Two Dimensional Interpolation Project

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Assignment:

Project 4 of Text; Page 185.

Goal of the Project:

To reconstruct multiple targets by using polar and hexagonal sampling of a signal, with a resulting reconstruction in the (x,y) rectangular coordinate domain. Also, use intentional aliasing to generate multiple versions of the reconstructed signals.

Hexagonal Sampling Parameters:

For the hexagonal case, I choose the a,b,c,d matrix as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.0 & 0.1 \\ 0.1 & 1.0 \end{bmatrix} \tag{1}$$

yielding an inverse matrix of:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} 1.01 & -0.101 \\ -0.101 & 1.01 \end{bmatrix} \tag{2}$$

Sampling Constraints:

We must make sure we use the correct sampling rate so as to avoid aliasing. The sampling constraint formulas are:

For the Polar case:

$$\Delta\rho \le \frac{\pi}{X_0} \tag{3}$$

$$\Delta\theta \le \frac{\pi}{\rho_0 X_0} = \frac{\pi}{X_0\sqrt{k_x^2 + k_y^2}} \tag{4}$$

We define a region of support, X_0, of 10. ρ_0 is the maximum value that ρ can take, which for this project is also 10.

For the Hexagonal case:

$$2\pi\left\|\frac{a}{\Delta\alpha}\right| - \left|\frac{c}{\Delta\beta}\right\| \ge 2X_0 \tag{5}$$

$$2\pi\left\|\frac{b}{\Delta\alpha}\right| - \left|\frac{d}{\Delta\beta}\right\| \ge 2X_0 \tag{6}$$

yielding the sampling constraints:

$$\Delta\alpha \le \frac{0.9091\pi}{X_0} \tag{7}$$

$$\Delta\beta \le \frac{0.9091\pi}{X_0} \tag{8}$$

Target Information:

Two targets were chosen for the imaging. Two dimensional rect and disc functions were used, each with the following spatial frequency formulas:

Rect function:

$$F(k_x, k_y) = 2a(\mathrm{sinc}(k_x a)) \times 2b(\mathrm{sinc}(k_y b)) \tag{9}$$

Disk function:

$$F(k_x, k_y) = \frac{RJ_0\left(R\sqrt{k_x^2 + k_y^2}\right)}{\sqrt{k_x^2 + k_y^2}} \tag{10}$$

Each was placed in a different location using the shifting properties of the fourier transform. i.e. multiplied by exp(j*kx*x_0)*exp(j*ky*y_0).

the value of the function $G(\alpha,\beta)$ is multiplied by the jacobian of the sampling function in the case of polar sampling, to counteract the effect that polar sampling has more sample points close to the origin and fewer as ρ increases. The jacobian for polar sampling is equal to 1/ρ

$$G(\alpha, \beta) = (F_{rect}(kx, ky) + F_{disk}(kx, ky)) \times jacobian \tag{11}$$

Reconstruction of Targets:

For EVERY sample point in the sampling domain, we do the following:

1. Find the (kx,ky) coordinates of the sample point (sampled in the (α,β) domain). I will call these coordinates (pkx,pky).
2. Find the CLOSEST point on the desired (kx,ky) grid. I will call this (ckx,cky)
3. For this closest point, and all (kx,ky) points within a 5x5 grid centered at this closest point, the contribution of (pkx,pky) on each (kx,ky) grid point is computed using the following formula:

$$F(kx, ky) = G(k\alpha, k\beta)\, w(kx - pkx)\, h(kx - pkx)\, w(ky - pky)\, h(ky - pky) \tag{12}$$

where h(t) is the sinc interpolating function:

$$h(t) = \sin c\left(\frac{\pi t}{\Delta t}\right) \tag{13}$$

and w(t) is a hamming window defined as:

$$w(t) = 0.54 + 0.46\cos\left(\frac{\pi t}{M\Delta t}\right) \tag{14}$$

Thus, for every sampled point, we have 25 contributions to add to our F(kx,ky) array which, after adding all the contributions from all the sample points will yield the spatial frequency transform of our reconstructed image.

The (x,y) image is recovered by taking the inverse fft (ifft) of this function.

In Figure 1, the “+” corresponds to the sample point of interest in the polar coordinate domain, denoted (pkx,pky). The “o” corresponds to the closest (kx,ky) grid point, denoted as (ckx,cky).

