b)$ improves in the sense of FOSD if b is lower. Partially differentiating the expression for K in equation (57) with respect to b yields $\partial K(h;b)/\partial b = -\partial K(h;p)/\partial p$, which is zero at $h = 0$ and positive for $0 < h \leq h^u(b)$. Moreover, $h^u(b)$ is independent of b.

I.10.3 Effect of λ

We show that $K(h;\lambda)$ improves in the sense of FOSD if λ is higher. The distribution function J satisfies $J(h;\lambda) = \int_0^h v(z)\, dK(z;\lambda)/\int_0^\infty v(z)\, dK(z;\lambda)$ for $0 \leq h < h^u(\lambda)$, which yields the following with some rearrangement after differentiating with respect to h:

$$k(h;\lambda) = \frac{j(h;\lambda)}{v(h)} \int_0^\infty v(z)\, dK(z;\lambda), \tag{59}$$

where:

$$\int_0^\infty v(z)\, dK(z;\lambda) = \lim_{h \uparrow h^u(\lambda)} \int_0^h v(z)k(z;\lambda)\, dz + \{1 - K[h^u(\lambda)^-;\lambda]\}v[h^u(\lambda)]. \tag{60}$$

Choose any $\lambda', \lambda'' > 0$ such that $\lambda' > \lambda''$. Note that $h^u(\lambda'') < h^u(\lambda')$. Since $j(h;\lambda)$ is independent of λ and $v(h) > 0$ for $0 \leq h < h^u(\lambda)$, the sign of $k(h;\lambda'') - k(h;\lambda')$ is the same as that of $\int_0^\infty v(z)\, dK(z;\lambda'') - \int_0^\infty v(z)\, dK(z;\lambda')$ for all $0 \leq h < h^u(\lambda'')$. Since $J(h;\lambda)$ is independent of λ for $0 \leq h < h^u(\lambda)$, we have $J[h^u(\lambda'')^-;\lambda''] = J[h^u(\lambda'');\lambda']$, which requires that:

$$\begin{aligned} &\frac{\int_0^{h^u(\lambda'')} v(z)k(z;\lambda')\, dz}{\lim_{h\uparrow h^u(\lambda')} \int_0^h v(z)k(z;\lambda')\, dz + \{1 - K[h^u(\lambda')^-;\lambda']\}v[h^u(\lambda')]} \\ &\qquad = \frac{\lim_{h\uparrow h^u(\lambda'')} \int_0^h v(z)k(z;\lambda'')\, dz}{\lim_{h\uparrow h^u(\lambda'')} \int_0^h v(z)k(z;\lambda'')\, dz + \{1 - K[h^u(\lambda'')^-;\lambda'']\}v[h^u(\lambda'')]} \end{aligned}. \tag{61}$$

Since $v(h)$ is decreasing in h for $0 \leq h < h^u(\lambda)$, we have:

$$\frac{\int_0^{h^u(\lambda'')} v(z)k(z;\lambda')\,dz}{\lim_{h\uparrow h^u(\lambda')}\int_0^h v(z)k(z;\lambda')\,dz + \{1-K[h^u(\lambda')^-;\lambda']\}v[h^u(\lambda')]} > \frac{\int_0^{h^u(\lambda'')} v(z)k(z;\lambda')\,dz}{\int_0^{h^u(\lambda'')} v(z)k(z;\lambda')\,dz + \{1-K[h^u(\lambda'');\lambda']\}v[h^u(\lambda'')]}. \tag{62}$$

Hence, the requirement in equation (61) is satisfied only if:

$$\lim_{h\uparrow h^u(\lambda'')}\int_0^h v(z)k(z;\lambda'')\,dz > \int_0^{h^u(\lambda'')} v(z)k(z;\lambda')\,dz, \tag{63}$$

which implies that $k(h;\lambda'') > k(h;\lambda')$ for all $0 \leq h < h^u(\lambda'')$, recalling that $k(h;\lambda'') - k(h;\lambda')$ must have the same sign for all $0 \leq h < h^u(\lambda'')$.

