IE 680 – Special Topics in Production Systems: Networks, Routing and Logistics*

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Lecture notes from Network Flows by Ahuja, Magnanti and Wong (1993)

Algorithm Design & Analysis

- Building blocks of computational problem solving
  - A recipe, or algorithm (a step-by-step procedure to solve a problem)
  - A means of encoding the procedure
  - Application of the method to problem data
- Data structures are important to represent informational elements
- And how does one measure effectiveness of an algorithm?

Algorithm Design & Analysis

- Computational complexity theory and computational analysis
- Different measures of complexity
- Definition of steps:
  - Assignment steps (assigning a variable a value)
  - Arithmetic steps (addition, multiplication, etc.)
  - Logical steps (comparison)
- We would like to measure how many steps an algorithm takes?

Complexity Measurement

- Empirical Analysis:
  - Simulation approach of testing the algorithm on sample data
- Average-case Analysis (statistics/asymptotic):
  - Expected steps the algorithm takes for a probability distribution of problem instances
- Worst-case Analysis:
  - Upper bound on the number of steps an algorithm can take on *any* problem instance

Problem size

- Measure of complexity of a problem instance
- This is often referred to as the "size" of the problem instance
- Usually, one likes to define a single (or two but not preferably more) "parameters" that indicate problem size "n"
- Two fundamental processing of "n":
  - n and its powers
  - log(n) because the way computers process binary numbers
- "Big O" notation
  - drops constants: e.g., 2n = O(n)

Worst-Case Complexity

- Worst-case bound or the upper bound on time taken by an algorithm
- With big-O notation we find the max # steps an algorithm takes
- Compute the function of # steps in terms of parameters
- Take the highest term
- Drop constants
- E.g., $n+20n^2 = O(n^2)$

Polynomial time algorithms

- If the worst case complexity of an algorithm is bounded by a polynomial function of the problems parameters it is said to be polynomial.
- Such algorithms are considered to be “good”

Exponential time algorithms

- An algorithms is exponential if its worst case running time grows as a function that cannot be polynomially bounded by the input length.

Search Algorithms

- Breadth-First
- Depth-First
- Best-First
  - "Opens" the smallest number of nodes in the search tree
- Block Depth-First Search (BDFS)

Topological Ordering

- It deals with the labeling of nodes in networks
- A network labeling is a topological ordering of nodes provided that every arc joins a lower labeled node to a higher labeled node

Flow Decomposition Theorem

- Every path and cycle flow has a unique representation as non-negative arc flows
- Conversely, every non-negative arc flow x can be represented as a path and a cycle flow (though not necessarily unique).
- With the following two properties:
  - Every directed path with positive flow connects a deficit node to an excess node
  - At most n+m paths and cycles have nonzero flow; of which at most m cycles have nonzero flow.