Online Appendix to
"Asymmetric Information and Search Frictions: A Neutrality Result"

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Abstract

The online appendix extends the analysis to the case where the arrival rate of job offers varies between employed and unemployed workers. Sections A and B explain how to generalize the model of search and the definition of equilibrium to accommodate this case. The extended version of the main theorem is stated in section C, and the corresponding proof is provided in section D.

A Model

The labor market comprises a continuum of agents in continuous time. The measures of firms and workers are respectively 1 and $m > 0$. Unemployed and employed workers obtain the respective flow payoffs b and w, where b and w correspond to the unemployment benefit and the current wage. The arrival of job offers to unemployed individuals follows a Poisson process with rate parameter $\lambda_o > 0$, and employed workers receive job offers at Poisson rate $\lambda_e > 0$. An employed searcher accepts an offer if and only if it exceeds his or her current wage, and an unemployed individual accepts a job if and only if it pays no less than his or her reservation wage. Matches between workers and firms are destroyed at Poisson rate $\delta > 0$, in which case a worker transits from employment to unemployment.

Workers vary in general ability. There is a finite number $N > 1$ of worker types, indexed by the set $S = \{1, 2, \ldots, N\}$. Let $m_i > 0$ signify the measure of type i workers, where $\sum_{i=1}^{N} m_i = m$. Let $\theta_i > b$ denote the flow of output produced by a type i worker, where $\theta_1 < \theta_2 < \ldots < \theta_N$. Define $\theta^l = \theta_1$ and $\theta^u = \theta_N$.

Firms differ in information about workers. Letting $p \in [0, 1]$, a fraction p of employers are informed I, and a fraction $1 - p$ are uninformed O. An informed firm observes a worker's ability and offers a wage w_i contingent on worker type. An uninformed firm does not know a worker's productivity and posts a wage w_O irrespective of worker type. The wage offered by an employer to a worker is constant over time.

Let H_i represent the distribution of wages that type I firms offer to workers of type $i \in S$. Denote the collection $\{H_1, H_2, \ldots, H_N\}$ by C_I. Let H_O be the distribution of wage offers across type O employers. Accordingly, type i workers sample wage offers from $F_i = pH_i + (1 - p)H_O$, which is the distribution of all wages offered to type i workers. Unemployed workers of type i have a common reservation wage R_i, which depends on F_i. In addition, define $F_i(\omega^-) = \lim_{\upsilon \uparrow \omega} F_i(\upsilon)$.

Several properties of a steady state can now be described.[1] Equating the flows into and out of employment, the fraction of workers unemployed is:

$$u_i = \frac{\delta}{\delta + \lambda_o[1 - F_i(R_i^-)]}. \tag{A}$$

The distribution of wages across employed workers of type $i \in S$ can be expressed as follows:

$$G_i(w) = \frac{\delta[F_i(w) - F_i(R_i^-)]/[1 - F_i(R_i^-)]}{\delta + \lambda_e[1 - F_i(w)]}, \tag{B}$$

which is obtained by equating the flow of workers from unemployment into jobs paying no more than w with the flow from such jobs into unemployment or higher paying jobs. Let $\ell_i(w|H_O, H_i)$ denote the measure of type $i \in S$ workers employed at a firm offering them a wage w. Equating the flow of workers recruited to and separating from an employer paying w, the following holds for $w \geq R_i$:

$$\ell_i(w|H_O, H_i) = \frac{\delta\lambda_o m_i\{\delta + \lambda_e[1 - F_i(R_i^-)]\}/\{\delta + \lambda_o[1 - F_i(R_i^-)]\}}{\{\delta + \lambda_e[1 - F_i(w)]\}\{\delta + \lambda_e[1 - F_i(w^-)]\}}, \tag{C}$$

and $\ell_i(w|H_O, H_i) = 0$ for $w < R_i$.

The payoff functions are next specified in steady state. The profit flow from type $i \in S$ workers employed at a firm paying them a wage w is:

$$\pi_i(w|H_O, H_i) = (\theta_i - w)\ell_i(w|H_O, H_i). \tag{D}$$

The profit of a type O firm offering the wage w_O to each worker is:

$$\Pi_O(w_O|H_O, C_I) = \sum_{i=1}^{N} \pi_i(w_O|H_O, H_i). \tag{E}$$

[1] The ensuing formulae for the unemployment rate, wage distribution, and employment level correspond to equations (7), (9), and (10) in Burdett and Mortensen (1998), who provide more details regarding their derivation.

The profit of a type I firm posting the wage vector $v_I = (w_1, w_2, \ldots, w_N)$ is:

$$\Pi_I(v_I|H_O, C_I) = \sum_{i=1}^{N} \pi_i(w_i|H_O, H_i), \tag{F}$$

where w_i is the wage paid by the firm to any worker of type $i \in S$.