I.10.4 Effect of δ

We show that $K(h;\delta)$ improves in the sense of FOSD if δ is lower. The distribution function J satisfies $J(h;\delta) = \int_0^h v(z;\delta)\,dK(z;\delta)/\int_0^\infty v(z;\delta)\,dK(z;\delta)$ for $0 \leq h < h^u(\delta)$, which yields the following with some rearrangement after differentiating with respect to h:

$$k(h;\delta) = \frac{j(h;\delta)}{v(h;\delta)}\int_0^\infty v(z;\delta)\,dK(z;\delta), \tag{64}$$

where:

$$\int_0^\infty v(z;\delta)\,dK(z;\delta) = \lim_{h\uparrow h^u(\delta)}\int_0^h v(z;\delta)k(z;\delta)\,dz + \{1-K[h^u(\delta)^-;\delta]\}v[h^u(\delta);\delta]. \tag{65}$$

Noting that $j(h;\delta)$ is independent of δ for $0 \leq h < h^u(\delta)$, it follows that:

$$k_\delta(h;\delta)/k(h;\delta) = y_\delta(0,\infty;\delta)/y(0,\infty;\delta) - v_\delta(h;\delta)/v(h;\delta), \tag{66}$$

where we denote $y(z^l, z^u;\delta) = \int_{z^l}^{z^u} v(z;\delta)\,dK(z;\delta)$. Since $v_\delta(h;\delta)/v(h;\delta)$ is increasing in h for $0 \leq h < h^u(\delta)$, there exists $\tilde{h}(\delta)$ with $0 \leq \tilde{h}(\delta) \leq h^u(\delta)$ such that $k(h;\delta)$ is increasing in δ for $0 < h < \tilde{h}(\delta)$ and decreasing in δ for $h^u(\delta) > h > \tilde{h}(\delta)$.

Choose any $\delta', \delta'' > 0$ such that $\delta' < \delta''$. Note that $h^u(\delta'') < h^u(\delta')$. Since $J(h;\delta)$ is independent of δ for $0 \leq h < h^u(\delta)$, we have $J[h^u(\delta'')^-;\delta''] = J[h^u(\delta'');\delta']$, which requires that:

$$\frac{\int_0^{h^u(\delta'')} v(z;\delta')k(z;\delta')\,dz}{\lim_{h\uparrow h^u(\delta')}\int_0^h v(z;\delta')k(z;\delta')\,dz + \{1-K[h^u(\delta')^-;\delta']\}v[h^u(\delta');\delta']} = \frac{\lim_{h\uparrow h^u(\delta'')}\int_0^h v(z;\delta'')k(z;\delta'')\,dz}{\lim_{h\uparrow h^u(\delta'')}\int_0^h v(z;\delta'')k(z;\delta'')\,dz + \{1-K[h^u(\delta'')^-;\delta'']\}v[h^u(\delta'');\delta'']}. \quad (67)$$

Since $v(h;\delta)$ is decreasing in h and $v_\delta(h;\delta)/v(h;\delta)$ is increasing in h for $0 \leq h < h^u(\delta)$, we have:

$$\frac{\int_0^{h^u(\delta'')} v(z;\delta')k(z;\delta')\,dz}{\lim_{h\uparrow h^u(\delta')}\int_0^h v(z;\delta')k(z;\delta')\,dz + \{1-K[h^u(\delta')^-;\delta']\}v[h^u(\delta');\delta']} > \frac{\int_0^{h^u(\delta'')} v(z;\delta'')k(z;\delta')\,dz}{\int_0^{h^u(\delta'')} v(z;\delta'')k(z;\delta')\,dz + \{1-K[h^u(\delta'');\delta']\}v[h^u(\delta'');\delta'']}. \quad (68)$$

Hence, the requirement in equation (67) is satisfied only if there exists h with $0 \leq h < h^u(\delta)$ such that $k(h,\delta'') > k(h,\delta')$.

Recalling that there exists $\tilde{h}(\delta)$ with $0 \leq \tilde{h}(\delta) \leq h^u(\delta)$ such that $k(h;\delta)$ is increasing in δ for $0 < h < \tilde{h}(\delta)$ and decreasing in δ for $h^u(\delta) > h > \tilde{h}(\delta)$, it must be that either $k(h;\delta'') > k(h;\delta')$ for all $0 < h < h^u(\delta'')$ or there exists $\hat{h}$ with $0 \leq \hat{h} \leq h^u(\delta'')$ such that $k(h;\delta'') > k(h;\delta')$ for $0 < h < \hat{h}$ and $k(h;\delta'') < k(h;\delta')$ for $h^u(\delta'') > h > \hat{h}$. In the former case, it follows immediately that K improves in the sense of FOSD when δ decreases from δ'' to δ'. In the latter case, this conclusion follows if $K[h^u(\delta'')^-;\delta''] \geq K[h^u(\delta'');\delta']$.

