Parthood and Multi-Location
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This paper is being prepared for publication in *Oxford Studies in Metaphysics*.

1. Introduction

An object is *exactly located at* (or, more briefly, *located at*) a spacetime region just in case the object has (at a time) the same shape, size, and position as the region.[1] In particular, I assume that an object has the same dimension as any spacetime region at which it is located. A number of philosophers have claimed that material objects are exactly located at more than one region of spacetime. See for example [van Inwagen, 1990], [Hudson, 2001], [Sattig, 2006], and [Gibson and Pooley, 2006]. Philosophers who, though perhaps not advocating multi-location, at least treat it as a viable option include [Sider, 2001, 3.4], [Beebee and Rush, 2003], [Bittner and Donnelly, 2004], [McDaniel, 2004], [Crisp and Smith, 2005], [Gilmore, 2006], [Balashov, 2008], and [Hawthorne, 2008].[2] While some philosophers ([Barker and Dowe, 2003, 2005], [Parsons, 2007]) claim not to understand multi-location, I take it that enough has been said to address their concerns elsewhere (particularly in [Beebee and Rush, 2003], [McDaniel, 2003], and [Sattig, 2006]) and proceed under the assumption that spatiotemporal multi-location counts at least as a serious possibility.

Multi-location raises the question of how location interacts with parthood. Let us say that an object is 'uniquely located' just in case it has exactly *one* spatiotemporal location (and thus is *not* multiply located). Over suitably delineated domains of *uniquely* located material objects[3], the logical link between location and parthood should be

[1] In this paper, I use 'is (exactly) located at' to mean what [Sattig, 2006] means by 'occupies', what [Hudson, 2001] and [Gilmore, 2006] mean by 'exactly occupies', and what [Hawthorne, 2008, p. 275-6] means by 'is wholly located at'. I have borrowed my gloss on 'exact location' from Gilmore's explanation of 'exact occupation': "This relation... is said to hold between a thing and a region just in case ... the thing *exactly fits* into the region, where this is meant to guarantee that the thing and the region have precisely the same shape, size, and position"[2006, p. 200].

[2] Most of the philosophers listed here treat multi-location as the default position for three-dimensionalists (or, endurantists) who are spacetime substantivalists. Given spacetime substantivalism, material objects must be located at spatiotemporal regions. If objects are three-dimensional, their locations must be three-dimensional. But then a persisting three-dimensional object has multiple locations in spacetime, since, taken together, its locations cut a four-dimensional path across multiple times.

[3] The 'suitably delineated' qualification is intended to accommodate philosophers such as [Doepke, 1982] or [Lowe, 2003] who think that material objects *x* and *y* may be such that at least one of *x*'s locations is included in one of *y*'s locations even though *x* is never (or, at no region) a part of *y*. For example [Lowe, 2003] claims that a statue and its structural parts (its arms, legs, etc) are never parts of the lump of bronze constituting the statue even though the lump of bronze and the statue are temporarily co-located. [Doepke, 1982] makes similar claims concerning a ship and the wood constituting it. I have a hard time making sense of this view. But I take it that even philosophers of the same mind as Doepke or Lowe would agree that on certain *restricted* object domains--e.g., domains limited to just artifacts or to just lumps of stuff--parthood is completely determined by inclusion relations among objects' locations.

See also [Saucedo, forthcoming] which assumes that parthood and location are wholly distinct relations and argues that it is possible that i) there are material objects *x* and *y* such that *x*'s location is included in *y*'s location but *x* is not (ever) part of *y*; and ii) there are material objects *x* and *y* such that *x* is part of *y* but *x*'s location is not included in *y*'s location. I have a hard time seeing why anyone would think that parthood and location are wholly distinct relations. (In particular, I cannot imagine why anyone would think that (ii) straightforward: object x is part of object y if and only if x's unique location is included in y's unique location. But if material objects are multiply located, things are more complicated. For then some, but not all, of x's locations may be included in some, but not all, of y's locations. We would in this case appear to have multiple possibilities for aligning a binary parthood relation with the location relation.

In fact, though, the most common move among proponents of multi-location is to abandon a simple binary parthood relation in favor of a more complex parthood relation that somehow narrows down the range of locations under consideration. [Hudson, 2001] [Bittner and Donnelly, 2004], [McDaniel, 2004], [Crisp and Smith, 2005], [Gilmore, 2006], and [Balashov, 2008] all make use of ternary *region-relative* parthood relations. These relations are intended to capture a sense in which x may be part of y at some spacetime regions but not at other spacetime regions. However, although most authors leave the details of their region-relative parthood relations underspecified, it is clear enough from what is said that the different authors assume different region-relative parthood relations. For example, one obvious source of variation among these relations is in different restrictions on the regions to which parthood is relativized. [Balashov, 2008] restricts the region argument of his parthood relation to *achronal regions*—regions which exclude spacetime points that are absolutely temporally separated. By contrast, [McDaniel, 2004] suggests that the region argument of his parthood relation should range only over maximally continuous three-dimensional slices of spacetime.[4] And neither [Hudson, 2001] nor [Crisp and Smith, 2005] place any global restrictions at all on the range of the region argument for their parthood relations.

This paper is an attempt to work out in detail some of the more promising options for interpreting and axiomatizing region-relative parthood relations. A particular objective of this paper is to evaluate different region-relative parthood relations in terms of, on the one hand, classical mereological principles and, on the other hand, ordinary assumptions about parthood. Here, we should not expect an exact match on either count. Whereas classical mereology assumes a binary parthood relation, all region-relative parthood relations considered in this paper are ternary. Still, we will see that the region-relative parthood relations considered below satisfy ternary counterparts of some classical principles. Also, although parthood is never explicitly relativized to spacetime regions in ordinary discourse, we do often relativize parthood to times. It is not unreasonable to expect that the ordinary time-indexed approach to parthood can be cashed out in the spacetime context in terms of a relation that links parthood to special regions—frame-relative time-slices or other achronal regions—that roughly correspond to ordinary times.

My line of approach in this paper is to first introduce specific parthood relations over classes of mathematical models and to then assess the properties of these relations, using classical mereology as a reference point. Because of the complexity of this task, I will, perhaps somewhat arbitrarily, narrow the scope of this study in the following ways. First, I assume that parthood relations among material objects are, in one way or another,

is possible.) But even Saucedo concedes that some possible worlds (including perhaps the actual world) are such that: for any objects x and y, x is part of y if and only if x's location is included in y's location.

[4] Note that whereas an achronal region could include just one point, a three-dimensional slice of spacetime must include a three-dimensional space. On the other hand, there are maximally continuous three-dimensional slices of spacetime that include absolutely temporally separated spacetime points and thus are not achronal.

entirely determined by inclusion relations among their locations. Without this assumption—and in the absence of any general principle detailing what exactly, besides some form of location inclusion, is required for parthood relations to hold—we could at best attribute only extremely weak logical properties to the relations introduced here.[5, 6] Second, for the most part, I will say little about *summation* (or, *fusion*) relations in this paper. In particular, I will not consider any version of a universal summation principle guaranteeing the existence of arbitrary sums of objects. I limit my discussion of summation relations because, though important, they bring along with them cumbersome baggage that would require too much digression from the main thread of this paper.[7] (However, I do note below some cases in which the introduction of an appropriate summation relation presents special difficulties for the parthood relation under consideration.) Third, although I allow (but do not require) that objects have multiple spacetime locations, I will throughout this paper place certain other restrictions on the way objects may be located in spacetime. Most controversially, except where noted otherwise, I assume that no more than one object is exactly located at any spacetime region. This and two other (more modest) restrictions are designed to lead to a fairly close match between the region-relative parthood relations and ternary counterparts of classical mereological principles. Though there is no reason to assume that any acceptable parthood relation must satisfy *all* classical mereological principles, many philosophers do assume that parthood behaves something like the classical mereological relation. It is thus worthwhile to see how close the region-relative parthood relations can come to supporting a classical mereological structure. But it is easy to see how, for those who want it, material coincidence can be accommodated (by discarding the prohibition on co-location) with only a localized adjustment in the formal axiomatization. I will discuss this alternative briefly in Section 4.

The remainder of this paper proceeds as follows. In Section 2, I introduce a general class of location models (L Models) as well as the four classical mereological principles that I use as a starting point for assessing the parthood relations. In Section 3, I consider possibilities for *binary* parthood relations on L Models. In Section 4, I suggest two alternatives for a (time-)slice-relative parthood relation. In Section 5, I focus on the very different region-relative parthood relation proposed in [Hudson, 2001].

