FORECASTING
1. Introduction

- For production, we need forecasts for demands
- Forecasting techniques - predict the future

Characteristics of Forecasts
- Usually wrong
- More than a number ($\mu$, $\sigma$)
- Aggregate forecasts are more accurate
- Longer the horizon, less accurate the forecast
- Forecasts shouldn’t be used to the exclusion of known information
Approaches to Forecasting

- **Subjective Methods** - *human judgment*
  
  - Sales force Composites
  - Customer Surveys
  - Expert Opinion
  - Delphi

- **Objective Methods** - *analysis of data*
  
  - Causal Methods - *Econometrics*
    
    \[ Y = f(X_1, X_2, \ldots, X_n) \]  
    
    \( X_i \) is a variable related to \( Y \)
  
  - Time Series Methods
Time Series (TS) Methods

1. Trend

2. Seasonality

3. Cycles - *length may vary*

4. Random
Notation

\[ D_i : \text{Demand during period } i \]
\[ F_{t,t+\tau} : \text{Forecast made in period } t \text{ for period } t + \tau \]

Simplification - *one period forecast*

\[ F_{t-1,t} = F_t \]

Basic idea of TS

\[ F_{t,t+\tau} = \sum_{n=0}^{\infty} a_n D_{t-n} \quad \text{for some weights of } a_0, a_1, a_2, \ldots \] (2)
2. EVALUATING FORECASTS
\[ e_t = \text{error in period } t \]
\[ = F_{t-t_i} - D_t \]
\[ = F_t - D_t(\text{onestep}) \]

- **MAD**: Mean Absolute Deviation
\[
MAD = \frac{1}{n} \sum_{i=1}^{n} |e_i| \tag{3}
\]

- **MSE**: Mean Squared Error
\[
MSE = \frac{1}{n} \sum_{i=1}^{n} e_i^2 : \sigma \approx 1.25 \text{ MAD for } \mathcal{N}(\ldots) \tag{4}
\]

- **MAPE**: Mean Absolute Percentage Error
\[
MAPE = \left( \frac{1}{n} \sum_{i=1}^{n} \frac{|e_i|}{D_i} \right) \times 100 \tag{5}
\]

**Desirable properties of forecasts** - Simplicity

- **Accuracy**: 
  a. *Unbiasedness*, \( E(e_i) = 0 \), b. *low variance*
3. METHODS OF FORECASTING STATIONARY SERIES
Stationary $\rightarrow$ no trend; no seasonality; no cycles

\[ D_t = \mu + \epsilon_t \] (6)
1. MA: Moving Averages (MA(\(N\)) - over \(N\) periods)

\[
F_t = \frac{1}{N} \sum_{i=t-n}^{t-1} D_i
\]  \hspace{1cm} (7)

**Drawbacks:** • N? • keep last N observations • lags trend
2. ES: Exponential Smoothing, \( ES(\alpha) \)

\( \alpha \) is a smoothing constant, \( 0 < \alpha \leq 1 \)
Weighted average of last forecast and current demand

\[
F_t = \alpha D_{t-1} + (1 - \alpha) F_{t-1}
\]

\[
= F_{t-1} - \alpha e_{t-1}
\]

for one step

\[
F_{t-1} = \alpha D_{t-2} + (1 - \alpha) F_{t-2}
\]
\[
F_t = \alpha D_{t-1} + (1 - \alpha) \alpha D_{t-2} + (1 - \alpha)^2 F_{t-2}
\]

- Declining set of weights for older periods
- \( \alpha \) near 1 \( \rightarrow \) less weight on old (spiky)
- \( \alpha \) small \( \rightarrow \) more weight on old (stable)

**Drawbacks:**
- Start-up? - MA
- lags trend
Suggested Reading: Similarities and differences between MA and ES
4. TREND BASED METHODS
Specifically account for trend; no seasonality; no cycles

\[ D_t = \mu_t + \epsilon_t; \text{ where } \mu_t = u_1 t + u_2 \]  \hspace{1cm} (8)
1. RA: Regression Analysis

\((x_i, y_i) - n \text{ points}\)

\[ \hat{Y} = a + bX \quad (9) \]

\(X: \text{time}; \hat{Y}: \text{estimate of demand} \)

\[ g(a, b) = \sum_{i=1}^{n} (y_i - (a + bx_i))^2 \quad (10) \]

\[ \frac{\partial g}{\partial a} = 0; \frac{\partial g}{\partial b} = 0 \quad (11) \]

\[ b = \frac{S_{xy}}{S_{xx}}; a = \bar{D} - \frac{b(n + 1)}{2} \quad (12) \]

where

\[ S_{xy} = n \sum_{i=1}^{n} iD_i - \frac{n(n+1)}{2} \sum_{i=1}^{n} D_i \]

\[ S_{xx} = \frac{n^2(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4} \]

\[ \bar{D} = \frac{1}{n} \sum_{i=1}^{n} D_i \]
2. DES: Double Exponential Smoothing

Extension of ES: two smoothing constants, $\alpha$, $\beta$

\[
S_t = \alpha D_t + (1 - \alpha)(S_{t-1} + G_{t-1}) \quad \text{“intercept”}
\]
\[
G_t = \beta (S_t - S_{t-1}) + (1 - \beta) G_{t-1} \quad \text{“slope”}
\]

New slope \quad \text{old slope}

\[
F_t = S_t + G_t \quad \text{(one step)}
\]
\[
F_{t,t+\tau} = S_t + \tau G_t \quad \text{(\(\tau\) step)}
\]

- Usually, $\alpha \geq \beta$
METHODS FOR SEASONAL SERIES
Pattern repeats every N periods

\underline{Representation of Seasonality} Set of multipliers, \( c_t \), for \( 1 \leq t \leq N \), such that:
\[ \sum c_t = N \]
Method for Computing Seasonal Factors

- Compute sample mean, $\mu$
- Divide each observation by $\mu$
- Average the factors for like periods