Suppose to the contrary that $K[h^u(\delta'')^-;\delta''] < K[h^u(\delta'');\delta']$. Recalling that $J(h;\delta) = y(0,h;\delta)/y(0,\infty;\delta)$ for all $0 \leq h < h^u(\delta)$ where $J(h;\delta)$ is independent of δ, it must be that for all $\delta \in [\delta',\delta'']$:

$$\frac{y_\delta(0,\infty;\delta)}{y(0,\infty;\delta)} = \frac{y_\delta[\hat{h},h^u(\delta'')^-;\delta]}{y[\hat{h},h^u(\delta'')^-;\delta]} = \frac{y_\delta[h^u(\delta'')^-,h^u(\delta');\delta]}{y[h^u(\delta'')^-,h^u(\delta');\delta]}, \quad (69)$$

which implies that:

$$\frac{\lim_{h\uparrow h^u(\delta'')}\int_{\hat{h}}^h v(z;\delta'')k(z;\delta'')\,dz}{\int_{\hat{h}}^{h^u(\delta'')} v(z;\delta')k(z;\delta')\,dz} = \frac{y[\hat{h},h^u(\delta'')^-;\delta'']}{y[\hat{h},h^u(\delta'');\delta']} = \frac{y[h^u(\delta'')^-,h^u(\delta');\delta'']}{y[h^u(\delta''),h^u(\delta');\delta']} = \frac{\{1-K[h^u(\delta'')^-;\delta'']\}v[h^u(\delta'');\delta'']}{\lim_{h\uparrow h^u(\delta')}\int_{h^u(\delta'')}^h v(z;\delta')k(z;\delta')\,dz + \{1-K[h^u(\delta')^-;\delta']\}v[h^u(\delta');\delta']}. \quad (70)$$

Since $v(h;\delta)$ is decreasing in h and $v_\delta(h;\delta)/v(h;\delta)$ is increasing in h for $0 \leq h < h^u(\delta)$, we have:

$$\frac{\lim_{h\uparrow h^u(\delta'')} \int_{\hat{h}}^{h} v(z;\delta'')k(z;\delta')\,dz}{\int_{\hat{h}}^{h^u(\delta'')} v(z;\delta')k(z;\delta')\,dz} < \frac{\{1-K[h^u(\delta'');\delta']\}v[h^u(\delta'');\delta'']}{\lim_{h\uparrow h^u(\delta')} \int_{h^u(\delta'')}^{h} v(z;\delta')k(z;\delta')\,dz + \{1-K[h^u(\delta')^-;\delta']\}v[h^u(\delta');\delta']}. \quad (71)$$

Hence, the requirement in equation (67) is satisfied only if $K[h^u(\delta'')^-;\delta''] > K[h^u(\delta'');\delta']$.

I.10.5 Effect of ρ

We show that $K(h;\rho)$ may neither improve nor worsen in the sense of FOSD if ρ is higher. Noting that c is convex, $c^{'-1}$ is an increasing function, so that $h^u(\rho)$ is decreasing in ρ.

Now set the parameter values at $\lambda = 1$, $\delta = 1$, $p = 1$, $b = 0$. Let $c : \mathbb{R}_+ \to \mathbb{R}$ be any cost function that satisfies the conditions in assumption 1 and has the form $c(h) = 99/100 - h + h^2$ for $0 \leq h \leq [1-\rho(2+\rho)]/2$ with $\rho \in (0, \sqrt{2}-1)$. Let $\rho' = 1/6$ and $\rho'' = 1/5$. Then we have $0.9026 \approx K(1/4;\rho') > K(1/4;\rho'') \approx 0.9015$.

It follows that neither $K(h,\rho')$ FOSD $K(h,\rho'')$ nor $K(h,\rho'')$ FOSD $K(h,\rho')$. ∎

I.11 Proof of Proposition 4

Consider a wage posting and skill investment equilibrium, in which F is as stated in theorem 1. Let ψ be a given parameter of the model. The notation $F(w;\psi)$ is used for the distribution function so as to express its dependence on ψ. By a change in variables, F satisfies the following for $h \in (0, h^u]$:

$$F[b + \rho h + c(h);\psi] = 1 + \frac{\rho(\delta+\rho)}{\lambda[c'(h)+\rho]}, \quad (72)$$

and also $F[b + c(0);\psi] = 1$. Let $w^l(\psi) = b + \rho h^u(\psi) + c[h^u(\psi)]$ and $w^u(\psi) = b + c(0)$ respectively denote the infimum and supremum of the support of F as a function of ψ, where $h^u(\psi)$ is as described in the proof of proposition 3.