2. Location, Regional Inclusion, and Classical Mereological Principles

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[5] This point is easy to illustrate in the simple case of unique location. Even if all objects had unique locations, we still could not infer, e.g., that parthood is transitive from *just* the assumption that, whenever x is part of y, x's unique location is included in y's unique location. If, on the other hand, we assume that location-inclusion is necessary *and sufficient* for parthood, then the transitivity of parthood follows immediately from the transitivity of the inclusion relation over the domain of spacetime regions.

[6] As stated in Note 3, I assume that even philosophers like Doepke and Lowe, who think that something more than location-inclusion is required for parthood, will allow that *on suitably restricted domains of objects*, parthood is entirely determined by inclusion relations among locations. If this is right, then these philosophers can take the results of this paper as valid at least on these proper sub-domains of material objects. (And [Saucedo, forthcoming] can take my results as valid at least in *some possible worlds*.) Even with these restrictions, the considerations raised in this paper still establish that there are important differences between the various region-relative relations proposed in recent literature.

[7] Baggage in the form of: i) additional machinery in the formal mereology for quantifying over sets, plurals, or something else along these lines and ii) controversy over universal summation principles.

I initially assume little more than that spacetime is a non-empty set of points, that regions are the non-empty subsets of spacetime, that every object is (exactly) located at some—but possibly at more than one—region, and that there is no region at which two objects are located. These assumptions are represented in the most general class of models introduced in this paper—Location (L) Models. L Models are ordered quadruples <**ST**, **R, OB, L**> where[8]

1. **ST** (*spacetime*) is any non-empty set of points;
2. **R** (*the region domain*) is the set of non-empty subsets of **ST** (i.e. **R** $= \wp(\mathbf{ST})\backslash\{\varnothing\}$)[9];
3. **OB** (*the object domain*) is any non-empty set disjoint from **R**;
4. **L** (*the location relation*) is any set of ordered pairs $<x, r>$ with $x \in \mathbf{OB}$ and $r \in \mathbf{R}$ such that
   i) for each $x \in \mathbf{OB}$, $<x, r> \in \mathbf{L}$ for at least one $r \in \mathbf{R}$ (i.e., **OB** is the domain of **L**);
   ii) if $<x, r>, <y, r> \in \mathbf{L}$, then $x = y$;
   iii) if $<x, r_x>, <y, r_y> \in \mathbf{L}$ and $r_x \backslash r_y \neq \varnothing$, then there is some $<z, r_z> \in \mathbf{L}$ such that $r_z \subseteq r_x \backslash r_y$;
   iv) if $<x, r_x>, <y, r_y> \in \mathbf{L}$ and $r_x \cap r_y \neq \varnothing$, then there is some $<z, r_z> \in \mathbf{L}$ such that $r_z \subseteq r_x \cap r_y$.

I will say that region r 'is a location' if and only if some object is located at r. Thus the class of all locations in an L Model <**ST**, **R, OB, L** > is the subclass of **R** which is the range of **L**.[10] Conditions (4.iii) and (4.iv) ensure that there are enough locations to 'represent' non-empty differences between locations and non-empty intersections of locations. Condition (4.iii) tells us that if the difference of location r_y in location r_x is non-empty, then there is some location r_z lying within this difference. Condition (4.iv) requires that if the intersection of location r_x and location r_y is non-empty, then some location r_z lies within this intersection. Conditions (4.iii) and (4.iv) will, for suitably defined parthood relations, guarantee that the mereological structure of the object domain at least *weakly* corresponds to the mereological structure of the region domain—objects

[8] I adopt the following notational conventions throughout this paper. I use **UPPER CASE TIMES BOLD** for the names of distinguished classes in the models (including relations—these are represented as sets of ordered pairs). I use *lower case times italics* for variables ranging over members of these classes. The mereological principles considered in this paper are presented in standard first-order predicate logic with identity. In the formal language, I use **BOLD UPPER CASE ARIEL** for predicates and lower case ariel for variables. The reader is strongly advised not to confuse the predicates of the formal language with the relations of the models—I will consider different model theoretic relations as alternative interpretations for the *same predicate* in the formal language.

[9] For any set **X**, $\wp(\mathbf{X})$ is the *power set* of **X**; i.e., $\wp(\mathbf{X}) = \{\mathbf{Z} : \mathbf{Z} \subseteq \mathbf{X}\}$. For any sets **X** and **Y**, **X****Y** is the *difference* of **Y** in **X**; i.e., $\mathbf{X}\backslash\mathbf{Y} = \{x : x \in \mathbf{X} \text{ and } x \notin \mathbf{Y}\}$. Thus, the region domain, $\wp(\mathbf{ST})\backslash\{\varnothing\}$, is the set of all subsets of **ST** except, $\varnothing$ (the sole member of $\{\varnothing\}$).

[10] Warning: I am using 'location' as a convenient technical term for a certain class of *spatiotemporal* regions—those spatiotemporal regions at which some object is exactly located. This usage is not intended to match the more common use of the term 'location' to pick out certain kinds of *spatial* regions through which objects might move.

need not have so many parts as their locations have subregions, but an object must have some part wherever its location is divided by intersecting locations.[11]

Notice that L Models are fairly general in that they leave open not only the specific geometric structure of spacetime, but also several important questions concerning the ways in which objects are distributed to locations in spacetime. As intended, L Models allow, but do not require, that objects are multi-located. In other words, an object *x* may stand in the location relation **L** to multiple regions. Or, *x* may stand in **L** to exactly one region. Further, in cases where an object has multiple locations, L Models place no restrictions on the spatiotemporal relations holding between these locations. For example, there is no requirement that all of an object's locations overlap as [Hudson, 2001] assumes.[12] Neither do L Models require that an object's locations are pairwise discrete, as they would be if they were distributed to discrete time-slices as is assumed in [Sattig, 2006, 2.1]. Also, there is no requirement that an object's locations are *additive* in the sense that x is located at both r and r^* only if x is also located at $r \cup r^*$.[13]

Following [Gilmore, 2006] and [Balashov, 2008], I use the location relation to define the *path* of an object x as that region which is the union of x's locations.

$$\mathbf{PATH}(x) = \cup_{<x,\, r> \in \mathbf{L}}\,(r).$$

Notice that since locations are not required to be additive, $\mathbf{PATH}(x)$ need not be one of x's locations (or, for that matter, the location of *any* object).

In the following sections, I introduce different parthood relations into L Models or extensions of L Models and use formal mereological principles as one way of highlighting important distinctions between these relations. The most familiar mereological principles are those used to axiomatize classical mereology. Though we will need more complex principles to evaluate the *ternary* region-relative relations, I take the following four formal principles for the *binary* parthood predicate **P** as a starting point from which to arrive at plausible ternary principles.[14]

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[11] Weaker or stronger options for regulating the distribution of locations are possible. Philosophers who think that objects are located only at three- (or four-) dimensional regions may opt for weaker versions of (4.iii)-(4.iv), requiring only that some object is located within all three- (or four-) dimensional intersections or differences of locations. At the other extreme, we might require that *every* subregion of a location is a location. Such a requirement would entail that all non-empty intersections of locations and all non-empty differences between locations are *themselves* locations (as are all of their proper subregions). For the most part, the particular choice of weaker or stronger versions of (4.iii) and (4.iv) is not relevant to our concerns.

[12] Regions r_1 and r_2 *overlap* if and only if r_1 and r_2 share some subregion. (Equivalently, r_1 and r_2 overlap if and only if $r_1 \cap r_2 \neq \varnothing$.) Regions r_1 and r_2 are *discrete* if and only if r_1 and r_2 do not overlap. (And r_1 and r_2 fail to overlap if and only if $r_1 \cap r_2 = \varnothing$.)

[13] The additivity assumption plays a key role in [Barker and Dowe, 2003], where it is used to derive a contradiction from the claim that objects persist by having multiple three-dimensional locations in spacetime. The additivity assumption is explicitly rejected in [Sattig, 2006], [Gilmore, 2006], and [Gilmore, 2007].

[14] In fact, by a 'classical mereology' most philosophers mean a rather stronger theory which, in addition to counterparts of (CM1)-(CM4), also includes a universal summation axiom requiring that every collection of individuals has a mereological sum (or, fusion). (See [Simons, 1987] for a detailed comparison of different mereologies and a discussion of universal summation axioms.) As stated in the introduction, I do not consider universal summation principles in this paper. Also, I do not introduce proper parthood,

(CM1) $\mathbf{P}xx$ (every individual is part of itself)
(CM2) $\mathbf{P}xy \;\&\; \mathbf{P}yz \rightarrow \mathbf{P}xz$ (if x is part of y and y is part of z, then x is part of z)
(CM3) $\sim\mathbf{P}xy \rightarrow \exists z\, (\mathbf{P}zx \;\&\; \sim\exists w(\mathbf{P}wz \;\&\; \mathbf{P}wy))$ (if x is not part of y, then x has a part that shares no parts with y)
(CM4) $\mathbf{P}xy \;\&\; \mathbf{P}yx \rightarrow x = y$ (if x is part of y and y is part of x, then x and y are identical)

I will call the theory axiomatized by (CM1)-(CM4) in standard first-order predicate logic '**CM**'. Classical mereologies generally introduce several different defined mereological predicates. Within **CM**, I introduce, besides the primitive **P**, only the overlap predicate **O**. Definition ($D_{\mathbf{O}}$) is standard.

