MATERIALS REQUIREMENTS PLANNING (MRP)
Production Models were good for replenishment from an outside vendor or internal

- Demand is independent
- Demand rate is constant (or mean is constant)

Previous Assumptions:

Introduction
Job-Shop Scheduling

Materials Requirements Planning (MRP)

Master Production Scheduling (MPS)

Aggregate Production Planning

Demand Forecasting

Hierarchy of Production Planning Decisions
Assumptions

- Lead-times known, constant and independent of lot-size
- Static planning over horizon
- No capacity constraints
- No backordering
- Demand is known
- Assumptions
  - Lot-sizing algorithms are applied
  - Lot-sizing algorithms (Bills-Of-Materials): Explosion Calculus
  - Translate into time-phased requirements at each level via BOM
  - Determine requirements for end items for each period via MPS

Outline of MRP

MPS is a means of converting independent demand into dependent demand

MPS is a means of meeting the MRP, or
Schedule for production

Capacity planning
Lot-sizing

Lead-times

MRP

BOM

MPS: for end items

- Internal orders
- Seasonal plan
- Safety stock requirements
- Forecast of future demand by item
- Firm customer orders

MPS consists of

Inputs to MRP

Materials Requirements Planning (MRP)
4. Proceed to next level (if any)
3. Compute ending inventory
2. Determine Planned Order Release (LO)
1. Compute time-phased requirements

Basic Algorithm

- Projected on-hand inventory
  Net Req. = Gross Req. - Scheduled Receipts

Basic Equation

Explosion Calculus: Example

at that level and requirements at lower levels:

Rules for translating gross requirements at one level to production schedule

Explosion Calculus
Schedule for Item C

Gross 84 84 64 24 52 224 90 28 152 68
Week 4 5 6 7 8 9 10 11 12 13

Schedule for Item B

Gross 42 42 32 26 112 45 14 76 34
Week 6 7 8 9 10 11 12 13 14 15

Net 42 42 32 26 112 45 14 76 34

Inv 23

Sch Rpt 12 6 9

End Item, A:

MATERIAL REQUIREMENTS PLANNING (MRP)
ALTERNATIVE LOT-SIZING SCHEMES
If not sufficient inventory, 0 otherwise

2. EoQ scheme:
\[ \text{Order Quantity} \]

1. Lot-for-lot scheme:
\[ \text{Order Quantity} \]

Examples

Note: No Backordering

- Holding, \( y \) per unit per period
- Set-up, \( K \) per order

Optimize costs:

- Determine ending inventory for each period

Given time-varying net requirements \( p_1, p_2, \ldots, p_n \)

- Determine planned order release \( y_1, y_2, \ldots, y_n \)

Purpose of lot-sizing:
\[
\begin{align*}
139 &= \frac{0.60}{132(43.9)} = \frac{\mu}{2k} = \theta \\
3.9 &= \frac{N}{p} = \chi
\end{align*}
\]

**Determination of EOG**

**Cost** = $132 (10) = $1320

L-T/L: 42 42 42 42 32 12 26 112 45 14 76 34

Cross: 42 42 42 32 12 26 112 45 14 76 34

Week: 6 7 8 9 10 11 12 13 14 15

Por for item B (Lot-for-Lot)

\[
\begin{align*}
\text{Example (1)}: \text{Lot-for-Lot vs EOG scheme} \\
\text{Por} &= \frac{0.60}{\chi} \\
132 &= \frac{\mu}{\chi}
\end{align*}
\]

Alternative Lot-Sizing Schemes
Cost = $132 (4) + 0.6 (653) = $ 919.80

Line  97  55 23 11 12 14 12 106 92 16 117
EOQ 139 0 0 0 139 0 0 139
Gross 42 42 12 12 12 12 45 14 76 34
Week  6 7 8 9 10 11 12 13 14 15
POR for Item B (EOQ)
\[
90 = \min \left( \frac{1}{(75)(57,52)} \right) = \ast \hat{O}
\]

\[
\frac{t_4}{218} = 4.5
\]

\[
\text{Cost (L-1-L)} = \$75 \quad \text{(4)} = D \quad \bullet
\]

\[
\text{I$} = \eta \quad \bullet
\]

\[
\text{775$} = K \quad \bullet
\]

