ZOMBIE APOCOLYPSE

MATH 309 - MATLAB LAB

BRIEFING

Top secret virus 749Z has been compromised. A world wide biological attack is imminent and without intervention will result in the complete annihilation of the human population. Your mission is to identify the cities in North America which are most likely to be attacked first. Once identified, these cities will be quarantined and a special forces team will be deployed to the area.

GRAPHS AND ADJACENCY MATRICES

A graph is a set of node (vertices) and edges between the nodes. Below is a simple graph

![Graph Image]

The adjacency matrix of a graph is a matrix whose entries describe the connections between nodes in the graph. Specifically if a graph has \( N \) nodes then the adjacency matrix is an \( N \times N \) matrix whose \( ij \) entry is 1 if there is an edge between \( i \) and \( j \) and 0 otherwise. The adjacency matrix of the graph above is:

\[
\begin{pmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 \\
0 & 1 & 1 & 0
\end{pmatrix}
\]

Mission 1

(1) Write down the adjacency matrix of the graph below

![Graph Image]

(2) Draw the graph whose adjacency matrix is

\[
\begin{pmatrix}
0 & 1 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0
\end{pmatrix}
\]
Mission 2

Degree of a Node. The degree of a node is the number of edges connected to a node. For example in the graph below node 2 has degree 3 since it is connected to 3 other nodes (1, 3, 4) while node 3 has degree 2 since it is connected to only 2 other nodes (2, 4).

(1) Write down the degree of each node in the graph below

(2) IMPORTANT! Describe a method for determining the degree of a node using the adjacency matrix.
MATLAB.

Transpose. Let $A$ be a matrix. The transpose of $A$ is written as $A^T$ and intuitively it means "flip the matrix." That is, the rows of $A$ become the columns of $A^T$. In MATLAB to take the transpose of a matrix you type $\text{'}$. There is special type of matrix called a symmetric matrix in which the matrix is equal to its transpose. That is $A$ is symmetric if $A = A^T$.

$$\text{>> A = rand(5,5);}
\text{>> AT = A.\text{'};}$$
% AT is the transpose of A

Sum. In MATLAB there is also command to sum the entries of a vector, or the columns of a matrix. The below code should help you understand how this function works:

$$\text{>> v = [1 1 2 3];}
\text{>> sum(v)}
\text{ans = 7}$$
$$\text{>> A = [1 2 3; 4 5 6; 7 8 9];}
\text{>> sum(A)}
\text{ans = [12 15 18]}$$

In the first case the sum function just sums the entries of the vector $v$. In the second case the sum function sums each column of $A$ and stores that information in a vector. So 12 corresponds to the sum of the first column of $A$. Similarly 15 corresponds to the sum of the second column of $A$ and so on.
Mission 3

(1) Use Matlab to find the transpose of the following matrix and record your results

\[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
13 & 14 & 15 & 16
\end{pmatrix}
\]

(2) Write down any 3 x 3 symmetric matrix

(3) Enter the adjacency matrix from Mission 1.1 into Matlab and then takes its transpose. Record your results

(4) Explain why the adjacency matrix of a graph is always symmetric

(5) Use the sum function to check your answer from Mission 2.1 and explain why summing the columns is allowed (instead of summing rows).
Final Mission

Below is an adjacency matrix of 8 North American Cities. An edge between two cities represents a significant threat of the virus spreading between the two cities if the virus outbroke in either city. Your mission is to identify the 3 most dangerous cities so that we can quarantine these cities and stop the virus from spreading.

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

(1) Define what it means for a city to be dangerous.

(2) Enter the adjacency matrix above into MATLAB

(3) Use the tools you’ve learned to identify the three most dangerous cities, and write them down. Use the legend below.

**Legend**
1. Toronto
2. Atlanta
3. Los Angeles
4. Mexico City
5. Miami
6. New York
7. Chicago
8. Vancouver

(4) Finally draw the graph corresponding to the adjacency matrix and visually check that your answers make sense.