($D_{\mathbf{O}}$) $\mathbf{O}xy$ (x overlaps y) $=_{def} \exists z(\mathbf{P}zx \;\&\; \mathbf{P}zy)$ (some individual is part of both x and y)

Notice that supplementation axiom (CM3) is equivalent to the following more compact formula:

$\sim\mathbf{P}xy \rightarrow \exists z\, (\mathbf{P}zx \;\&\; \sim\mathbf{O}zy)$ (if x is not part of y, then x has a part that does not overlap y).

We already have one suitable interpretation for **CM**'s primitive **P**. It is easy to verify that when **P** is interpreted as the set inclusion relation $\subseteq$ on the region domain **R** (i.e., as the set of ordered pairs $\subseteq_{\mathbf{R}} = \{<r, r^*> : r, r^* \in \mathbf{R} \text{ and } r \subseteq r^*\}$), each of (CM1)-(CM4) is satisfied.[15] On this interpretation of **P**, the overlap predicate **O** is interpreted as the relation that holds between $r, r^* \in \mathbf{R}$ if and only if r and r^* overlap. More precisely, **O** is interpreted as $\mathbf{O}_{\mathbf{R}} = \{<r, r^*>: r, r^* \in \mathbf{R} \text{ and } r \cap r^* \neq \varnothing\}$.

It should come as no surprise that the inclusion relation on **R** satisfies the axioms of classical mereology. Classical mereologies were designed with the set theoretic inclusion relation (or, what for our purposes amounts to the same thing, the partial ordering of a Boolean algebra) as an intended model theoretic interpretation for the parthood predicate (see, e.g., [Simons, 1987], [Tarski, 1956]). But unless objects can be put in correspondence with regions (or, at least, with the subclass of regions that includes all *locations*) in a way that exactly aligns a candidate parthood relation on the object domain with the inclusion relation on locations, it is not obvious that there is an appropriate parthood relation for objects that satisfies (CM1)-(CM4). And multi-location makes any such a correspondence between objects and their locations particularly unlikely. For, given multi-location, the way in which objects are arranged in spacetime must be radically different from the way in which regions are arranged in spacetime—regions definitely do *not* have multiple positions in spacetime. Thus, given multi-location, we cannot assume that there is a binary parthood relation on objects which 'mirrors' the

differences, or intersections into **CM**. For these reasons, **CM** should be considered a *weak version* of classical mereology. It is strong enough, however, to lead us to interesting comparisons between the parthood relations considered in this paper.

[15] The only one of **CM**'s axioms whose verification on this interpretation is not entirely trivial is (CM3). To see that (CM3) is satisfied, suppose that $r, r^* \in \mathbf{R}$ and $r \not\subseteq r^*$. Then there is some $s \in \mathbf{ST}$ such that $s \in r$ and $s \notin r^*$. $\{s\} \subseteq r$ and (since $\varnothing$ is not a member of **R**) there is no region which is included in both $\{s\}$ and r^* (i.e., $\{s\}$ and r^* do not overlap).

inclusion relation on locations. We will see in the next section that none of most reasonable potential candidates for a binary parthood relation on the object domains of L Models satisfies (CM1)-(CM4). So multi-location seems to require some kind of revision of classical mereological principles.[16] The ternary region-relative parthood relations that are the focus of this paper retain much of the strength of a classical binary relation by restricting parthood ascriptions to regions.

3. Binary Parthood Relations for Object Domains

Just what is it about multi-location that drives us away from a more traditional binary parthood relation and motivates us to look for appropriate ways of relativizing parthood to times or to spacetime regions? In fact, even if we assume that objects have multiple locations in spacetime, there are a number of different ways we *might* reasonably attempt to introduce binary parthood relations over object domains. Here are the four most promising possibilities.

(**Occasional Parthood**) $<x, y> \in \mathbf{P_{OC}}$ if and only if there are regions r_x and r_y such that $<x, r_x>, <y, r_y> \in \mathbf{L}$ and $r_x \subseteq r_y$.
(**Bound Parthood**) $<x, y> \in \mathbf{P_{BD}}$ if and only if, for any region r_x such that $<x, r_x> \in \mathbf{L}$, there is some region r_y such that $<y, r_y> \in \mathbf{L}$ and $r_x \subseteq r_y$.
(**Constant Parthood**) $<x, y> \in \mathbf{P_{CT}}$ if and only if, for any region r_y such that $<y, r_y> \in \mathbf{L}$, there is some region r_x such that $<x, r_x> \in \mathbf{L}$ and $r_x \subseteq r_y$.
(**Path Parthood**) $<x, y> \in \mathbf{P_{PT}}$ if and only if $\mathbf{PATH}(x) \subseteq \mathbf{PATH}(y)$.

The differences between these four binary parthood relations are most easily illustrated if we assume for the moment that spacetime is partitioned by a unique collection of instantaneous time-slices and that each ordinary object—each person, table, car, and so on—has a unique location within each time-slice through which it persists.[17] On this view, for example, Jane Austen (JA) is located only at subregions of three-

[16] A caveat: The fact that the binary relations considered in the next section fail to satisfy (CM1)-(CM4) over L Models depends on the restrictions i) – iv) on the location relation. As we will see at the end of the next section, one obvious way of forcing the binary relations to satisfy (CM1)-(CM4) is to eliminate multi-location by requiring that each object stand in **L** to no more than one region. But, instead of prohibiting multi-location, we *could* reformulate conditions ii)-iv) so that that they are stated in terms of objects' paths instead of their locations. I think such reformulated conditions seem less natural, but are not obviously untenable. With this change in the structure of L Models, the fourth of the binary relations considered in the next section, $\mathbf{P_{PT}}$ (path parthood), would satisfy each of (CM1)-(CM4) even on models in which objects have multiple locations. But, even so, the path parthood relation would suffer the same inadequacy as the standard four-dimensionalist binary parthood relation—it does not preserve common-sense assumptions about which parts objects have. Thus, even if by introducing more complex restrictions on the way objects are located in spacetime, we can come up with a binary relation for multiply located domains which satisfies classical principles, we still have reasons (the same reasons four-dimensionalists have) for finding an alternative parthood relation that better fits common sense.

[17] Something like this view of objects' locations in spacetime is presented in [van Inwagen, 1990] and in [Sattig, 2006, 2.1]. But whereas [van Inwagen, 1990] leaves open (but does not advocate) the possibility that an object is located *both* at subregions of instantaneous time-slices *and* at its four-dimensional path, I assume in these examples that ordinary objects are located only at three-dimensional subregions of instantaneous time-slices.

dimensional time-slices and has a unique three-dimensional location within each instantaneous time-slice between 1775 and 1817.

An *occasional part* of JA is any object having at least one location included within one or more of JA's locations. For example, Jane's teeth, hair, cells, head, and hands are all occasional parts of JA.

To be not just an occasional part of JA, but also a *bound part* of JA, an object cannot have any location that is not included in one of JA's locations. This condition is *not* satisfied by objects, such as JA's head, hands, nose, and eyes, that we ordinarily think of as JA's most salient parts. These objects all, however briefly, survive JA's death. But some cells are bound parts of JA. For example, any red blood cell that remains within JA throughout its short life is a bound part of JA. (However, cells that survive removal from JA are not bound parts of JA.)

On the other hand, JA's head, hands, nose, and eyes *are* constant parts of JA. A *constant part* of JA has at least one location within each of JA's locations and JA (fortunately) retained her head, hands, nose, and eyes throughout her life. By contrast, most cells did not remain with JA for all of her 41 years and thus are not constant parts of JA. Similarly, none of JA's teeth or hairs is a constant part of JA. Nor are any of the molecules and atoms that passed in and out of JA during her lifetime.

A *path part* of JA is any object whose path is included in JA's path. It is easy to see that any bound part of JA must also be a path part of JA. Moreover, if ordinary objects such as hands, cells, and so on, are located only at disjoint three-dimensional subregions of time-slices (as we for the moment assume they are), then any one of *these* objects will be a path part of JA *if and only if* it is a bound part of JA (since in this case its path is included in JA's path if and only if, within each time-slice through which it persists, its location is included in JA's location). To see that path parthood need not always imply bound parthood (and thus that path parthood is *strictly weaker than* bound parthood), imagine that, besides objects located at multiple three-dimensional regions, there are other objects—call them 'events'—each of which is located at a unique four-dimensional region. Then an event—a particular clenching of one of JA's hands or a blinking of her eyes—may be a path part of JA even though it cannot be a bound part of JA, or even an occasional part of JA, since its four-dimensional location is not included in any of JA's three-dimensional locations.

