PATHS OF A GIVEN LENGTH

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Given a graph we would like to count the number of paths of a given length between two nodes. For example consider the graph below:

```
3  2
  1  4
```

If we want the number of paths of length 3 from 1 to 2 we find that there are 3 paths of length 3:

1 → 2 → 1 → 2
1 → 2 → 3 → 2
1 → 2 → 4 → 2

Length 3 just mean we need to cross exactly 3 edges to get from 1 to 2, which is accomplished above. For another example consider the paths of length 2 from 4 to 1. In this case there is only 1 path:

4 → 2 → 1

Our goal is to write a MATLAB function that counts the number of paths of length n from node i to j given an adjacency matrix for a graph, A (see zombie apocalypse lab for definitions). Throughout the course of 2 recitations we discovered the following simple formula:

The $i, j$ entry of $A^n$ tells us the number of paths of length $n$ from $i$ to $j$. Thus our functions is:

```matlab
function p = num_paths(A,n,i,j)
B = A^n;
p = B(i,j);
end
```

The function first raises $A$ to the $n$th power and then stores that new matrix into a variable called $B$. Then it takes the $i, j$ entry of $B$ and stores it in a variable called $p$. When we declared the function we said that $p$ was a special variable which holds the final piece of information we want, in other words whatever is stored in $p$ is returned by our function to us. Since the $i, j$ entry of $A^n$ is stored in $p$ we are done; this is exactly the information we want our function to tell us.