IE 680 – Special Topics in Production Systems: Networks, Routing and Logistics

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About the course

This graduate level course is a comprehensive coverage of a trinity of topics from:

(i) *graph and network theory* that provide the modeling constructs to specify and algorithms to solve a large class of practical problems,

(ii) *routing* that helps determine the sequence and timing when traversing these network structures, and

(iii) the *business logistics* decisions that coordinate management of storing (inventory), handling, locating (location-allocation), distributing and mode/carriers selection.

About the course

- The objective is to expose participants to NRL issues, and in a participatory setting, enable them to discuss and creatively synthesize these ideas to research projects of choice.
- It blends quantitative and qualitative material, from multiple disciplines of industrial and management engineering.
- The course will be conducted in a beneficial cooperative learning setting. Lectures, group discussions, research projects and participant presentations will constitute this course.

About the course

- Considered as an informal course this year:
  You will need to fill an informal course description along with your application to candidacy (NB: Limit of 2 informal courses for MS)
- Meets OR as well as PS elective requirements

Course Topics

- Network Optimization
  - Paths, Trees, and Cycles; Shortest Paths
  - Maximum Flows, Minimum Cost Flows
  - Multicommodity Flows; Lagrangian Relaxation.
- Routing
  - TSP, VRP, Inventory Routing
- Business Logistics
  - Analyzing, Designing, and Implementing Logistics Systems
  - Analytical Models for One-to-One/Many Distribution and Transshipment
  - Information Exchange and Supporting Technologies
  - Core competencies from a Business standpoint
- Several Research Articles and Case Studies

Course prerequisites

- IE 505 Production Planning and Control or similar course
- Advanced Graduate standing in engineering or management
- Optimization (linear IE 572, discrete IE 573 are highly recommended)

Course Elements

Homework	- 4-5 assignments	15%
Class presentations	- 2 lectures	10%
Research project report, presentation	- progress report, final	50%
Programming project (C/C++, Java or VB)	- High level languages	10%
Exam	- one midterm	15%

(+/- Grading scheme will be employed)

Chapter 1

- We touch many types of networks in daily life
  - Electrical and power
  - Telecommunications (Telephone, Wireless)
  - Transportation (highways, roads, airline, etc.)
  - Manufacturing and distribution
  - Computers (local and internet)
- Basic questions
  - Who is connected to whom (possibly through someone)
  - How to get to somewhere
  - How to make something (message, material) flow/get there
- Networks/Graphs help mathematically model systems

3 Basic Questions

- Shortest Path Problem. What is the best way to traverse the network to get from one point to another as cheaply as possible?
- Maximum Flow Problem. When network arcs are capacitated, how to get the max flow between a pair of points?
- Minimum Cost Problem. When there is cost per unit flow on capacitated arcs, how to supply commodities resident at some points?

Network Flow Problems

- Minimum Cost Flow Problem

Let $G = (N,A)$ be a directed network. Let arc $(i,j) \in A$ have associated

- *cost* per unit flow c_{ij}
- *capacity* u_{ij} (max amount of flow possible)
- *lower bound* l_{ij} (min amount of flow necessary)

Let each node $i \in N$ be associated with an integer number $b(i)$ representing supply/demand

- If $b(i) > 0$, node i is a *supply node*
- If $b(i) < 0$, node i is a *demand node*
- If $b(i) = 0$, node i is a *transshipment node*

The decision variable x_{ij} is the arc flow on arc $(i,j) \in A$.

Network Flow Problems

- Minimum Cost Flow Problem

$$\text{Minimize} \sum_{(i,j)\in A} c_{ij}x_{ij} \qquad (1.1a)$$

subject to

$$\sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i) \quad \text{for all } i \in N, \qquad (1.1b)$$

$$l_{ij} \le x_{ij} \le u_{ij} \quad \text{for all } (i,j) \in A, \qquad (1.1c)$$

Mass-flow balance

Flow bound constraint

where $\sum_{i=1}^{n} b(i) = 0$. In matrix form, we represent the minimum cost flow problem as follows:

$$\text{Minimize } cx \qquad (1.2a)$$

subject to

$$\mathcal{N}x = b, \qquad (1.2b)$$

$$l \le x \le u. \qquad (1.2c)$$

In this formulation, $\mathcal{N}$ is an $n \times m$ matrix, called the *node–arc incidence matrix* of the minimum cost flow problem. Each column $\mathcal{N}_{ij}$ in the matrix corresponds to the variable x_{ij}. The column $\mathcal{N}_{ij}$ has a $+1$ in the ith row, a -1 in the jth row; the rest of its entries are zero.

Network Flow Problems

- Shortest Path Problem

  Special version of the min cost flow problem

  - *Source node* s (single)
  - *Sink node* t (single)
  - *Cost (length)* for each arc (i,j) is c_{ij}
  - If b(s) =1, for the *source node*
  - If b(t) = -1, for the *sink node*
  - If b(i) = 0, for all other *nodes*

  The decision is to send a unit of flow from s to t.

Network Flow Problems

- Maximum Cost Flow Problem

  In some sense is a complementary model to the shortest path problem.

  - In shortest path, cost is incurred, but there is no arc capacity
  - Whereas here, cost is not incurred but there is arc capacity.

  Can be reformulated as the min cost flow problem as follows:

  - $b(i) = 0$, for all *nodes* $i \in N$
  - $c_{ij} = 0$, for all $(i,j) \in A$
  - Introduce an additional arc (t,s) with
  - cost $c_{ts} = -1$ and flow bound $u_{ts} = \infty$.
  - Min cost flow solution maximizes the flow on arc (t,s); but this flow must travel through the network from node s to node t. This is what we want!

Network Flow Problems

- Assignment Problem

  Consider two equal-sized sets N_1 and N_2 ($|N_1| = |N_2|$), and a collection of pairs $A \subseteq N_1 x\ N_2$ representing possible assignments, and a cost c_{ij} associated with each element $(i,j) \in A$. We wish to pair at min cost an element in N_1 with another in N_2.

  Can be reformulated as the min cost flow problem as follows:

  - $G=(N_1 \cup N_2 , A)$
  - $b(i) = 1$, for all *nodes* $i \in N_1$
  - $b(i) = -1$, for all *nodes* $i \in N_2$
  - $u_{ij} = 1$, for all $(i,j) \in A$

Network Flow Problems

- Transportation Problem

  Consider two possibly unequal-sized sets N_1 and N_2

  Elements in N_1 are supply nodes and elements in N_2 are demand nodes.

  The arc set is $A \subseteq N_1 x\ N_2$ representing possible distribution channels.

  Can be reformulated as the min cost flow problem as follows:

  - $G=(N_1 \cup N_2 , A)$
  - b(i) = supply quantity, for all *nodes* $i \in N_1$
  - b(i) = -desired demand, for all *nodes* $i \in N_2$
  - u_{ij} = channel capacity, for all $(i,j) \in A$
  - c_{ij} = unit transportation cost, for all $(i,j) \in A$

Network Flow Problems

- Other Problems
- Circulation Problems
- Convex cost flow problems
- Generalized flow problems
- Multicommodity flow problems
- Minimum spanning tree problems
- Matching problems