One immediate dilemma in attempting to introduce a binary parthood relation over the object domains of L Models is that no one of these four binary relations clearly distinguishes itself as *the* parthood relation. There is no obvious reason for, say, preferring constant parthood over the three other relations.

Further, taken individually, each of $\mathbf{P_{OC}}$, $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ has shortcomings that should motivate us to look for an alternative treatment of parthood (even if we concede, as I think we should, that each of the four binary relations is useful on its own terms). The most obvious problem with $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ is that, at least on the three-dimensionalist interpretation adopted for the Jane Austen example, none of these relations preserve uncontroversial common-sense assumptions about the parts of objects like JA. We saw this above--Jane's head does not stand in $\mathbf{P_{BD}}$ or in $\mathbf{P_{PT}}$ to JA, most of Jane's cells do not stand in $\mathbf{P_{CT}}$ to JA, and so on. Less obviously, $\mathbf{P_{OC}}$ also fails to match ordinary usage in that it provides no mechanism for specifying the different times at which objects have specific parts. Objects that never comprise JA at the same time—her baby teeth and a mole that appears in her fortieth year—all count in *the same way* as occasional parts of JA. In ordinary usage, we can distinguish these objects as parts of JA at different times.

A different sort of issue with $\mathbf{P_{OC}}$, $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ is that each of these relations is relatively weak, with $\mathbf{P_{OC}}$ (perhaps the least problematic from the point of view of common sense) being the weakest of the bunch. I take it that this is a separate concern from the failure of these relations to preserve intuitive assumptions about what parts objects have. In non-philosophical contexts, parthood relations are not usually put to work in complicated reasoning or organizational tasks. And I cannot see that non-philosophers have intuitions about what logical properties a parthood relation is supposed to have. But many philosophers do use parthood relations to introduce structure into object domains. Temporal parts, mereological sums (or fusions), and other key players in current philosophical debates are typically defined in terms of a parthood relation.[18] Moreover, philosophers do often use the assumption that mereological relations have certain fairly strong logical properties—in particular, that parthood is transitive and satisfies some version of a supplementation principle—to support controversial claims.[19] It would be an unfortunate thing for these philosophers if the only available parthood relation turned out to be much too weak to support interesting reasoning about the way objects are structured.

I will try to make this point clearer by using the weakest of our binary relations as an example. Of **CM**'s four axioms, $\mathbf{P_{OC}}$ satisfies only (CM1) over all L Models—the occasional parthood relation is reflexive and not much else. In particular, $\mathbf{P_{OC}}$ is not transitive. For example (recurring to our three-dimensionalist picture of JA), a certain protein molecule may be an occasional part of a blood cell and the blood cell may, in turn, be an occasional part of JA, even though the protein molecule is *not* an occasional part of JA. This would be so if the blood cell acquired the protein molecule only after leaving JA (while it was, say, lying in a vial in her doctor's laboratory). To see that $\mathbf{P_{OC}}$ does not generally satisfy the supplementation axiom, (CM3), imagine that a certain almond is never located within JA. But, after the almond is destroyed by being smashed into a marzipan paste, all of its component molecules are incorporated into JA's body.[20] In this case, the almond is not an occasional part of JA, but all of its occasional parts share occasional parts with JA (so, substituting into (CM3), we get a conditional with a true antecedent and a false consequent).

Because it is itself so very weak, the occasional parthood relation yields an extremely flimsy overlap relation. For x and y to overlap in the $\mathbf{P_{OC}}$ way, it is sufficient for some object z to be located within one of x's locations and also within one of y's locations. But z does not have to be located in the *intersection* of any location of x with a location of y. Thus, x and y may overlap in the $\mathbf{P_{OC}}$ sense even if all of their locations are

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[18] Temporal parthood relations are defined in terms of mereological relations in, e.g., [Sider, 2001, Ch 3]. [Sider, 2001] also includes mereological definitions of summation relations (as does, e.g., [Simons, 1987]).

[19] Recent examples of assumptions about the logical properties of (binary or ternary) parthood at work in an argument are found in: [Sider, 2001, p. 65], [Crisp and Smith, 2005, p. 335], and [Olson, 2006, p. 742]. In each of these cases it is a variant of the classical supplementation principle, (CM3), which is used as a premise in an argument.

[20] Let's avoid complications by assuming that the almond never loses molecules or other microscopic parts. If necessary, it can be a very short-lived almond.

disjoint. In practical terms, JA may $\mathbf{P_{OC}}$-overlap with a man she has never met if just one atom is at one time located in her body and later incorporated into his body. Another weakness in $\mathbf{P_{OC}}$-overlap is that it is not transferred from parts to wholes as is the overlap relation of classical mereology. More precisely, z may $\mathbf{P_{OC}}$-overlap an occasional part, x, of y even though z does not $\mathbf{P_{OC}}$-overlap y. For example, JA's red blood cell in the laboratory vial $\mathbf{P_{OC}}$-overlaps a protein molecule which (we may presume) does not $\mathbf{P_{OC}}$-overlap JA.

What kind of a summation relation would we have in such a murky mereology? Using plural quantification, the standard definition of (atemporal) summation runs something like this:

x is a sum of the ys $=_{\text{def}}$ each of the ys is part of x and any object that is part of x overlaps at least one of the ys.

Plugging the $\mathbf{P_{OC}}$ relations into this definition, we end up with sums that need not extend over the entirety of their summands. In particular, x may be a $\mathbf{P_{OC}}$-sum of the ys even though some (or even all) of the ys have $\mathbf{P_{OC}}$-parts that are not also $\mathbf{P_{OC}}$-parts of x. For example, JA is a $\mathbf{P_{OC}}$-sum of all of the molecules that ever make up her body. But any of these molecules may have $\mathbf{P_{OC}}$-parts that are not $\mathbf{P_{OC}}$-parts of JA—this could be so if, say, a particular water molecule is at one time incorporated in JA and acquires a new electron after leaving JA. Also, not only may one plurality have two or more $\mathbf{P_{OC}}$-sums (so that, in general, there is no such thing as *the* $\mathbf{P_{OC}}$-sum of a given plurality), but the different $\mathbf{P_{OC}}$-sums of a fixed plurality need not even coincide. In fact, a given plurality may have $\mathbf{P_{OC}}$-sums whose spacetime paths are completely disjoint. Suppose, for example, I build a shed from a bunch of sticks and later dismantle the shed to build a fence from the same sticks. Then the shed is a $\mathbf{P_{OC}}$-sum of the sticks and the fence is also a $\mathbf{P_{OC}}$-sum of the sticks, even though the shed and the fence are never in this world at the same time.

And there is worse still. Nothing in our very weak mereology prevents a whole from being a $\mathbf{P_{OC}}$-sum of just *one* of its proper $\mathbf{P_{OC}}$-parts, where a *proper* $\boldsymbol{P_{OC}}$*-part* of x is any occasional part of x other than x itself. Suppose there is some organism x which is so constructed that every microscopic particle ever entering its body is incorporated at some time into a special organ designed to arrange particles into a suitable form. (I have no idea whether there actually is such an organism. But surely it is possible.) Let z be x's 'rearrangement' organ. Then x is a $\mathbf{P_{OC}}$-sum of the plurality consisting of just z—z is an occasional part of x and z $\mathbf{P_{OC}}$-overlaps every occasional part of x.[21] But this is really odd. Summands are supposed to, in some sense, make up *all* of the object to which they sum. But z is just one organ within x. We may assume that x includes other organs which are spatially separated from z. Insofar as z does not (ever) extend over these other organs, z does not seem to make up all of x.

I hope the discussion above suffices to convince readers that, if such a weak relation as $\mathbf{P_{OC}}$ were what philosophers refer to when they make claims about parthood, overlap,

[21] To see that z $\mathbf{P_{OC}}$-overlaps every occasional part of x, suppose that y is an object which is at some time included within x. Then, while within x, y is made up of particles within x. By assumption, each of these particles is an occasional part of z (since each is located within z at some time). But then z and y $\mathbf{P_{OC}}$-overlap—each of these particles is an occasional part of both z and y.

summation, and so on, then much of what they say would be false or confused. We should hope that there is a reasonable parthood relation which is stronger than $\mathbf{P_{OC}}$. Now, each of $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ is *slightly* stronger than $\mathbf{P_{OC}}$. But not much stronger. Given our restrictions on L Models, each of $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ is (unlike $\mathbf{P_{OC}}$) transitive. But besides being reflexive and transitive, I cannot see that $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ have any other useful properties. In particular, none of these relations satisfies anything like **CM**'s supplementation principle, (CM3). And none satisfies the antisymmetry principle (CM4).[22] Also, importantly, like $\mathbf{P_{OC}}$, each of $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ generates odd overlap and summation relations. For example, given that a certain stick is a constant part of both my shed and the fence that I built after dismantling the shed, the fence and the shed would $\mathbf{P_{CT}}$-overlap even though these two objects are never in the world at the same time.