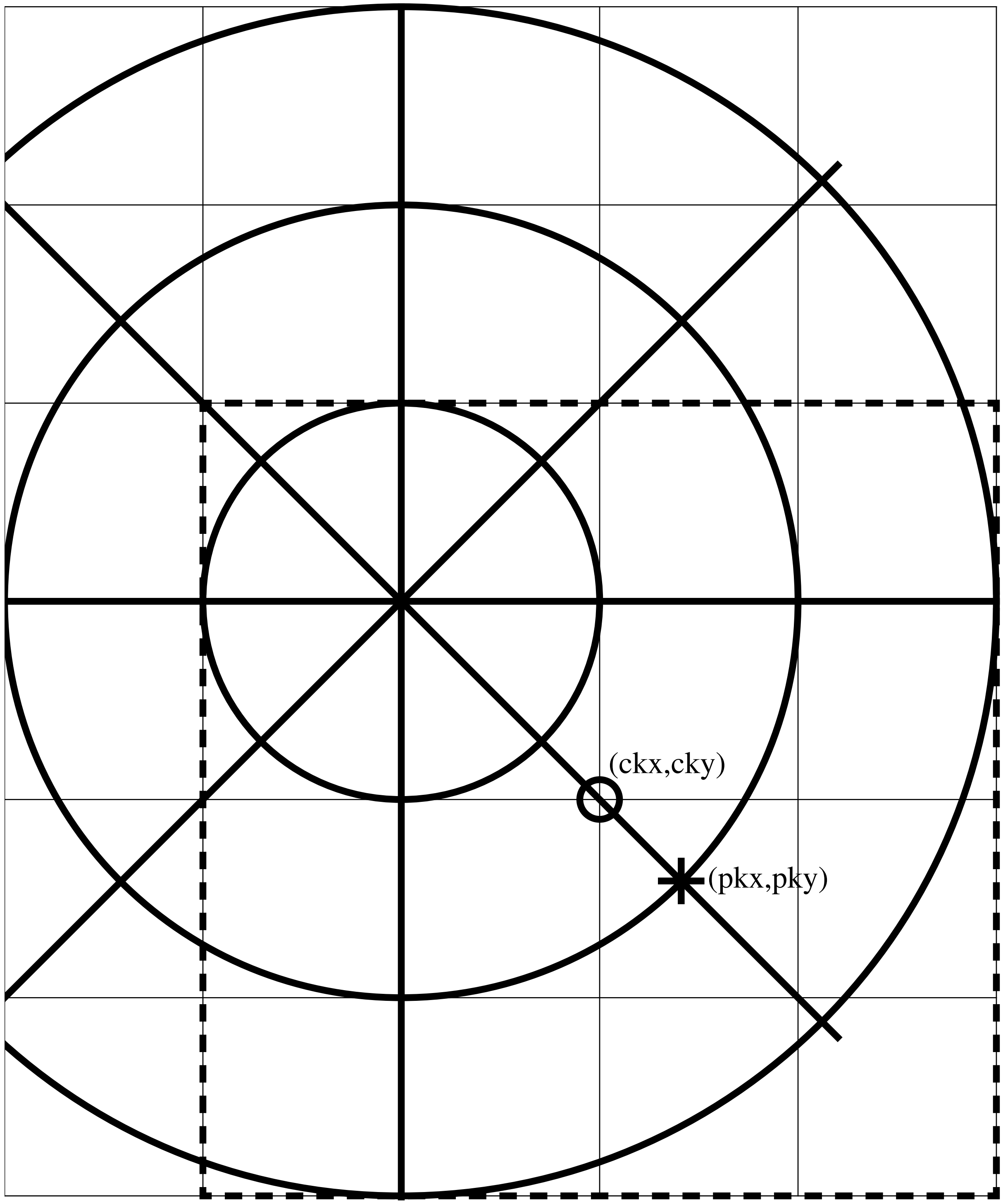

Figure 1

The dashed line surrounding 25 grid points corresponds to the 25 (kx,ky) points that are affected by the contribution from polar sample (pkx,pky). By increasing the size of this box, some improvement in the accuracy of the reconstruction can be obtained, but at the price of increased processing power. Beyond a small distance, the value of the windowing function will become insignificant, as h(t) rapidly decreases with the distance between the sample point and the grid point.

Results:

Targets were imaged using spatial domain sampling and interpolated onto the (kx,ky) axis. Plotted are the 3 cases considered;

1. Rectangular Sampling
2. Hexagonal Sampling
3. Polar Sampling

In order to create images similar to Figures 2.12-2.14, another parameter, X_{00} was used to denote the entire range shown, which was larger than the covered (sampled) region. Thus, aliasing replicated the images several times in the X_{00} region. In this project, $X_{00} = 4\ X_0$.

For the polar sampling case, the reconstructed image has severe distortion due to nonlinear aliasing.

It should also be mentioned that this program was not efficiently written as for as speed of execution is concerned. For a faster program, more matrix operations should be used. However, for illustrative purposes, the method chosen was used.