I.11.1 Effect of p

It follows by inspection of the expression for the equilibrium wage offer distribution that $F(w;p') = F(w;p'')$ for any p' and p''.

I.11.2 Effect of b

Clearly, $w^u(b)$ is increasing in b. Noting that $h^u(b)$ is independent of b, $w^l(b)$ is increasing in b. Partial differentiation of equation (72) with respect to b yields the following for $0 < h < h^u(b)$:

$$\frac{\partial F[b + \rho h + c(h); b]}{\partial w} + \frac{\partial F[b + \rho h + c(h); b]}{\partial b} = 0, \tag{73}$$

where $\partial F[b + \rho h + c(h); b]/\partial w > 0$ and so $\partial F[b + \rho h + c(h); b]/\partial b < 0$ for $0 < h < h^u(b)$. Hence, $\partial F(w; b)/\partial b < 0$ for $w^l(b) < w < w^u(b)$. It follows that $F(w; b')$ FOSD $F(w; b'')$ if $b' > b''$.

I.11.3 Effect of λ

Clearly, $w^u(\lambda)$ is independent of λ. Noting that $h^u(\lambda)$ is increasing in λ, $w^l(\lambda)$ is decreasing in λ as $c^{'}[h^u(\lambda)] = -\rho(\delta + \lambda + \rho)/\lambda < -\rho$. Partial differentiation of equation (72) with respect to λ yields the following for $0 < h < h^u(\lambda)$:

$$\frac{\partial F[b + \rho h + c(h); \lambda]}{\partial \lambda} = -\frac{\rho(\delta + \rho)}{\lambda^2[c^{'}(h) + \rho]}, \tag{74}$$

which is positive because $c^{'}(h) \leq -\rho(\delta + \lambda + \rho)/\lambda < -\rho$ for $0 \leq h \leq h^u(\lambda)$. Hence, $\partial F(w; \lambda)/\partial \lambda > 0$ for $w^l(\lambda) < w < w^u(\lambda)$. It follows that $F(w; \lambda'')$ FOSD $F(w; \lambda')$ if $\lambda' > \lambda''$.

I.11.4 Effect of δ

Clearly, $w^u(\delta)$ is independent of δ. Noting that $h^u(\delta)$ is decreasing in δ, $w^l(\delta)$ is increasing in δ as $c^{'}[h^u(\delta)] = -\rho(\delta + \lambda + \rho)/\lambda < -\rho$. Partial differentiation of equation (72) with respect to δ yields the following for $0 < h < h^u(\delta)$:

$$\frac{\partial F[b + \rho h + c(h); \delta]}{\partial \delta} = \frac{\rho}{\lambda[c^{'}(h) + \rho]}, \tag{75}$$

which is negative because $c^{'}(h) \leq -\rho(\delta + \lambda + \rho)/\lambda < -\rho$ for $0 \leq h \leq h^u(\delta)$. Hence, $\partial F(w; \delta)/\partial \delta < 0$ for $w^l(\delta) < w < w^u(\delta)$. It follows that $F(w; \delta')$ FOSD $F(w; \delta'')$ if $\delta' > \delta''$.

I.11.5 Effect of ρ

Clearly, $w^u(\rho)$ is independent of ρ. Moreover, $w^l(\rho)$ is increasing in ρ since $\rho h + c(h)$ is increasing in ρ for all $h > 0$ and since $\rho + c^{'}[h^u(\rho)] = -\rho(\delta + \rho)/\lambda < 0$ where $h^u(\rho)$ is decreasing in ρ. Partial differentiation of equation (72) with respect to ρ yields the following for $0 < h < h^u(\rho)$:

$$h\frac{\partial F[b+\rho h+c(h);\rho]}{\partial w}+\frac{\partial F[b+\rho h+c(h);\rho]}{\partial \rho}=\frac{\rho^2+(\delta+2\rho)c^{'}(h)}{\lambda[c^{'}(h)+\rho]^2}<0, \tag{76}$$

where the inequality follows from the fact that $c^{'}(h) \leq -\rho(\delta+\lambda+\rho)/\lambda < -\rho$ for $0 \leq h \leq h^u(\rho)$. Since $\partial F[b+\rho h+c(h);\rho]/\partial w > 0$ in the preceding inequality, we have $\partial F[b+\rho h+c(h);\rho]/\partial \rho < 0$ for $0 < h < h^u(\rho)$. Hence, $\partial F(w;\rho)/\partial \rho < 0$ for $w^l(\rho) < w < w^u(\rho)$. It follows that $F(w;\rho')$ FOSD $F(w;\rho'')$ if $\rho' > \rho''$. ∎

I.12 Proof of Proposition 5

Consider a wage posting and skill investment equilibrium, in which K is as stated in theorem 1. The notation is the same as in the proof of proposition 3. In addition, we specify $c(h) = \chi + \kappa\hat{c}(h)$.