Example (2): Lot-for-Lot vs EOQ scheme

Alternative lot-sizing schemes
Demand rate is not constant

Conclusion: E0Q tries to minimize average holding and set-up, but

\[ \text{Cost} = \$75 \times (3) + \$149 = \$374 \]

<table>
<thead>
<tr>
<th>Item</th>
<th>38</th>
<th>41</th>
<th>18</th>
<th>52</th>
</tr>
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<tbody>
<tr>
<td>E0Q</td>
<td>90</td>
<td>90</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>Gross</td>
<td>52</td>
<td>87</td>
<td>23</td>
<td>56</td>
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</tbody>
</table>

POR for item B (E0Q)
Thus, optimal policy can be specified as 0-1 integer programming problem.

\[ \sum_{i=1}^{n-1} x_i \]

\[ y_i = 0, \text{ or } y_i = 1 \]

Theorem of Optimality

Optimal values of \( y_i \)

Optimal Lot-Sizing for Time-Varying Demand

Alternative Lot-Sizing Schemes
3. Cost of holding cost up until period $j$:

$$C_j = \sum_{i=1}^{j} K$$

2. The ordered amount in period $i$:

$$\text{order} \leftarrow 1, 2, \ldots, n, n + 1$$

1. Nodes: $I, 2, \ldots, n, n + 1$

Translate to Minimal Cost Path Problem (MCP)

2. Solution by Dynamic Programming (DP):

<table>
<thead>
<tr>
<th>Node</th>
<th>Cost</th>
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<tr>
<td>1</td>
<td>376</td>
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<tr>
<td>1, 0</td>
<td>283</td>
</tr>
<tr>
<td>1, 0, 0</td>
<td>312</td>
</tr>
<tr>
<td>1, 1</td>
<td>293</td>
</tr>
<tr>
<td>1, 0, 0</td>
<td>285</td>
</tr>
<tr>
<td>1, 1</td>
<td>281</td>
</tr>
<tr>
<td>1, 1, 0</td>
<td>300</td>
</tr>
<tr>
<td>1, 1, 1</td>
<td>248</td>
</tr>
</tbody>
</table>

1. Explicit Enumeration
Step 2: Backward DP

\[ 98, c_2 = 210, c_3 = 75, c_4 = 131, c_{15} = 162, c_{16} = 75, c_{17} = 376, c_{18} = 75 + (166+79+56) = 376 \]

D: \( (52, 87, 23, 56) \), \( C_{15} = 75 \)

Step 1: Compute costs:

\[
0 = u^T + \sum\limits_{n=1}^{N-1} f_n \text{ for } n = 1, 2, \ldots, \text{subject to } N \left( f + \frac{d}{c} (C < f) \right) \]

\[ N \text{ and } f \text{ are minimum cost starting at node } k \text{, assuming order is placed in period} \]

Optimality Equation

\[ f_k \text{ is minimum cost starting at node } k \text{, assuming order is placed in period} \]
\[ 248 = 1f \quad 5 \quad 4 \quad 3 \quad 2 \quad 1. \]

\[ 173 = 2f \quad 5 \quad 4 \quad 3 \quad 2. \]

\[ 131 = 3f \quad 5 \quad 4 \quad 3 \quad . \]

\[ 72 = 4f \quad 5 \quad 4 \quad . \]
\[ \frac{C_3}{K + \frac{2K^2}{C_2} + \frac{C_1}{C_3}} \]

Determine \( C(1), C(2), \ldots, C(L) \) until \( C \) increases.

A. Silver-Meal Heuristic

\[ C \] average cost/period, where \( L \) is the number of periods.

Optimal LS is a complex problem.