But even if $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, or $\mathbf{P_{PT}}$ had nicer logical properties, they would still conflict with important intuitive assumptions about parthood (that JA's head and hands are part of JA) in the ways noted above. It is thus worthwhile investigating region-relative parthood relations in the hopes that some such alternative to the binary parthood relations will do better at preserving intuitive assumptions about parthood while maintaining a structural power comparable to that of classical mereology.

Before proceeding to the investigation of region-relative parthood relations, it is worth noting that two of the major obstacles in the way of binary parthood—that no one binary relation distinguishes itself as *the* parthood relation and that the most obvious candidate relations have weak logical properties—disappear if multi-location is eliminated. I think that this helps account for the fact that nearly all four-dimensionalists (at least among those who deny multi-location) assume that parthood is fundamentally a *binary* relation.

We can see how unique location makes a crucial difference in some of the issues raised above if we consider, not all L Models, but only those in which there is no multi-location. A *Unique Location (UL) Model* is an L Model in which every object has a unique location. In other words, the UL Models are L Models in which the relation **L** is a function from **OB** into **R**. In UL Models, each object is located only at its path.

$\mathbf{P_{OC}}$, $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, and $\mathbf{P_{PT}}$ collapse into a single relation on UL Models. This is because, for any objects *x* and *y* in a UL Model, the following are equivalent: i) *one* of *x*'s locations is included in *one y*'s locations; ii) *all* of *x*'s locations are included in *all* of *y*'s

[22] Given suitable ontological assumptions, plausible additional restrictions on how objects are located in spacetime might render one (or all) of $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, or $\mathbf{P_{PT}}$ antisymmetric. But I cannot think of any reasonable additional restrictions that would force $\mathbf{P_{BD}}$ or $\mathbf{P_{CT}}$ to satisfy a supplementation principle. And the supplementation principle is really the more important of **CM**'s axioms. Philosophers who endorse material coincidence may want to reject antisymmetry anyway (as does as, e.g., Simons in [1987, p. 177-187]). But philosophers of many different persuasions have assumed that a parthood relation must satisfy some version of a supplementation principle (see note 19 above).

To see that, e.g., $\mathbf{P_{CT}}$ does not satisfy (CM3), suppose that, at different times, the same small particles make up two oxygen atoms, O_1 and O_2. Suppose further that O_1 and O_2 have each of the particles as constant parts—they are each made of the same particles throughout their lives. But their lives are confined to separate times. So neither of O_1 or O_2 is a constant part of the other. But every constant part of O_1 shares a constant part (one of the particles) with O_2. (And, likewise, each of O_2's constant parts $\mathbf{P_{CT}}$-overlaps O_1). Plugging into (CM3), we have a conditional with a true antecedent and a false consequence.

locations; iii) x's path is included in y's path. Let $\mathbf{P_{UL}}$ be the binary relation defined on UL Models as follows:

(Unique Location Parthood) $<x, y> \in \mathbf{P_{UL}}$ if and only if $\mathbf{PATH}(x) \subseteq \mathbf{PATH}(y)$.[23]

It is easy to verify that $\mathbf{P_{UL}}$ satisfies (CM1), (CM2), (CM3), and (CM4) over all UL Models.[24] Thus, the only obvious candidate for a binary parthood relation on UL Models turns out to have nice logical properties (or, in any case, the logical properties that philosophers whose thinking about parthood has been guided by classical mereology would expect it to have).

Moreover, $\mathbf{P_{UL}}$ serves as the basis for a plausible overlap relation. Plugging into **CM**'s definition ($\mathbf{D_O}$), we get the relation $\mathbf{O_{UL}}$ that holds between objects x and y in UL Models just in case some object stands in $\mathbf{P_{UL}}$ to both x and y. It follows from Condition (4.iv) on L Models that $<x, y> \in \mathbf{O_{UL}}$ if and only if x and y are located at overlapping spatiotemporal regions (i.e., if and only if $\mathbf{PATH}(x) \cap \mathbf{PATH}(y) \neq \emptyset$). Thus, we avoid the ugly cases of 'overlapping' objects located in disjoint regions of spacetime (like JA and the man who inherited her atom) that plague the $\mathbf{P_{OC}}$ and $\mathbf{P_{CT}}$ versions of overlap.[25]

However, $\mathbf{P_{UL}}$ suffers from the same sorts of clashes with intuition as do $\mathbf{P_{BD}}$ and $\mathbf{P_{PT}}$. For, if ordinary objects have unique locations in spacetime, then each ordinary object must be located at a four-dimensional region which extends exactly as long as that object persists. On this view, Jane Austen is located *not* at multiple three-dimensional regions (as assumed earlier), but at a single four-dimensional region which extends from December 1775 to July 1817. But in this four-dimensionalist picture, JA's hands, head, teeth, and so on, do not stand in the $\mathbf{P_{UL}}$ relation to JA, since their unique locations all extend somewhat beyond July 1817.[26] Also, many artifacts—bicycles, computers, tables—would, on this view, lack what we ordinarily take to be their most salient parts. For example, the wheels, frame, and gears of my bicycle all pre-date my bicycle. Thus, on the four-dimensionalist account, none of these objects have locations that are included in my bicycle's location and, as a result, none stands in $\mathbf{P_{UL}}$ to my bicycle.

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[23] Notice that $<x, y> \in \mathbf{P_{UL}}$ if and only if: i) for some $<x, r_x>, <y, r_y> \in \mathbf{L}$ and $r_x \subseteq r_y$; ii) for any $<x, r_x> \in \mathbf{L}$, there is some $<y, r_y> \in \mathbf{L}$ such that $r_x \subseteq r_y$; or iii) for any $<y, r_y> \in \mathbf{L}$, there is some $<x, r_x> \in \mathbf{L}$ such that $r_x \subseteq r_y$. Thus, if instead of introducing $\mathbf{P_{UL}}$ along the lines of $\mathbf{P_{PT}}$ we had followed the format for the definitions of $\mathbf{P_{OC}}$, $\mathbf{P_{BD}}$, or $\mathbf{P_{CT}}$, we would have ended up with the same relation.

[24] To see that $\mathbf{P_{UL}}$ satisfies the supplementation principle (CM3), suppose that x and y are objects in a UL Model and that $<x, y> \notin \mathbf{P_{UL}}$. Then $\mathbf{PATH}(x)$ is not included in $\mathbf{PATH}(y)$. Since $\mathbf{PATH}(x)$ and $\mathbf{PATH}(y)$ are locations (the unique locations of x and y), it follows from condition (4.iii) on L Models that some object z is located within the difference $\mathbf{PATH}(x)/\mathbf{PATH}(y)$. Since z's unique location (its path) lies within $\mathbf{PATH}(x)$, $<z, x> \in \mathbf{P_{UL}}$. Also, since no object can have a path that is included in both $\mathbf{PATH}(x)/\mathbf{PATH}(y)$ and $\mathbf{PATH}(y)$, no object can stand in $\mathbf{P_{UL}}$ to both z and y. Thus, $<x, y> \notin \mathbf{P_{UL}}$ implies that some object standing in $\mathbf{P_{UL}}$ to x shares no $\mathbf{P_{UL}}$-parts with y.

[25] More good news: It is easy to see that a reasonable summation relation can be introduced in terms of $\mathbf{P_{UL}}$ and that a universal summation principle is satisfied over the subclass of UL Models in which the union of any collection of locations is also a location. Thus $\mathbf{P_{UL}}$ satisfies *all* of the axioms of the strongest version of classical mereology on the expected subclass of UL Models.

[26] Notice that the ordinary objects which are $\mathbf{P_{UL}}$-parts of JA on this four-dimensionalist account are just those objects which are $\mathbf{P_{BD}}$-parts and $\mathbf{P_{PT}}$-parts of JA on the three-dimensionalist multi-location account assumed at the beginning of this section.