I.12.1 Effect of χ

We show that $K(h;\chi)$ improves in the sense of FOSD if χ is lower. Partially differentiating the expression for K in equation (57) with respect to χ yields $\partial K(h;\chi)/\partial\chi = -\partial K(h;p)/\partial p$, which is zero at $h = 0$ and positive for $0 < h \leq h^u(\chi)$, where $\partial K(h;p)/\partial p$ is as characterized in the proof of proposition 3. Moreover, $h^u(\chi)$ is independent of χ.

I.12.2 Effect of κ

We show that $K(h;\kappa)$ may neither improve nor worsen in the sense of FOSD if κ is higher. Noting that $\hat{c}$ is convex, $\hat{c}^{'-1}$ is an increasing function, so that $h^u(\kappa)$ is increasing in κ.

Now set the parameter values at $\lambda = 1$, $\delta = 1$, $\rho = 1/5$, $p = 2$, $b = 0$, $\chi = 1$. Let $\hat{c} : \mathbb{R}_+ \to \mathbb{R}$ be any function that satisfies the conditions in assumption 1 and has the form $\hat{c}(h) = -h + h^2$ for $0 \leq h \leq 1/2 - 11\kappa/50$ with $\kappa \in (0, 11/25)$. Let $\kappa' = 1$ and $\kappa'' = 2$. Then we have $0.0894 \approx K(1/4,\kappa') < K(1/4,\kappa'') \approx 0.1396$.

It follows that neither $K(h,\kappa')$ FOSD $K(h,\kappa'')$ nor $K(h,\kappa'')$ FOSD $K(h,\kappa')$. ∎

I.13 Proof of Proposition 6

Consider a wage posting and skill investment equilibrium, in which F is as stated in theorem 1. The notation is the same as in the proof of proposition 4. In addition, we specify

$c(h) = \chi + \kappa\hat{c}(h)$.

I.13.1 Effect of χ

Clearly, $w^u(\chi)$ is increasing in χ. Noting that $h^u(\chi)$ is independent of χ, $w^l(\chi)$ is increasing in χ. Partial differentiation of equation (72) with respect to χ yields the following for $0 < h < h^u(\chi)$:

$$\frac{\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \chi]}{\partial w} + \frac{\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \chi]}{\partial \chi} = 0, \tag{77}$$

where $\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \chi]/\partial w > 0$ and so $\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \chi]/\partial \chi < 0$ for $0 < h < h^u(\chi)$. Hence, $\partial F(w; \chi)/\partial \chi < 0$ for $w^l(\chi) < w < w^u(\chi)$. It follows that $F(w; \chi')$ FOSD $F(w; \chi'')$ if $\chi' > \chi''$.

I.13.2 Effect of κ

Clearly, $w^u(\kappa)$ is independent of κ. Moreover, $w^l(\kappa)$ is decreasing in κ since $\rho h + \kappa\hat{c}(h)$ is decreasing in κ for all $h > 0$ and $\rho > 0$ where $\hat{c}(h) < 0$ for all $h > 0$ and since $\rho + \kappa\hat{c}'[h^u(\kappa)] = -\rho(\delta + \rho)/\lambda < 0$ where $h^u(\kappa)$ is increasing in κ. Partial differentiation of equation (72) with respect to κ yields the following for $0 < h < h^u(\kappa)$:

$$\hat{c}(h)\frac{\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \kappa]}{\partial w} + \frac{\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \kappa]}{\partial \kappa} = -\frac{\rho(\delta + \rho)\hat{c}'(h)}{\lambda[\kappa\hat{c}'(h) + \rho]^2} > 0, \tag{78}$$

where the inequality follows from the fact that $\hat{c}'(h) \leq -\rho(\delta + \lambda + \rho)/(\lambda\kappa) < -\rho/\kappa$ for $0 \leq h \leq h^u(\kappa)$. Since $\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \kappa]/\partial w > 0$ and $\hat{c}(h) < 0$ in the preceding inequality, we have $\partial F[b + \rho h + \chi + \kappa\hat{c}(h); \kappa]/\partial \kappa > 0$ for $0 < h < h^u(\kappa)$. Hence, $\partial F(w; \kappa)/\partial \kappa > 0$ for $w^l(\kappa) < w < w^u(\kappa)$. It follows that $F(w; \kappa'')$ FOSD $F(w; \kappa')$ if $\kappa' > \kappa''$. ∎

I.14 Proof of Theorem 2

Let c be any disutility function satisfying the assumptions in section 2.1. If (R, K, F) is a wage posting and skill investment equilibrium, then K and R must be as defined in the statement of theorem 1. Assume, therefore, that K with support $[h^l, h^u]$ and R are as defined in the statement of theorem 1.

