Asymmetric Information and Search Frictions: A Neutrality Result

Neel Rao*
University at Buffalo, SUNY

August 26, 2016

Abstract

This paper integrates asymmetric information between firms into a canonical model of on-the-job search. Workers are heterogeneous in ability, but not all employers observe a worker's type. Wage dispersion caused by search frictions makes the equilibrium wage distribution insensitive to informational asymmetries. Hence, the equilibrium outcome may be the same as when a worker's ability is known to every firm. The supportability of the full information outcome depends on market parameters related to productivity, knowledge, and search. The theoretical results elucidate an empirical puzzle about demographic differences in asymmetric information between employers.

JEL Classification: D82, D83, J31, J63
Keywords: asymmetric information, human capital, equilibrium search, wage dispersion

*E-mail: neelrao@buffalo.edu. Tel: 716-645-8674. Address: 423 Fronczak Hall, Buffalo, NY 14260.
I thank Lawrence Katz, Alan Manning, and an anonymous referee for helpful comments.

1 Introduction

Informational frictions can have significant implications for the efficiency of labor markets and the equilibrium distribution of wages. A seminal paper by Stigler (1961) ascribes price dispersion to search frictions, and a classic article by Akerlof (1970) illustrates market failure from asymmetric information. Depending on how such mechanisms interact, there may be a role for government intervention to improve welfare. In some cases, adverse selection radically alters market outcomes. In others, informational imperfections may not constrain the attainable equilibria.

This paper studies asymmetric information between employers in a search model based on Burdett and Mortensen (1998). Workers differ in productivity, but only some firms can detect such variation. A necessary and sufficient condition is derived for the existence of an equilibrium achieving the full information outcome, which is the wage distribution that arises when all firms observe worker ability. This outcome is supportable because there exists a nonempty set of wages that are optimal for an employer to offer workers regardless of their types. When search is random, such a result can be obtained, even though no signalling or screening occurs, and workers are employed at both informed and uninformed firms in equilibrium.

The analysis contributes to research on asymmetric information in search models. Albrecht and Vroman (1992) demonstrate how private information causes the nonexistence of an equilibrium in symmetric pure strategies. Guerrieri, Shimer, and Wright (2010) examine competitive search with adverse selection. Carrillo-Tudela and Kaas (2015) present a search model in which asymmetric information affects wages and mobility. The last paper considers only two types of workers and assumes that ability becomes contractible. By contrast, ability is noncontractible in the current model, which accommodates an arbitrary number of worker types. Moreover, the neutrality result derived here is novel to the literature. The implementability of the full information outcome is an important question in mechanism design and contract theory.

The findings also illuminate a puzzle in empirical studies. Schönberg (2007) detects asymmetric information between employers of college graduates but not less educated workers. Likewise, Hu and Taber (2011) observe adverse selection in the labor market for white males but not women or blacks. The model here can generate differential evidence of asymmetric information across groups with dissimilar search parameters. In a population with high unemployment due to a low offer arrival rate or a high job destruction rate, informational asymmetries may not affect the equilibrium wage distribution. In the opposite situation, the full information outcome may not be supportable.

2 Model

The labor market comprises a continuum of agents in continuous time. The measures of firms and workers are respectively 1 and $m > 0$. Unemployed and employed workers receive the respective flow payoffs $b \geq 0$ and $w \geq 0$, where b and w correspond to the unemployment benefit and the current wage. The arrival of job offers to workers follows a Poisson process with rate parameter $\lambda > 0$. An employed searcher accepts an offer if and only if it exceeds the current wage, and an unemployed individual accepts a job if and only if it pays at least b. Matches between workers and firms are destroyed at Poisson rate $\delta > 0$, in which case a worker transits from employment to unemployment.