This mismatch between the four-dimensionalist's binary parthood relation and our ordinary assumptions about parthood is noted elsewhere (in, e.g., [Thomson, 1983], [Sider, 2001], [Sattig, 2006]). In response, four-dimensionalists have introduced supplementary ternary parthood relations—usually linking parthood to times through temporal parts—that are much closer to the ordinary notion of parthood. Thus, even though an advocate of unique location may adopt a binary parthood relation with useful logical properties, he still has an interest in finding an alternative parthood relation that better fits ordinary usage. At the end of the next section, I suggest one region-relative parthood relation—a generalization of the time-relative parthood relation of [Sider, 2001, p. 53]—which, under appropriate conditions, can serve this purpose for four-dimensionalists.

4. Slice-Relative Parthood Relations

There are two general types of strategies for introducing region-relative parthood relations. The first sort of approach—the focus of the current section—is to restrict the region argument of the ternary parthood relation to the members of a *special subclass* of regions. Examples of this approach are found in: [Bittner and Donnelly, 2004], which restricts the region argument to absolute time-slices[27]; [McDaniel, 2004], which (tentatively) restricts the region argument to maximal three-dimensional slices of spacetime; and [Balashov, 2008], which restricts the region argument to achronal regions. The second strategy, which will be considered in Section 5, places no global restrictions on the sorts of regions at which objects may have parts. Examples of the second approach to region-relative parthood are found in [Hudson, 2001] and [Crisp and Smith, 2006].

The first strategy assumes that there are special kinds of regions—I will call them 'slices'—within which objects have a limited number of locations. As we will see, ideally this should be no more than one location per object per slice. In the ideal case, relativizing parthood to a slice amounts to limiting the scope of parthood claims to regions of spacetime in which objects have unique locations. Here, we should expect that *at a fixed slice* the logical properties of slice-relative parthood relations more or less match those of the unique location parthood relation $\mathbf{P}_{\mathbf{UL}}$. And the primary slice-relative parthood relation introduced below does indeed satisfy natural ternary counterparts of **CM**'s axioms.

The primary slice-relative parthood relation considered in this section, $\mathbf{P}_{\mathbf{S\text{-}3D}}$, assumes that *all* locations are included in some slice. If we take slices to be three-dimensional regions roughly corresponding to times (e.g., frame-relative hyperplanes of simultaneity or other achronal regions), then $\mathbf{P}_{\mathbf{S\text{-}3D}}$ is viable only on the assumption that all objects are located at regions of no more than three dimensions. But I will also briefly consider an alternative slice-relative parthood relation, $\mathbf{P}_{\mathbf{S\text{-}4D}}$, which is compatible with some versions of four-dimensionalism or with mixed ontologies that include both three- and four-dimensional objects.

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[27] In [Bittner and Donnelly, 2004], it is assumed that spacetime is the sum of a unique set of non-overlapping (instantaneous) time-slices. As it stands, this approach does not accommodate relativistic treatments of spacetime. But it could easily be generalized to allow that any region consisting of those points which are simultaneous *relative to any frame* counts as a time-slice. This requires only that we drop the assumption that time-slices are pairwise discrete.

A Slice$_{3D}$ Model (S$_{3D}$ Model)[28] is an ordered quintuple <**ST**, **R, OB, L**, **S**> where <**ST**, **R, OB, L**> is an L Model (satisfying conditions 1-4 on L Models) and where, in addition, the following condition is satisfied:

5. **S** (the set of slices) is a subset of **R** such that i) for any $<x, r> \in$ **L**, there is some $s \in$ **S** such that $r \subseteq s$; ii) for any $x \in$ **OB** and $s \in$ **S**, if $< x, r >, < x, r^* > \in$ **L** and $r, r^* \subseteq s$, then $r = r^*$.

Condition 5 tells us that i) every location is included in some slice and ii) no object has more than one location within any given slice.

The ternary slice-relative parthood relation $\mathbf{P_{S\text{-}3D}}$ is defined over S$_{3D}$ Models as follows:

$$<x, y, s> \in \mathbf{P_{S\text{-}3D}} \text{ if and only if } s \in \mathbf{S} \text{ and for some } r_x , r_y \in \mathbf{R},$$
$$<x, r_x >, < y, r_y > \in \mathbf{L} \text{ and } r_x \subseteq r_y \subseteq s.$$

In other words, x is part of y at slice s if and only if both x and y have locations within s and x's unique location in s is included in y's unique location in s.

We can introduce additional slice-relative relations on S$_{3D}$ Models. The *exists at* relation $\mathbf{E_{S\text{-}3D}}$ holds between an object and a slice just in case the object has a location within the slice.

$$<x, s> \in \mathbf{E_{S\text{-}3D}} \text{ if and only if } s \in \mathbf{S} \text{ and for some } r \in \mathbf{R}, <x, r > \in \mathbf{L} \text{ and } r \subseteq s.$$

The *overlaps* relation $\mathbf{O_{S\text{-}3D}}$ holds between objects x and y at slice s if and only if both x and y exist at s and x's unique location in s overlaps y's unique location in s.

$$<x, y, s> \in \mathbf{O_{S\text{-}3D}} \text{ if and only if } s \in \mathbf{S} \text{ and for some } r_x , r_y \in \mathbf{R},$$
$$<x, r_x >, < y, r_y > \in \mathbf{L}, r_x , r_y \subseteq s, \text{ and } r_x \cap r_y \neq \varnothing.$$

Though less common than formal treatments of binary parthood, axiomatizations of ternary parthood predicates have been proposed in several works including [Thomson, 1983], [Sider, 2001], and [Bittner and Donnelly, 2004].[29] **SM** (Slice Mereology) is

[28] The indices '**S-3D**' and '**S-4D**' distinguish between the slice-relative parthood relation which is *intended for* a three-dimensionalist ontology and the slice-relative parthood relation which is *intended for* mixed or four-dimensionalist ontologies. The corresponding indices ('3D' and '4D') distinguish between the different classes of models into which these parthood relations are introduced. *However*, nothing in the abstract conditions on S$_{3D}$ Models or S$_{4D}$ Models requires that slices are three-dimensional regions. (This should be obvious, since I do not in this paper introduce any device for distinguishing the dimension of a region. Within the models, I do not even assume that regions are the sorts of things that can have dimensions.) But, as I will emphasize below, in the sorts of S$_{3D}$ Models or S$_{4D}$ Models which offer plausible counterparts of ordinary *temporalized* parthood, the slices are roughly three-dimensional subregions of a four-dimensional spacetime.

[29] See also the Mereology of Continuants in [Simons, 1987, 5.2]. Instead of using a ternary parthood predicate, Simons introduces a multitude of binary parthood predicates, each of which is indexed to a distinct time.

similar to these mereologies, but is somewhat stronger in that it assumes a ternary version of **CM**'s antisymmetry axiom. We will see that $\mathbf{P}_{\mathbf{S\text{-}3D}}$ satisfies this principle over all S_{3D} Models. This is not necessarily a good thing. Three-dimensionalists who endorse material coincidence (including [Thomson, 1983] and [Simons, 1987]) tend to reject the ternary version of the antisymmetry axiom and will find $\mathbf{P}_{\mathbf{S\text{-}3D}}$ too strong. I will say something later about how S_{3D} Models can easily be modified to accommodate material coincidence. But it is worthwhile first seeing how close we can come to preserving the full power of **CM** in a slice-relative setting.

SM is developed in a standard sorted first-order predicate logic. Its domain of quantification is partitioned into two sorts: objects (over which the variables w, x, y, z range) and regions (over which the variables s, r range). All quantification in **SM** is restricted to a single sort. In the presentation below, quantifier restrictions are conveyed implicitly through the conventions on variable usage. **SM**'s only non-logical primitive is the ternary predicate **P** (parthood) which takes two object variables and one region variable as its arguments.[30] Two additional predicates are defined in terms of **P**.

($DS_{\mathbf{E}}$) **E**xs (x exists at s) $=_{def}$ **P**xxs (x is part of itself at s)

($DS_{\mathbf{O}}$) **O**xys (x overlaps y at s) $=_{def}$ ∃z (**P**zxs & **P**zys) (some object is part of both x and y at s)

The following five axioms govern **SM**'s mereological predicates.

(SM0) ∃s **E**xs (every object exists at some region)

(SM1) **P**xys → **E**xs & **E**ys (if x is part of y at s, then both x and y exist at s)

(SM2) **P**xys & **P**yzs → **P**xzs (if x is part of y at s and y is part of z at s, then x is part of z at s)

(SM3) **E**xs & ~**P**xys → ∃z(**P**zxs & ~**O**zys) (if x exists at s and is not part of y at s, then some object is part of x at s but does not overlap y at s)

(SM4) **P**xys & **P**yxs → x = y (if x is part of y at s and y is part of x at s, then x and y are identical)

When the ternary predicate **P** is interpreted in S_{3D} Models as the ternary relation $\mathbf{P}_{\mathbf{S\text{-}3D}}$, the defined predicate **E** is interpreted as $\mathbf{E}_{\mathbf{S\text{-}3D}}$[31], the defined predicate **O** is interpreted as $\mathbf{O}_{\mathbf{S\text{-}3D}}$[32], and **SM**'s axioms (SM0)-(SM4) are satisfied over all S_{3D} Models[33].