The triple (R, K, F) is a wage posting and skill investment equilibrium if and only if c

satisfies the following for all $w \in [w^l, w^u)$:

$$F(w) = 1 + \rho(\delta+\rho)/\big(\lambda\{c^{'}[R^{-1}(w)] + \rho\}\big), \tag{79}$$

where the infimum and supremum of the support of F are respectively given by $w^l = b + \rho h^u + c(h^u)$ and $w^u = b + c(0)$. Through a change of variables the preceding condition can be expressed as:

$$F[b + \rho h + c(h)] = 1 + \rho(\delta+\rho)/\{\lambda[c^{'}(h) + \rho]\}, \tag{80}$$

which is equivalent to:

$$\int_{b+\rho h+c(h)}^{b+c(0)} 1 - F(z)dz = h\rho(\delta+\rho)/\lambda, \tag{81}$$

where $h \in [0, h^u]$. Given that c satisfies $w^l = b + \rho h^u + c(h^u)$ and $w^u = b + c(0)$, the equation above implies that $h^u = \lambda \int_{w^l}^{w^u} 1 - F(z)dz/[\rho(\delta+\rho)]$. Letting h^u be equal to this value, the triple (R, K, F) is an equilibrium if and only if c satisfies the following for all $h \in [0, h^u]$:

$$\int_{b+\rho h+c(h)}^{w^u} 1 - F(z)dz = h\rho(\delta+\rho)/\lambda. \tag{82}$$

For every $h \in [0, h^u]$, the left-hand side of equation (82) is continuously decreasing in $c(h)$ from $h^u\rho(\delta+\rho)/\lambda$ when $c(h)$ equals $w^l - b - \rho h$ to 0 when $c(h)$ equals $w^u - b - \rho h$, and the right-hand side does not depend on $c(h)$. Hence, there is a unique value of $c(h)$ that solves this equation given any $h \in [0, h^u]$, and this equation yields the explicit solution for c on $[0, h^u]$ in the statement of the theorem. Moreover, given any value of $c(h)$, the left-hand side of equation (82) is continuous in h, and so is the right-hand side. Hence, the solution c to this equation is continuous in h.

For $h \geq h^u$, c can, for example, be defined as:

$$c(h) = w^l - b - \rho h^u + \lambda/[2\rho(\delta+\lambda+\rho)] - \sqrt{h - h^u + \{\lambda/[2\rho(\delta+\lambda+\rho)]\}^2}, \tag{83}$$

the derivative of which is increasing from $-\rho(\delta+\lambda+\rho)/\lambda$ for $h = h^u$ to 0 in the limit as h goes to ∞.

Differentiating each side of equation (82) yields the following result for $h \in (0, h^u]$ after some simplification and rearrangement:

$$c^{'}(h) = -\rho\big[1 + (\delta+\rho)/\big(\lambda\{1 - F[b + \rho h + c(h)]\}\big)\big]. \tag{84}$$

Since $F[b + \rho h + c(h)] < 1$ is decreasing in h for $h \in (0, h^u]$ with $F(w^l) = 0$, $c^{'}(h) > -\infty$ is increasing in h for $h \in (0, h^u]$ with $c^{'}(h^u) = -\rho(\delta + \lambda + \rho)/\lambda$. Given the definition of c for $h > h^u$, it follows that c is convex and differentiable on $\mathbb{R}_{++}$ with $\lim_{h\uparrow\infty} c^{'}(h) = 0$.

If F does not have an atom at w^u, then we have $\lim_{h\downarrow 0} c^{'}(h) = -\infty$ from equation (84). If F has an atom at w^u, then differentiation of equation (82) results in the following after some algebraic manipulation:

$$c^{'+}(0) = -\rho\big(1 + (\delta + \rho)/\{\lambda[1 - F(w^{u-})]\}\big), \tag{85}$$

which is greater than $-\infty$ but less than $-\rho(\delta + \lambda + \rho)/\lambda$.