Motivation: Heuristic Lot-Sizing Schemes
\begin{align*}
\zeta_2 &= \gamma_1 \iff \zeta_2 < \frac{1}{2} \left( \left( \gamma_7 \right)^2 + \zeta_5 \right) = \left( \gamma_7 \right)^2 \cdot \zeta_5 \\
\zeta_2 &= \left( \gamma_1 \right)^2 \\
\left( 2, 5, 23, 56 \right) &= D \\
I \gamma &= \gamma \cdot I \\
I \gamma &= \gamma \cdot I \\
\left( I - \gamma \right) \zeta_2 < \left( \gamma \right) \zeta_2 \\
\gamma \gamma (I - \gamma) + \cdots + \gamma \gamma \gamma + \gamma \gamma (I - \gamma) &= \left( \gamma \right) \zeta_2 \\
\text{Example:} \\
\gamma \gamma (I - \gamma) + \cdots + \gamma \gamma \gamma + \gamma \gamma (I - \gamma) &= \left( \gamma \right) \zeta_2 \\
\text{Stop when} \quad \zeta_2 < \left( \gamma \right) \zeta_2 \\
\text{In general,} \\
\gamma \gamma (I - \gamma) + \cdots + \gamma \gamma \gamma + \gamma \gamma (I - \gamma) &= \left( \gamma \right) \zeta_2 \\
\end{align*}
Performance better when demand variability increases

Generally performs better than EOE

Critique:

Note: Solution is optimal

\[
\begin{align*}
\frac{I_8}{87} + \frac{(0,4\% \times 33)}{51} = \frac{96}{72} \left(\frac{I_8}{87} + \frac{(0,4\% \times 33)}{51}\right) = (\mu_3)C
\end{align*}
\]

\[
\begin{align*}
\frac{I_2}{(23) + (75)} = (\mu_2)C
\end{align*}
\]

\[
\begin{align*}
75 = (\mu_1)C
\end{align*}
\]
we set $I - \ell \gamma < (\ell) C$

\[
p \prod_{i=1}^{\ell} (1 - (1 - \ell) \gamma) + \cdots + \ell \rho = (\ell) C
\]

In general,

\[
(\ell \rho + \gamma + I p)/(\ell \rho + \gamma + \gamma + X) = (\ell) C
\]

\[
(\ell \rho + I p)/(\ell \rho + X) = (\ell) C
\]

\[
I p/X = (1) C
\]

Define $C(\ell)$ average cost/unit for a $\ell$-period order horizon

Instead of cost per period, it is cost per unit

Similar to Silver-Meal

B. Least unit cost (LUC) Heuristic
87 < 75 and 75 is closer to 87 than 0, so order horizon = 2

\[
\begin{array}{c|c}
87 & 2 \\
0 & 1 \\
\end{array}
\]

Order Horizon Total Holding Cost

Example: \( R = 575, \theta = 7 \) $D = (52, 87, 23, 56) \]

\[
(\frac{52 + 87}{2}) / (52 + 87) = 1.44 \approx 1.45
\]

C. Part Period Balancing

Example: \( R = 575, \theta = 7 \) $D = (52, 87, 23, 56) \]

\[
(52, 87, 23, 56)
\]
Solution $X = (1, 0, 1, 0)$, or $X = (139, 0, 79, 0)$

$56 > 75$, but no more data, so order horizon $= 2$

<table>
<thead>
<tr>
<th></th>
<th>Order</th>
<th>Horizon</th>
<th>Total holding cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>56</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Restart:
INCOMPATIBLE LOT-SIZING IN EXPLOSION

CALCULUS
\[ c(4) = 1.86; \quad c(5) = 1.89; \]
\[ c(3) = 1.88; \quad c(2) = (132 + 6 \times (26)/(12 + 26)) = 3.88; \]
\[ c(1) = (132/12 = 11; \quad c(3) = (132/42 = 3.14; \]

Restart in week 7:

Since \( c(4) < c(3) \), stop; set \( \bar{y} = 42 + 42 = 116 \)

\[ c(4) = (132 + 6 [42 + 2(32)] + 32)/(12 + 116 + 12) = 1.69; \]
\[ c(3) = (132 + 6 [42 + 2(32)] + 84 + 32)/(12 + 42 + 42) = 1.87; \]
\[ c(2) = (132 + 6 \times 42)/(42 + 42) = 3.4; \]
\[ c(1) = (132/42 = 3.14; \]