B Equilibrium

Employers make wage offers so as to maximize their profits in steady state, given the distribution of wages across workers of each type. An uninformed employer is constrained to post a uniform wage, whereas an informed employer may condition the wage on worker type. Formally, an equilibrium consists of a wage offer distribution H_O for type O firms and a collection $C_I = \{H_1, H_2, \ldots, H_N\}$ of wage offer distributions for type I firms such that: $w_O \in \operatorname{argmax}_{w\in\mathbb{R}} \Pi_O(w|H_O, C_I)$ for all w_O in the support of H_O; $v_I = (w_1, w_2, \ldots, w_N) \in \operatorname{argmax}_{v\in\mathbb{R}^N} \Pi_I(v|H_O, C_I)$ if w_i is in the support of H_i for all $i \in S$.

Suppose that all employers observe worker ability, in which case $p = 1$ representing full information. The resulting model is analytically equivalent to Burdett and Mortensen (1998), who derive a unique equilibrium outcome. From their equation (16), the equilibrium distribution of all wages offered to type $i \in S$ workers is as follows for $w \in (w_i^l, w_i^u)$:

$$K_i(w) = \frac{\delta + \lambda_e}{\lambda_e}\left(1 - \sqrt{\frac{\theta_i - w}{\theta_i - w_i^l}}\right), \tag{G}$$

where $K_i(w) = 0$ for $w \leq w_i^l$, $K_i(w) = 1$ for $w \geq w_i^u$, and w_i^u and w_i^l respectively denote the supremum and infimum of its support. The supremum from their equation (17) is:

$$w_i^u = \theta_i - (\theta_i - w_i^l)\left(\frac{\delta}{\delta + \lambda_e}\right)^2, \tag{H}$$

and the infimum from their equation (18) is:

$$w_i^l = \frac{b(\delta+\lambda_e)^2 + \theta_i(\lambda_o - \lambda_e)\lambda_e}{(\delta+\lambda_e)^2 + (\lambda_o - \lambda_e)\lambda_e}. \tag{I}$$

In equilibrium, w_i^l is the reservation wage of unemployed workers of type i.

Now let $p \in [0,1]$. An equilibrium (H_O, C_I) is said to achieve the full information outcome if and only if $pH_i(w) + (1-p)H_O(w) = K_i(w)$ for all $w \in \mathbb{R}$ and any $i \in S$. That is, the distribution of all wages offered to workers of each type is the same as when every firm is informed about worker type.

C Existence

The theorem below identifies a threshold for invariance to asymmetric information. See the next section for the proof.

Theorem A *In the model with $p < 1$, an equilibrium achieving the full information outcome exists if and only if $p \geq \phi$, where the cutoff ϕ is given by:*

$$\phi = \begin{cases} \sqrt{\dfrac{\delta^2(\theta^l - b) + (\delta^2 + 2\delta\lambda_e + \lambda_e\lambda_o)(\theta^u - \theta^l)}{\lambda_e^2(\theta^u - b)}} - \dfrac{\delta}{\lambda_e} & \text{if } \lambda_e < \lambda_o \\ \sqrt{\dfrac{\delta^2(\theta^l - b) + (\delta^2 + 2\delta\lambda_e + \lambda_e\lambda_o)(\theta^u - \theta^l)}{\lambda_e^2(\theta^u - b)}} + 1 & \\ \quad - \sqrt{\dfrac{(\delta+\lambda_e)^2(\theta^l - b) + (\delta^2 + 2\delta\lambda_e + \lambda_e\lambda_o)(\theta^u - \theta^l)}{\lambda_e^2(\theta^u - b)}} & \text{if } \lambda_e \geq \lambda_o \end{cases}. \tag{J}$$

The condition $\phi < 1$ is necessary and sufficient for there to exist $p < 1$ such that the full information outcome can be implemented in equilibrium. With some algebra, it can be shown that $\phi < 1$ is equivalent to $w_1^u > w_N^l$. This inequality means the full information outcome is such that the supremum of the support of the distribution of wages offered to the least productive type of worker is greater than the infimum of the support of the distribution of wages offered to the most productive type of worker. Note that $w_1^u > w_N^l$ for $\lambda_e \geq \lambda_o$. If $\lambda_e < \lambda_o$, then $\phi < 1$ holds for some parameter values but not for others.

D Proof

Below is the proof of the extended version of the main theorem. Let $p < 1$. The first part demonstrates by construction the existence of an equilibrium achieving the full information outcome when $p \geq \phi$. Nonexistence of such an equilibrium for $p < \phi$ is shown in the second part.