Hence, the function c as defined above satisfies assumption 1. ∎

I.15 Proof of Theorem 3

I.15.1 Uniform Convergence of Reservation Wage Function

Let R^* be as defined in the statement of the theorem. Denote the equilibrium reservation wage function in theorem 1 by R_λ so as to make its dependence on λ explicit, and let h^u_λ be the supremum of the support of the equilibrium skill distribution when the offer arrival rate is λ . The reservation wage satisfies $R_\lambda(h) = R^*(h)$ for all $\lambda > 0$ and any $h \in [0, h^u_\lambda]$.

Noting that $\rho + \lambda c^{'}(h^u_\lambda)/(\delta + \lambda + \rho) = 0$ by the definition of h^u_λ where c is convex, the expression $\rho h + \lambda c(h)/(\delta + \lambda + \rho)$ is increasing in h on $[h^u_\lambda, \infty)$. Hence, for all $\lambda > 0$ and any $h \in [h^u_\lambda, h^*]$, we have:

$$\begin{aligned} \mid R_\lambda(h) - R^*(h) \mid &= \rho(h - h^u_\lambda) + \lambda[c(h) - c(h^u_\lambda)]/(\lambda + \delta + \rho) \\ &\leq \rho(h^* - h^u_\lambda) + \lambda[c(h^*) - c(h^u_\lambda)]/(\lambda + \delta + \rho). \end{aligned} \tag{86}$$

In the limit as λ tends to ∞, h^u_λ converges to h^* where c is continuous, so the expression on the right-hand side of the preceding inequality goes to 0. Hence, R_λ converges uniformly to R^* on the interval $[h^u_\lambda, h^*]$ as λ goes to ∞.

Moreover, given any $E \subset \mathbb{R}_+$, the following holds for all $\lambda > 0$ and any $h \in E \cap [h^*, \infty)$:

$$\begin{aligned} \mid R_\lambda(h) - R^*(h) \mid &= \rho(h^* - h^u_\lambda) + \{\lambda[c(h^*) - c(h^u_\lambda)] + (\delta + \rho)[c(h^*) - c(h)]\}/(\lambda + \delta + \rho) \\ &\leq \rho(h^* - h^u_\lambda) + \{\lambda[c(h^*) - c(h^u_\lambda)] + (\delta + \rho)[c(h^*) - c^l]\}/(\lambda + \delta + \rho), \end{aligned} \tag{87}$$

where $c^l = \inf\{c(h) : h \in E \cap [h^*, \infty)\}$. In the limit as λ tends to ∞, h^u_λ converges to h^* where c is continuous, so the expression on the right-hand side of the preceding inequality goes to 0 if $c^l > -\infty$. Hence, R_λ converges uniformly to R^* on the set $E \cap [h^*, \infty)$ as λ goes to ∞.

Since any $E \subset \mathbb{R}_+$ with $\inf\{c(h) : h \in E\} > -\infty$ can be expressed as a finite union of sets on which R_λ converges uniformly to R^*, R_λ converges uniformly to R^* on E as λ goes to ∞.

I.15.2 Convergence in Probability of Skills and Wage Offers

Let the distribution function K^* be defined as follows:

$$K^*(h) = \begin{cases} 0, & \text{for } \ h < h^* \\ 1, & \text{for } \ h \geq h^* \end{cases}, \tag{88}$$

where h^* is as specified in the statement of the theorem. Denote by K_λ the equilibrium skill distribution in theorem 1 so as to make explicit its dependence on the parameter λ.

In the limit as λ goes to ∞, h^u_λ converges to h^*. Moreover, $R_\lambda(h^u_\lambda)$ converges to $b + \rho h^* + c(h^*) < p$ in the limit by the continuity of c. Letting S_λ represent the function S in theorem 1 for a given value of λ, we have $S_\lambda(h^u_\lambda) > 0$ for all $\lambda > 0$, and for any $h < h^u_\lambda$, we have $S_{\hat{\lambda}}(h) = S_\lambda(h) > 0$ for all $\hat{\lambda} > \lambda$. Hence, taking the limit of the expression for the skill distribution in theorem 1, we have $\lim_{\lambda \uparrow \infty} K_\lambda(h) = K^*(h)$ for all $h \in \mathbb{R}_+$. Thus, skills converge in distribution to the constant h^*.

Let the distribution function F^* be defined as follows:

$$F^*(w) = \begin{cases} 0, & \text{for } \ w < w^* \\ 1, & \text{for } \ w \geq w^* \end{cases}, \tag{89}$$

where w^* is as specified in the statement of the theorem. Denote by F_λ the equilibrium wage offer distribution in theorem 1 so as to make explicit its dependence on the parameter λ, and let w^l_λ be the infimum of the support of the equilibrium wage offer distribution when the offer arrival rate is λ.