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[30] Of course **SM**'s ternary parthood predicate is not to be confused with **CM**'s binary parthood predicate. I might have introduced a new symbol for the ternary parthood predicate (and yet another symbol for **SM**'s ternary overlap predicate), but I do not see that this extra bit of notational complication is necessary because in what follows we will work within only one mereological theory at a time.

[31] To see this:
(⇒) If $\langle x, s\rangle \in \mathbf{E}_{\mathbf{S\text{-}3D}}$, then for some $r \in \mathbf{R}$, $\langle x, r\rangle \in \mathbf{L}$ and $r \subseteq s$. In this case, $\langle x, r\rangle, \langle x, r\rangle \in \mathbf{L}$, $r \subseteq r \subseteq s$, and $\langle x, x, s\rangle \in \mathbf{P}_{\mathbf{S\text{-}3D}}$. (⇐) Conversely, if $\langle x, x, s\rangle \in \mathbf{P}_{\mathbf{S\text{-}3D}}$, then for some $r, r^* \in \mathbf{R}$, $\langle x, r\rangle, \langle x, r^*\rangle \in \mathbf{L}$,

The principles (SM0) – (SM4) are, taken together, natural ternary counterparts of the axioms of **CM** in that any parthood relation satisfying these axioms behaves as a classical binary parthood relation *within a fixed slice*. More precisely, let <OB, S, P> be any model for **SM,** where OB is the object domain, S is a subset of the region domain, and P is an interpretation of **P** over OB × OB × S satisfying (SM0)-(SM4). For example, given any S_{3D} Model <**ST**, **R, OB, L**, **S**>, <OB, S, P> may be <**OB**, **S**, $\mathbf{P_{S\text{-}3D}}$ >. Now let $\boldsymbol{s}$ be any member of S and define the *binary* relation $P_{\boldsymbol{s}}$ as:

$$<x, y> \in P_{\boldsymbol{s}} \text{ if and only if } <x, y, \boldsymbol{s}> \in P.$$

Let $OB_{\boldsymbol{s}} = \{x \in OB: <x, x, \boldsymbol{s}> \in P\}$ (i.e., $OB_{\boldsymbol{s}}$ is the subset of OB consisting of those objects existing at $\boldsymbol{s}$). Then $<OB_{\boldsymbol{s}}, P_{\boldsymbol{s}}>$ is a model of **CM**. In particular, for any slice $\boldsymbol{s}$ in an S_{3D} Model <**ST**, **R, OB, L**, **S**>, the binary slice-indexed parthood relation $(\mathbf{P_{S\text{-}3D}})_{\boldsymbol{s}}$ satisfies each of (CM1)-(CM4) over $\mathbf{OB}_{\boldsymbol{s}}$.[34]

To this extent, we should be satisfied that, *relative to a fixed slice*, $\mathbf{P_{S\text{-}3D}}$ replicates the structure of classical mereology. But, as mentioned above, three-dimensionalists who favor material coincidence will think that **SM** is too strong. For, it is a theorem of **SM** that no two objects can be composed of the same parts at the same slice. Taking slices as time-slices, this would prohibit a statue from temporarily coinciding with, while remaining distinct from, the lump of clay from which it is formed. But pro-coincidence three-dimensionalists claim that the statue and lump are distinct objects which share a location and parts (at least their molecular parts) for as long as the lump constitutes the statue.

The issue here is, at bottom, with the structure of the models. Condition (4.ii) on L Models and S_{3D} Models prohibits distinct objects from standing in the exact location relation **L** to the same spacetime region. But proponents of material coincidence (at least those who endorse spacetime substantivalism) hold that distinct objects may be exactly located at the same spatio-temporal region. These philosophers will think that S_{3D} Models

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and $r^* \subseteq r \subseteq s$. It follows immediately that $<x, s> \in \mathbf{E_{S\text{-}3D}}$. (⇔) Thus, $<x, s> \in \mathbf{E_{S\text{-}3D}}$ if and only if $<x, x, s> \in \mathbf{P_{S\text{-}3D}}$.

[32] To see this:
(⇒) Suppose $<x, y, s> \in \mathbf{O_{S\text{-}3D}}$. Then x and y have, respectively, locations r_x and r_y in slice s where $r_x \cap r_y \neq \varnothing$. By Condition (4.iv) on L Models, there is a location $r \subseteq r_x \cap r_y$. Let z be the object that is located at r. Since $r \subseteq r_x \subseteq s$ and $r \subseteq r_y \subseteq s$, $<z, x, s> \in \mathbf{P_{S\text{-}3D}}$ and $<z, y, s> \in \mathbf{P_{S\text{-}3D}}$. (⇐) Suppose $<z, x, s> \in \mathbf{P_{S\text{-}3D}}$ and $<z, y, s> \in \mathbf{P_{S\text{-}3D}}$. Let r, r_x, and r_y be, respectively, z's, x's, and y's unique locations within s. Then since $r \subseteq r_x$ and $r \subseteq r_y$, $r_x \cap r_y \neq \varnothing$. (⇔) Thus, $<x, y, s> \in \mathbf{O_{S\text{-}3D}}$ if and only if there is some object z such that $<z, x, s> \in \mathbf{P_{S\text{-}3D}}$ and $<z, y, s> \in \mathbf{P_{S\text{-}3D}}$.

[33] To see that $\mathbf{P_{S\text{-}3D}}$ satisfies (SM3), suppose that $<x, s> \in \mathbf{E_{S\text{-}3D}}$ and $<x, y, s> \notin \mathbf{P_{S\text{-}3D}}$. Let r_x be x's unique location in s. (Case 1) y does not have a location within s. Then y does not overlap any object at s. Thus, $<x, x, s> \in \mathbf{P_{S\text{-}3D}}$ and $<x, y, s> \notin \mathbf{O_{S\text{-}3D}}$. (Case 2) y has a location $r_y \subseteq s$. Since $<x, y, s> \notin \mathbf{P_{S\text{-}3D}}$, $r_x \nsubseteq r_y$ and $r_x \backslash r_y \neq \varnothing$. By Condition (4.iii), some location r is included in $r_x \backslash r_y$. Let z be the object located at r. Then since $r \subseteq r_x \subseteq s$, $<z, x, s> \in \mathbf{P_{S\text{-}3D}}$. Since $r \cap r_y = \varnothing$, $<z, y, s> \notin \mathbf{O_{S\text{-}3D}}$.

In both cases, the supposition that $<x, s> \in \mathbf{E_{S\text{-}3D}}$ and $<x, y, s> \notin \mathbf{P_{S\text{-}3D}}$, implies that some part of x at s does not overlap y at s.

[34] Furthermore, (SM1)-(SM4) are clearly also *necessary* for establishing that each of the reduced 'single-slice' domains is a model of **CM**. The additional axiom (SM0) just ensures that every member of the original object domain shows up in at least one of the single-slice domains.

misrepresent the way objects are located in spacetime since the models prohibit spatio-temporal coincidence. But the models can be easily modified to accommodate coincidence. Let us introduce the term 'S_{3D}* Model' for any ordered quintuple <**ST**, **R, OB, L**, **S**> which satisfies all of the requirements of an S_{3D} Model except Condition (4.ii). Let **SM*** be that mereology which is just like **SM** *except* that **SM*** omits axiom (SM4). **SM*** is equivalent to the time-relative mereology introduced at [Sider, 2001, p. 58] and is quite similar to the time-relative mereologies of [Thomson, 1983] and [Simons, 1987, 5.2].[35] It is trivial to verify that the S_{3D}* counterpart of $\mathbf{P_{S\text{-}3D}}$ satisfies all axioms of **SM***, but does *not* satisfy **SM**'s (SM4).

