Allocating Inspection Effort on a Production Line

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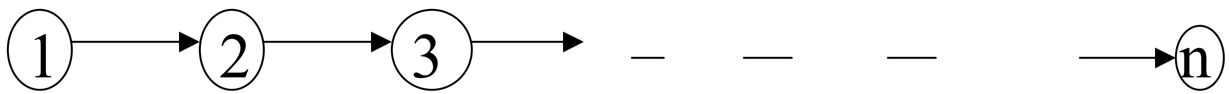
Consider a production line with n stages and each stage has a manufacturing operation followed by a potential inspection.



The product enters stage 1 in batches of size $B \geq 1$

As the batches move through the stages, the operations might introduce defects.

P(Producing a defective unit at any stage i) = α_i

We have the following assumptions:

- All defects are non-repairable
- At each stage we can either inspect all of the items or none of them (i.e. we cannot sample the items)
- The production line must end with an inspection so that we do not send any defective items

<u>Objective:</u>

To find an optimal inspection plan that specifies at which stages we should inspect the items so that we minimize the total cost of production and inspection.

The optimal no. of inspection stations will achieve an appropriate trade-off between these two conflicting cost considerations.

Let us suppose the following cost parameters are known:

(1) p_i = Manufacturing cost per unit in stage i;

(2) f_{ij} = Fixed cost of inspecting a batch after stage j, given the last inspection occurred at stage i;

(3) g_{ij} = Variable per unit cost of inspection

The inspection costs at any station j depend on when the batch was inspected last, say at station i, because the inspector needs to look for the defects incurred at any of the intermediate stages i+1, i+2, ….., j.

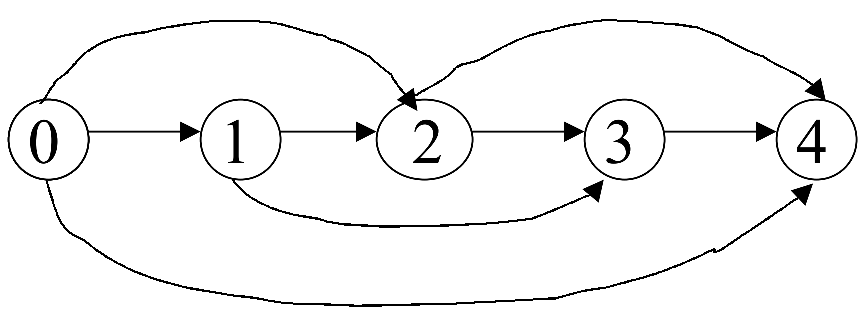
Formulation:

We formulate the inspection problem as a shortest path problem on a network with (n+1) nodes numbered as 0,1,2,.….,n

The arc (i,j) indicates that there is an arc from node i to node j with i < j

For Example:

Consider the following network-



Let $B(i) = B\prod_{k=i+1}^{j}(1-\alpha_k)$, denote the expected no. of non-defective units at the end of stage i

Each path in the graph indicates an inspection plan.

Consider the path 0-2-4, which indicates that we inspect after the stage 2nd and the 4th.

We define the total cost c_{ij} with any arc (i,j) in the network as follows:

$$c_{ij} = f_{ij} + B(i)g_{ij} + B(i)\sum_{k=i+1}^{j} p_k$$

where, the first two terms in the equation are the fixed and the variable inspection costs, and the third term is the production cost incurred in the stages i+1, i+2,…..,j

Thus, the shortest path formulation permits us to solve the inspection application as a network flow problem.