In the limit as λ goes to ∞, h^u_λ converges to h^*, and so w^l_λ converges to w^* by the continuity of c. Moreover, for any $\lambda > 0$ and all $w \in (w^l_\lambda, w^u)$, we have $R^{-1}_{\hat{\lambda}}(w) = R^{-1}_\lambda(w) \in (0, h^u_\lambda)$ for all $\hat{\lambda} > \lambda$, and so $c'[R^{-1}_{\hat{\lambda}}(w)] = c'[R^{-1}_\lambda(w)] < -\rho$ for all $\hat{\lambda} > \lambda$ given the convexity of c. Hence, taking the limit of the expression for the skill distribution in theorem 1, we have $\lim_{\lambda \uparrow \infty} F_\lambda(w) = F^*(w)$ for all $w \in \mathbb{R}$ such that $w \neq w^*$, where F^* has a discontinuity. Thus, wage offers converge in distribution to the constant w^*.

Note that convergence in distribution to a constant implies convergence in probability to the same constant.

I.15.3 Worker and Firm Welfare in the Limit

In the limit as λ goes to ∞, h^u_λ converges to h^*, and so w^l_λ converges to $b + \rho h^* + c(h^*) < p$ by the continuity of c. Moreover, $S_\lambda(h^u_\lambda)$ converges to $[p - b - c(0)] \int_0^{h^*} [\rho^2 - \delta c^{'}(z)]/[p - b - \rho h - c(h)]^2 dz < \infty$ as λ tends to ∞. Hence, taking the limit as λ goes to ∞ of the expression in corollary 1 for the maximum equilibrium profits yields $n[p - b - \rho h^* - c(h^*)]$.

From lemma 4, the expected payoff of a worker satisfies $\max_{h \in \mathbb{R}_+} Y(h) = b/\rho$. ∎

II Calibration

II.1 Parameter Values

Each parameter is calibrated as specified below using the 1980 employer survey database from the EOPP along with standard government statistics. The sample weights in the EOPP are applied to account for differences in the sampling probability across employers. The database contains information on 5918 employers.

II.1.1 Labor Productivity p

The gross sales receipts in the previous quarter of each employer in the EOPP are annualized by multiplying by four. This is then divided by the current employment of a firm to obtain the annual sales per employee. The median of this value is taken across firms. The GDP deflator is used to express this value in current dollars.

II.1.2 Value of Leisure b

The average weekly unemployment benefit amount in 1980 is obtained from the Monthly Program and Financial Data for the UI system, which are released by the Employment and Training Administration of the US Department of Labor. This value is annualized by multiplying by 52 and expressed in current dollars using the CPI.

II.1.3 Discount Rate ρ

The discount rate is set equal to 0.05 on an annual basis as is common in the literature (e.g., Farboodi *et al.* 2021).

II.1.4 Job Destruction Rate δ

The number of employees who quit or were fired, permanently laid off, or temporarily let go in the previous quarter is computed for each employer in the EOPP and annualized by multiplying by four. This value is then divided by the current number of employees at a firm to obtain a separation rate for each firm. The average of this value is taken across firms.

II.1.5 Offer Arrival Rate λ

The number of new hires in the previous quarter is computed for each employer in the EOPP and annualized by multiplying by four. This value is then multiplied by the number of offers that the firm made to fill its last vacancy, which yields the number of job offers per firm. This is converted to the number of job offers per worker by multiplying by the ratio of the number of firms to the number of workers, where the number of firms is calculated by dividing the total number of employed persons by the average number of employees at a firm in the EOPP, and the number of workers is set equal to the number of persons in the labor force. The employment level and labor force size are obtained from the Bureau of Labor Statistics.

II.2 Wage Density

The annual wage of the typical worker at each employer is calculated as follows. Each employer in the EOPP is asked to report the number of workers in six broad occupational groups as well as the minimum and maximum hourly wage for workers in each group. The midrange of the wage distribution for each occupational category is calculated by averaging the minimum and maximum wages. The weighted average of the resulting values is then calculated, where the weights are proportional to the number of employees in each occupational category. This is then expressed in current dollars using the CPI and converted to annual terms by multiplying by 40 times 52.

A probability density function is fitted to the respective wage data in Baton Rouge, Louisiana and Outagamie, Wisconsin. Adaptive kernel density estimation is used as in Abramson (1982) with an Epanechnikov kernel. The EOPP contains information on 377 employers in Baton Rouge and 63 employers in Outagamie.