Workers vary in general ability. There is a finite number $N > 1$ of worker types, indexed by the set $S = \{1, 2, \ldots, N\}$. Let $m_i > 0$ signify the measure of type i workers, where $\sum_{i=1}^{N} m_i = m$. Let $\theta_i > b$ denote the flow of output produced by a type i worker, where $\theta_1 < \theta_2 < \ldots < \theta_N$. Define $\theta^l = \theta_1$ and $\theta^u = \theta_N$.

Firms differ in information about workers.[1] Letting $p \in [0, 1]$, a fraction p of employers are informed I, and a fraction $1 - p$ are uninformed O. An informed firm observes

[1]Firms might vary in the screening and monitoring of workers. See Mansour (2012) for an analysis of occupational differences in employer learning.

a worker's ability and offers a wage w_i contingent on worker type. An uninformed firm does not know a worker's productivity and posts a wage w_O irrespective of worker type. The wage offered by an employer to a worker is constant over time.[2]

Let H_i represent the distribution of wages that type I firms offer to workers of type $i \in S$. Denote the collection $\{H_1, H_2, \ldots, H_N\}$ by C_I. Let H_O be the distribution of wage offers across type O employers. Accordingly, type i workers sample wage offers from $F_i = pH_i + (1-p)H_O$, which is the distribution of all wages offered to type i workers. In addition, define $F_i(\omega^-) = \lim_{\upsilon \uparrow \omega} F_i(\upsilon)$ for $\omega > 0$, and let $F_i(0^-) = 0$.

Several properties of a steady state can now be described.[3] Equating the flows into and out of employment, the fraction of workers unemployed is:

$$u_i = \frac{\delta}{\delta + \lambda[1 - F_i(b^-)]}. \tag{1}$$

The distribution of wages across employed workers of type $i \in S$ can be expressed as follows for $w \in \mathbb{R}_+$:

$$G_i(w) = \frac{\delta[F_i(w) - F_i(b^-)]}{[1 - F_i(b^-)]\{\delta + \lambda[1 - F_i(w)]\}}, \tag{2}$$

which is obtained by equating the flow of workers from unemployment into jobs paying no more than w with the flow from such jobs into unemployment or higher paying jobs. Let $\ell_i(w|H_O, H_i)$ denote the measure of type $i \in S$ workers employed at a firm offering them a wage $w \in \mathbb{R}_+$. Equating the flow of workers recruited to and separating from

[2]Burdett and Coles (2003) as well as Stevens (2004) examine wage contracts that depend on tenure.

[3]The ensuing formulae for the unemployment rate, wage distribution, and employment level correspond to equations (7), (9), and (10) in Burdett and Mortensen (1998), who provide more details regarding their derivation.

an employer paying w, the following holds for $w \geq b$:

$$\ell_i(w|H_O, H_i) = \frac{\delta\lambda m_i}{\{\delta + \lambda[1 - F_i(w)]\}\{\delta + \lambda[1 - F_i(w^-)]\}}, \tag{3}$$

and $\ell_i(w|H_O, H_i) = 0$ for $w < b$.

The payoff functions are next specified in steady state. The profit flow from type $i \in S$ workers employed at a firm paying them a wage $w \in \mathbb{R}_+$ is:

$$\pi_i(w|H_O, H_i) = (\theta_i - w)\ell_i(w|H_O, H_i). \tag{4}$$

The profit of a type O firm offering the wage w_O to each worker is:

$$\Pi_O(w_O|H_O, C_I) = \sum_{i=1}^{N} \pi_i(w_O|H_O, H_i). \tag{5}$$

The profit of a type I firm posting the wage vector $v_I = (w_1, w_2, \dots, w_N)$ is:

$$\Pi_I(v_I|H_O, C_I) = \sum_{i=1}^{N} \pi_i(w_i|H_O, H_i), \tag{6}$$

where w_i is the wage paid by the firm to any worker of type $i \in S$.