Least Unit Cost Heuristic

<table>
<thead>
<tr>
<th>Net</th>
<th>42</th>
<th>42</th>
<th>32</th>
<th>12</th>
<th>26</th>
<th>112</th>
<th>14</th>
<th>76</th>
<th>34</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

Example: \( Y = $1,32, 60, \quad D = time\ Phased\ net\ \text{or} = D = \text{time} \)

Example: \( Y = $1,32, 60, \quad D = time\ Phased\ net\ \text{or} = D = \text{time} \)
\[
\text{EOQ costs} = \$919.80 \\
\text{L-T-L costs} = 132 (10) = 1320 \\
\text{Total costs (S-M)} = 132 (3) + 6 (424) = 650.40 \\
\text{Total costs} = 132 (3) + 6 (635) = \$777 \\
\]

<table>
<thead>
<tr>
<th>Inventory</th>
<th>74 32 183 157 45 0 110 34</th>
</tr>
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<tbody>
<tr>
<td>Pl. Deliv.</td>
<td>116 0 0 195 0 0 12 13</td>
</tr>
<tr>
<td>Week</td>
<td>8 9 6 7 8 9 10 11 12 13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>POR (LUC)</th>
<th>42 42 32 12 26 112 45 14 76 34</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net</td>
<td>8 9 10 11 12 13 14 15 16 17</td>
</tr>
<tr>
<td>Week</td>
<td></td>
</tr>
</tbody>
</table>

**Summary of MRP Calculations**

Restate in week II: set \( Yi \) = 14 + 76 + 34 = 124

Since \( C(5) > C(4) \), stop; set \( Yi = 122 + 122 + 45 = 195 \)
LOT-SIZING WITH CAPACITY CONSTRAINTS
Back Shift demand from periods in which $p_i < c_i$.

Backward procedure (starts from $n$)

2. Initial feasible solution (lot sizing technique)

$$u \geq l \geq 1 \text{ for } i \neq 1$$

I. Feasibility:

Heuristic procedure

Ending inventory of order period not zero

$$u \geq l \geq 1 \text{ for } i \neq 1$$

Capacities constraints:

$y_i \geq c_i$ for $i \neq 1$
Holding costs are less than one setup cost.

- Backshift entire demand to a previous period with enough capacity if

\[ n \]

Backward procedure (starts from \( n \) = 0)

<table>
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<tr>
<th>Improvement Step</th>
<th>( n )</th>
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<tr>
<td>100 109 200 105</td>
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<tr>
<td>100 79 230 105</td>
<td>0</td>
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<tr>
<td>100 79 230 105</td>
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<td>100 79 230 105</td>
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<td>100 200 200 400</td>
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Example: 1
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<th></th>
<th>EX. Cap</th>
<th>20</th>
<th>91</th>
<th>0</th>
<th>295</th>
<th>142</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
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<th>100</th>
<th>109</th>
<th>200</th>
<th>105</th>
<th>158</th>
<th>0</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Check</td>
<td>2 x 50 x 1</td>
<td>=</td>
<td>$100 &gt; 450</td>
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<td></td>
<td>EX. Cap</td>
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Example: \( Y = \$450, \; \gamma = \$2 \)
SHORTCOMINGS AND ADDITIONAL CONSIDERATIONS
JIT and MRP: advanced topics
Order Picking
Data Integrity
Variable Yield

Includes finance, accounting & marketing functions, and MPS & CRP

Manufacturing Resources Planning, MRP II: Bigger picture

Lead-times can be computed from routings!
Lead-times depend on lot size

Regenerative Versus Net Change MRP runs
Rolling Horizons and System Nervousness

Capacity Requirements Planning (CRP) for Iterative Correction

Attempted to be corrected by safety stock & safety lead times

Uncertainty: Forecasts

Shortcomings and Additional Considerations