Sufficiency

Assume that $p \geq \phi$, where ϕ is given by equation (J). Let ξ represent the constant $1/[K_N(w_1^u) - K_N(w_1^l)]$, where w_1^l, w_1^u, and K_N are given by equations (I), (H), and (G). Define the distribution function $\tilde{H}_O$ by $\tilde{H}_O(w) = 0$ if $w < w_1^l$, $\tilde{H}_O(w) = 1$ if $w > w_1^u$, and $\tilde{H}_O(w) = \xi[K_N(w) - K_N(w_1^l)]$ for $w \in [w_1^l, w_1^u]$. For $i \in S$, the distribution function $\tilde{H}_i$ is specified as $\tilde{H}_i(w) = 0$ if $w < w_i^l$, $\tilde{H}_i(w) = 1$ if $w > w_i^u$, and $\tilde{H}_i(w) = [K_i(w) - (1-p)\tilde{H}_O(w)]/p$ for $w \in [w_i^l, w_i^u]$. Denote $\tilde{C}_I = \{\tilde{H}_1, \tilde{H}_2, \ldots, \tilde{H}_N\}$.

It follows directly from the definition above that $\tilde{H}_O(w) = 0$ for $w \leq w_1^l$, $\tilde{H}_O(w) = 1$ for $w \geq w_1^u$, and $\tilde{H}_O$ is continuous and nondecreasing on the interval $[w_1^l, w_1^u]$. Recall that $p \geq \phi$. For $i \in S$, it is straightforward to confirm from the specification above that $\tilde{H}_i$ is continuous and nondecreasing on the interval $[w_i^l, w_i^u]$ with $\tilde{H}_i(w) = 0$ for $w \leq w_i^l$ and $\tilde{H}_i(w) = 1$ for $w \geq w_i^u$. Therefore, the distribution functions are well defined. Moreover, they are constructed such that $p\tilde{H}_i(w) + (1-p)\tilde{H}_O(w) = K_i(w)$ for all $w \in \mathbb{R}$ and any $i \in S$, where K_i is the equilibrium distribution of all wages offered to type $i \in S$ workers in the case where $p = 1$.

For any $i \in S$, the preceding sentence implies that $w_i \in \operatorname{argmax}_{w \in \mathbb{R}} \pi_i(w|\tilde{H}_O, \tilde{H}_i)$ if $w_i \in [w_i^l, w_i^u]$. Note that for each $i \in S$, the support of $\tilde{H}_i$ is a subset of $[w_i^l, w_i^u]$. Hence, $v_I = (w_1, w_2, \ldots, w_N) \in \operatorname{argmax}_{v \in \mathbb{R}^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$ if w_i belongs to the support of $\tilde{H}_i$ for all $i \in S$. Let x_w be an N-vector each of whose elements is $w \in \mathbb{R}$. If $w \in [w_i^l, w_i^u]$ for all $i \in S$, then $x_w \in \operatorname{argmax}_{v \in \mathbb{R}^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$. Note that $\max_{w \in \mathbb{R}} \Pi_O(w|\tilde{H}_O, \tilde{C}_I) \leq \max_{v \in \mathbb{R}^N} \Pi_I(v|\tilde{H}_O, \tilde{C}_I)$, $\Pi_O(w|\tilde{H}_O, \tilde{C}_I) = \Pi_I(x_w|\tilde{H}_O, \tilde{C}_I)$ if $w \in \bigcap_{i=1}^N [w_i^l, w_i^u]$, and the support of $\tilde{H}_O$ is $\bigcap_{i=1}^N [w_i^l, w_i^u]$. It follows that $w_O \in \operatorname{argmax}_{w \in \mathbb{R}} \Pi_O(w|\tilde{H}_O, \tilde{C}_I)$ for all w_O in the support of $\tilde{H}_O$.

Thus, $(\tilde{H}_O, \tilde{C}_I)$ is an equilibrium achieving the full information outcome for $p \geq \phi$.

Necessity

Assume that (H_O, C_I) is an equilibrium achieving the full information outcome. Recall that $p < 1$. It must be that $H_O(w_1^u) = 1$. Otherwise, if $H_O(w_1^u) < 1$, then $p + (1-p)H_O(w_1^u) < 1$, which would contradict the requirement that $pH_1(w_1^u) + (1-p)H_O(w_1^u) = K_1(w_1^u)$. It must be that $H_O(w_1^l) = 0$. Otherwise, if $H_O(w_1^l) > 0$, then $(1-p)H_O(w_1^l) > 0$, which would contradict the requirement that $pH_1(w_1^l) + (1-p)H_O(w_1^l) = K_1(w_1^l)$.

It follows that $pH_N(w_1^u) + (1-p) = K_N(w_1^u)$ and $pH_N(w_1^l) = K_N(w_1^l)$, which implies that $p[H_N(w_1^u) - H_N(w_1^l)] + (1-p) = K_N(w_1^u) - K_N(w_1^l)$. Since $H_N(w_1^u) - H_N(w_1^l) \geq 0$, it must be that $1 - p \leq K_N(w_1^u) - K_N(w_1^l)$. Using equations (I), (H), and (G) to substitute for w_1^l, w_1^u, and K_N in the previous inequality results in $p \geq \phi$ after some algebraic manipulation, where ϕ is defined in equation (J).