3 Equilibrium

Employers make wage offers so as to maximize their profits in steady state, given the distribution of wages across workers of each type. An uninformed employer is constrained to post a uniform wage, whereas an informed employer may condition the wage on worker type. Formally, an equilibrium consists of a wage offer distribution H_O for type O firms and a collection $C_I = \{H_1, H_2, \dots, H_N\}$ of wage offer distributions for type I firms such that: $w_O \in \operatorname{argmax}_{w \in \mathbb{R}_+} \Pi_O(w|H_O, C_I)$ for all w_O in the support of H_O;

$v_I = (w_1, w_2, \ldots, w_N) \in \text{argmax}_{v \in \mathbb{R}_+^N} \Pi_I(v|H_O, C_I)$ if w_i is in the support of H_i for all $i \in S$.

Suppose that all employers observe worker ability, in which case $p = 1$ representing full information. The resulting model is analytically equivalent to Burdett and Mortensen (1998), who derive a unique equilibrium outcome. From their equation (16), the equilibrium distribution of all wages offered to type $i \in S$ workers is as follows for $w \in (w_i^l, w_i^u)$:

$$K_i(w) = \frac{\delta + \lambda}{\lambda} \left(1 - \sqrt{\frac{\theta_i - w}{\theta_i - b}}\right), \tag{7}$$

where $K_i(w) = 0$ for $w \leq w_i^l$, $K_i(w) = 1$ for $w \geq w_i^u$, and the infimum and supremum of the support are respectively $w_i^l = b$ and

$$w_i^u = \theta_i - (\theta_i - b) \left(\frac{\delta}{\delta + \lambda}\right)^2. \tag{8}$$

Now let $p \in [0, 1]$. An equilibrium (H_O, C_I) is said to achieve the full information outcome if and only if $pH_i(w) + (1 - p)H_O(w) = K_i(w)$ for all $w \in \mathbb{R}_+$ and any $i \in S$. That is, the distribution of all wages offered to workers of each type is the same as when every firm is informed about worker type.

4 Existence

The theorem below identifies a threshold for invariance to asymmetric information. See the appendix for the proof.

Theorem 1 *An equilibrium achieving the full information outcome exists if and only if $p \geq \phi$, where the cutoff ϕ is given by:*

$$\phi = \sqrt{\frac{\delta^2(\theta^l - b) + (\delta + \lambda)^2(\theta^u - \theta^l)}{\lambda^2(\theta^u - b)}} - \frac{\delta}{\lambda}. \tag{9}$$

The full information outcome is such that any wage in the interval $[b, w_1^u]$ is optimal for an employer to offer a worker irrespective of ability. Hence, this outcome can be implemented in equilibrium as follows if p is not excessively small. The distribution H_O of wages offered by uninformed employers is specified so that its support is the interval $[b, w_1^u]$. The distribution H_i of wages offered to workers of type $i \in S$ is defined so that the mixture distribution $pH_i + (1-p)H_O$ is the same as the full information distribution K_i.

The resulting probability $pH_N(w_1^u) + (1-p)H_O(w_1^u)$ of a type N worker being offered a wage no greater than w_1^u is at least $(1-p)$. However, this probability exceeds the full information level $K_N(w_1^u)$ if p is too low, in which case the preceding construction is infeasible.

The online appendix generalizes the preceding theorem to allow the arrival rate of job offers to differ between employed and unemployed individuals. Although more complex, the statement and proof are similar, involving the existence of a nondegenerate interval of wages that are optimal for an employer to offer workers regardless of their types. The existence of such an interval under the full information outcome provides a necessary and sufficient condition for the full information outcome to be supportable when a positive but sufficiently small fraction of firms are uninformed. The full information outcome has this property whenever employed workers have an arrival rate of job offers no lower than that of unemployed workers. This property also holds in some but not all cases in which the arrival rate is greater for unemployed than for employed workers.

5 Properties

The ensuing corollary presents comparative statics for the threshold ϕ. Since the proof simply involves differentiation, it is omitted.

Corollary 1 *The cutoff* $\phi \in (0,1)$ *is decreasing in* δ *and* θ^l *as well as increasing in* λ, θ^u, *and* b. *Moreover,* $\lim_{\theta^l \to \theta^u} \phi = 0$ *and* $\lim_{\theta^l \to b} \phi = 1$. *Finally,* $\lim_{\lambda \to 0} \phi = \lim_{\delta \to \infty} \phi = \kappa$ *and* $\lim_{\lambda \to \infty} \phi = \lim_{\delta \to 0} \phi = \sqrt{\kappa}$, *where* $\kappa = (\theta^u - \theta^l)/(\theta^u - b) \in (0,1)$.

Under full information, there is a nonempty set of wages that are optimal for a firm to offer all workers, no matter their types. A higher job destruction rate or minimum ability level increases the probability of a worker being offered a wage in this set, enabling the full information outcome to be supported with fewer informed employers. The opposite holds for the job arrival rate, maximum ability level, and unemployment benefit.

With asymmetric information, the full information outcome is sustainable as the least and greatest abilities converge but not as minimum productivity approaches the unemployment benefit. This outcome may or may not be achievable for extreme values of the job arrival or destruction rate.

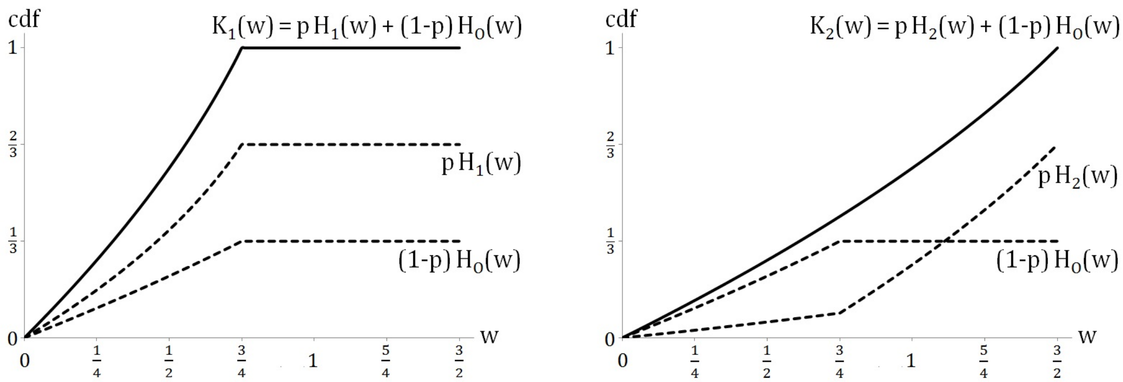
6 Example

An equilibrium insensitive to informational asymmetries is specified and illustrated. Setting $N = 2$, assume that $\theta_1 = 1$, $\theta_2 = 2$, and $b = 0$. Let $\delta = \lambda = 1$. From equation (7) along with expression (8), the full information outcome is as follows. The distribution of all wages offered to type 1 workers is $K_1(w) = 2 - 2\sqrt{1-w}$ for $w \in [0, \frac{3}{4}]$, with $K_1(w) = 1$ for $w > \frac{3}{4}$. The distribution of all wages offered to type 2 workers is $K_2(w) = 2 - 2\sqrt{1 - w/2}$ for $w \in [0, \frac{3}{2}]$, with $K_2(w) = 1$ for $w > \frac{3}{2}$.

Given these parameter values, $\phi = \sqrt{\frac{5}{2}} - 1$ in equation (9). Choose $p = \frac{2}{3}$, whence the existence condition $p \geq \phi$ is satisfied. The following is an example of an equilibrium achieving the full information outcome. The distribution of wage offers across type O employers is $\tilde{H}_O(w) = K_2(w)/K_2(\frac{3}{4})$ for $w \in [0, \frac{3}{4}]$, with $\tilde{H}_O(w) = 1$ for $w > \frac{3}{4}$. The distribution of wages offered by type I firms to workers of type $i \in \{1, 2\}$ is $\tilde{H}_i(w) = \frac{3}{2}K_i(w) - \frac{1}{2}\tilde{H}_O(w)$ for $w \in \mathbb{R}_+$.

Figure 1: Equilibrium $(H_O, \{H_1, H_2\})$ Achieving Full Information Outcome (K_1, K_2)

(a) CDF of Wages Offered to Type 1 Workers (b) CDF of Wages Offered to Type 2 Workers



Market Parameters: $N = 2$, $\theta_1 = 1$, $\theta_2 = 2$, $b = 0$, $\delta = 1$, $\lambda = 1$, $p = \frac{2}{3}$.

The relationship among these distributions is depicted in Figure 1. In such an equilibrium, the sampling distribution $pH_i + (1 - p)H_O$ coincides with the full information outcome K_i, where $i \in \{1, 2\}$. Moreover, the support of H_O is contained in the intersection of the supports of K_1 and K_2.

7 Conclusion

The implementation of the full information outcome is examined in a search model with asymmetric information between firms. A pertinent question for future research concerns the existence and uniqueness of equilibria that do not achieve this outcome. The analysis is complicated by potential discontinuities in the profit functions of firms.[4] Possible extensions include heterogeneity among workers in search parameters or among firms in job productivity. More complex wage contracts and information structures might also be investigated.

[4] Dasgupta and Maskin (1986) study the equilibria of games with discontinuous payoff functions.

References

Akerlof, G. A. (1970): "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84(3), 488–500.

Albrecht, J. W., and S. B. Vroman (1992): "Non-Existence of Single-Wage Equilibria in Search Models with Adverse Selection," *Review of Economic Studies*, 59(3), 617–624.

Burdett, K., and M. Coles (2003): "Equilibrium Wage-Tenure Contracts," *Econometrica*, 71(5), 1377–1404.

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39(2), 257–273.

Carrillo-Tudela, C., and L. Kaas (2015): "Worker Mobility in a Search Model with Adverse Selection," *Journal of Economic Theory*, 160, 340–386.

Dasgupta, P., and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory," *Review of Economic Studies*, 53(1), 1–26.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse Selection in Competitive Search Equilibrium," *Econometrica*, 78(6), 1823–1862.

Hu, L., and C. Taber (2011): "Displacement, Asymmetric Information, and Heterogeneous Human Capital," *Journal of Labor Economics*, 29(1), 113–152.

Mansour, H. (2012): "Does Employer Learning Vary by Occupation?," *Journal of Labor Economics*, 30(2), 415–444.

Schönberg, U. (2007): "Testing for Asymmetric Employer Learning," *Journal of Labor Economics*, 25(4), 651–691.

Stevens, M. (2004): "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention," *Review of Economic Studies*, 71(2), 535–551.

Stigler, G. J. (1961): "The Economics of Information," *Journal of Political Economy*, 69(3), 213–225.

Appendix

Below is the proof of the main theorem. The first part demonstrates by construction the existence of an equilibrium achieving the full information outcome when $p \geq \phi$. Nonexistence of such an equilibrium for $p < \phi$ is shown in the second part.

I Sufficiency

Let ξ represent the constant $1/K_N(w_1^u)$, where K_N and w_1^u are given by equations (7) and (8). Define the distribution function $\tilde{H}_O$ by $\tilde{H}_O(w) = 0$ if $w < w_1^l$, $\tilde{H}_O(w) = 1$ if $w > w_1^u$, and $\tilde{H}_O(w) = \xi K_N(w)$ for $w \in [w_1^l, w_1^u]$. For $i \in S$, the distribution function $\tilde{H}_i$ is specified as $\tilde{H}_i(w) = 0$ if $w < w_i^l$, $\tilde{H}_i(w) = 1$ if $w > w_i^u$, and $\tilde{H}_i(w) = [K_i(w) - (1-p)\tilde{H}_O(w)]/p$ for $w \in [w_i^l, w_i^u]$. Denote $\tilde{C}_I = \{\tilde{H}_1, \tilde{H}_2, \ldots, \tilde{H}_N\}$.

It follows directly from the definition above that $\tilde{H}_O(w) = 0$ for $w \leq w_1^l$, $\tilde{H}_O(w) = 1$ for $w \geq w_1^u$, and $\tilde{H}_O$ is continuously increasing on the interval $[w_1^l, w_1^u]$. Assume that $p \geq \phi$, where ϕ is given by equation (9). For $i \in S$, it is straightforward to confirm from the specification above that $\tilde{H}_i$ is continuous and nondecreasing on the interval $[w_i^l, w_i^u]$ with $\tilde{H}_i(w) = 0$ for $w \leq w_i^l$ and $\tilde{H}_i(w) = 1$ for $w \geq w_i^u$. Therefore, the distribution functions are well defined. Moreover, they are constructed such that $p\tilde{H}_i(w) + (1-p)\tilde{H}_O(w) = K_i(w)$ for all $w \in \mathbb{R}_+$ and any $i \in S$, where K_i is the equilibrium distribution of all wages offered to type $i \in S$ workers in the case where

$p = 1$.

For any $i \in S$, the preceding sentence implies that $w_i \in \text{argmax}_{w \in \mathbb{R}_+} \pi_i(w|\tilde{H}_O, \tilde{H}_i)$ if $w_i \in [w_i^l, w_i^u]$. Note that for each $i \in S$, the support of $\tilde{H}_i$ is a subset of $[w_i^l, w_i^u]$. Hence, $v_I = (w_1, w_2, \ldots, w_N) \in \text{argmax}_{v \in \mathbb{R}_+^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$ if w_i belongs to the support of $\tilde{H}_i$ for all $i \in S$. Let x_w be an N-vector each of whose elements is $w \in \mathbb{R}_+$. Because $[w_1^l, w_1^u]$ is a subset of $[w_i^l, w_i^u]$ for all $i \in S$, it follows that $x_w \in \text{argmax}_{v \in \mathbb{R}_+^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$ if $w \in [w_1^l, w_1^u]$. Note that $\max_{w \in \mathbb{R}_+} \Pi_O(w|\tilde{H}_O, \tilde{C}_I) \leq \max_{v \in \mathbb{R}_+^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$, $\Pi_O(w|\tilde{H}_O, \tilde{C}_I) = \Pi_I(x_w|\tilde{H}_O, \tilde{C}_I)$ if $w \in [w_1^l, w_1^u]$, and the support of $\tilde{H}_O$ is $[w_1^l, w_1^u]$. Thus, $w_O \in \text{argmax}_{w \in \mathbb{R}_+} \Pi_O(w|\tilde{H}_O, \tilde{C}_I)$ for all w_O in the support of $\tilde{H}_O$.

Hence, $(\tilde{H}_O, \tilde{C}_I)$ is an equilibrium that achieves the full information outcome for $p \geq \phi$.

II Necessity

Assume that (H_O, C_I) is an equilibrium achieving the full information outcome. Let $p < 1$. It must be that $H_O(w_1^u) = 1$. Otherwise, if $H_O(w_1^u) < 1$, then $p + (1-p)H_O(w_1^u) < 1$, which would contradict the requirement that $pH_1(w_1^u) + (1-p)H_O(w_1^u) = K_1(w_1^u)$. It follows that $pH_N(w_1^u) + (1-p) = K_N(w_1^u)$, which implies that $p \geq 1 - K_N(w_1^u)$. Using equations (7) and (8) to substitute for K_N and w_1^u in the previous inequality results in $p \geq \phi$ after some algebraic manipulation, where ϕ is defined in equation (9). Since $\phi < 1$, note that $p \geq \phi$ in case $p = 1$.