Whether we make room for coincident objects or not, $\mathbf{P_{3D\text{-}S}}$ is fairly robust, at least from a logical point of view. Relative to a slice, it is reflexive and transitive and satisfies a supplementation principle. Thus, $\mathbf{P_{3D\text{-}S}}$ supports much of the reasoning philosophers would like to do with a parthood relation, as long as they are willing to relativize parthood claims appropriately. Moreover, whether or not we leave room for coincident objects, $\mathbf{P_{3D\text{-}S}}$ generates a plausible overlap relation—objects *x* and *y* share a $\mathbf{P_{3D\text{-}S}}$-part at slice *s* if and only if *x*'s unique location within *s* overlaps *y*'s unique location within *s*. With $\mathbf{P_{3D\text{-}S}}$-overlap, we cannot have 'overlapping' objects that are separated in space and time (as are JA and the $\mathbf{P_{OC}}$-overlapping man who inherits her atom). Also, $\mathbf{P_{3D\text{-}S}}$ serves as the basis for a reasonable summation relation. We can introduce slice-relative sums as

x is a sum of the *y*s at *s* $=_{\text{def}}$ each of the *y*s is part of *x* at *s* and any object that is part of *x* at *s* overlaps at least one of the *y*s at *s*. [36]

It then turns out that $\mathbf{P_{3D\text{-}S}}$-sums must extend exactly so far as their summands within the slice in question. More precisely, if *x* is a sum of the *y*s at *s*, then *x*'s unique location within *s* is the union of the locations of the *y*s within *s*. With $\mathbf{P_{3D\text{-}S}}$-summation in a fixed slice, we cannot have summands that trail off past their sum (as the molecules that $\mathbf{P_{OC}}$-sum to JA extend past JA) and or sums that bulge out beyond their summands (as our possible organism from Section 3 extends beyond the single organ of which it is a $\mathbf{P_{OC}}$-sum).

In these formal and structural respects, $\mathbf{P_{3D\text{-}S}}$ works out better than the binary relations $\mathbf{P_{BD}}$, $\mathbf{P_{CT}}$, $\mathbf{P_{PT}}$, and (especially) $\mathbf{P_{OC}}$. But what about our other important criterion—how well can a relation like $\mathbf{P_{3D\text{-}S}}$ preserve intuitive assumptions about the parts of ordinary objects like JA? Obviously we do not normally think of ourselves as relativizing parthood to regions of spacetime. But we do explicitly link parthood to time (a certain cell is part of JA at some times but not others and the wheel that is now part of my bike was not part of it last year). I think $\mathbf{P_{3D\text{-}S}}$ can be made to fit common sense thinking about parthood quite well *if* (and only if) there is a slice set consisting of regions that correspond roughly to times—slices that extend over only those spacetime points that

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[35] Thomson's time-relative mereology is somewhat stronger than **SM*** in that it prohibits distinct objects from being parts of one another at *all* times at which at least one of them exists (see her axiom (CCL_1) [1983, p. 216]). Besides being formulated in a free logic (instead of standard predicate logic as is **SM***), Simons' time-relative mereology uses multiple time-indexed *binary* parthood relations and a weaker supplementation principle than (SM3) (see his CTA10 [1987, p. 179]).

[36] Except for its use of plurals instead of sets of summands, this matches the time-relative summation relation introduced at [Sider, 2001, p. 58].

might (at least from an appropriate frame-relative perspective) count as simultaneous. For such time-slices, the set of all objects existing at a slice includes all objects existing at the corresponding (frame-relative) time and the parthood relation holds between objects *x* and *y* at a slice just in case *x* is included in *y* at the corresponding time. So, for example, there will be some slice at which all of my current cells are part of me, another slice at which all of my cells from five years ago are part of me, another slice at which all of my cells from ten years ago are part of me, and so on.

But there can be S_{3D} Models in which the slices do not look anything at all like time-slices—models in which the slices zig-zag randomly all over spacetime but happen to pick up no more than one location per object as they do so. Although the slice-relative parthood relation for this sort of model would still enjoy **SM**'s or **SM***'s nice logical properties, it clearly would *not* fit ordinary usage. There is no ordinary sense in which, say, all of my current cells are part of me at the same time (or region, or anything else) at which all of your cells from ten years ago are part of you.

I will assume that the ideal spatiotemporal correlates of times are *maximal achronal regions*—spacetime regions that are achronal (i.e. include no absolutely temporally separated spacetime points) and are not properly included in larger achronal regions.[37] For example, assuming Special Relativity, for each inertial frame *F*, there is a partition of spacetime consisting of the equivalence classes of spacetime points which are simultaneous *relative to F*. Each of these frame-relative time-slices is a maximal achronal region. When wondering whether $\mathbf{P_{S\text{-}3D}}$ parthood corresponds appropriately to ordinary temporalized parthood, the main question we should be asking then is:

> (*) Is any set **S** of *maximal achronal regions* such that i) every object is located *only at* subregions of the members of **S** and ii) no object has more than one location within any member of **S**?

It is easy to name circumstances in which the answer to (*) is definitely 'no'. This will be so if some (or all) objects are located at four-dimensional regions or if objects travel through time in a way that locates at least one object in multiple positions within an achronal region. But even if we assume that objects have only three-dimensional achronal locations and cannot travel backwards in time, it is not clear that the answer to (*) is 'yes'. It *might* be, but then again it might not. It all depends on what exactly the three-dimensionalist can say about where objects are located in a relativistic spacetime.

In a Galilean spacetime, there is an absolute simultaneity relation on spacetime points and the only maximal achronal regions are absolute time-slices (where each absolute time-slice consists of all points absolutely simultaneous with a given point). Importantly, absolute time-slices are pair-wise discrete. On *this* picture of spacetime, it makes sense for three-dimensionalists who eschew time travel to hold that an object *x* is located at region *r* if and only if *r* is the intersection of *x*'s path with an absolute time-

[37] See [Gilmore, 2006] and [Balashov, 2008] for more detailed discussions of absolute temporal separation and maximal achronal regions. Notice that we need not assume that *every* maximal achronal region is a natural spacetime correlate of some time instant. For example, [Balashov, 2008] argues that, in the context of Special Relativity, only flat frame-relative time-slices (and not *curved* maximal achronal regions) play this role. Also, I think that spatiotemporal regions that are only *roughly* as spatially 'long' and temporally 'short' as maximal achronal regions could also play the role of time-slices. But the points made below about maximal achronal regions apply equally well to regions that are almost, but not quite, maximal achronal regions.

slice.[38, 39] Since absolute time-slices are pairwise discrete, no absolute time-slice could include more than one location per object. Since, in addition, each location is included in some absolute time-slice, the set of all absolute time-slices clearly satisfies the S_{3D} criteria for slice sets.

But in a relativistic spacetime, there are no absolute time-slices. There are at best only frame-relative time-slices. Importantly, regions that are time-slices relative to different inertial frames may have nonempty intersections. More generally, distinct maximal achronal regions in relativistic spacetimes may overlap. And, as Gilmore describes in his 'corner slice' example [Gilmore, 2006, p. 212-213], there may be an object x and overlapping frame-relative time-slices s_1 and s_2 such that **PATH**(x) ∩ s_1 is properly included in **PATH**(x) ∩ s_2. Here, s_1 cuts through a small end 'corner' of x's path while s_2 cuts through a larger swatch of x's path—one that includes both **PATH**(x) ∩ s_1 and **PATH**(x) ∩ s_2. If, following the example of non-relativistic three-dimensionalism, we assume that every object is located at any non-empty intersection of its path with a maximal achronal region, we must conclude that both **PATH**(x) ∩ s_1 and **PATH**(x) ∩ s_2 are locations of x. But then, since any region which includes **PATH**(x) ∩ s_2 also includes **PATH**(x) ∩ s_1, no set of regions would satisfy the S_{3D} criteria for slice sets.

Of course, one could always argue that in transferring three-dimensionalism to a relativistic framework, we should expect more complicated rules for locating objects within their paths. On behalf of the three-dimensionalist, Gilmore suggests that objects might be located only at all maximal achronal subregions of their paths (achronal subregions of their paths which are not properly included in larger achronal subregions of their paths) [Gilmore, 2006, p. 212-213]. On this modified location principle, maximal achronal regions cannot include more than one location per object. But it is not clear to me what reasons the three-dimensionalist has for thinking that objects are located *only* at maximal achronal subregions of their paths.[40] Significantly, Gilmore himself ends up rejecting three-dimensionalism because he does not believe that the three-dimensionalist can provide any general criteria for determining which subregions of an object's path are its locations. While I do not see the difficulty in formulating a general location principle as a reason for rejecting three-dimensionalism, I do agree that it is not obvious what the three-dimensionalist who takes relativity seriously should say about where objects are located in spacetime.[41] In particular, I do not think that the three-dimensionalist can simply *assume* that there must be a set of regions satisfying the S_{3D} criteria for slice sets whose members correspond appropriately to ordinary time instants.

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[38] This is the characterization of location in spacetime which I initially assumed for the extended Jane Austen example in Section 3. Again, see [van Inwagen, 1990] and [Sattig, 2006, 2.1] for this sort of take on location in spacetime.

[39] The three-dimensionalist who allows for time-travel will presumably want to say something different. Where x's path crosses slice s twice, x has two separate locations within s and is not located at the intersection of its path with s (i.e., at the union of its two locations in s).

[40] See [Gibson and Pooley, 2006, p. 186] for doubts over the proposal that objects are located only at maximal achronal subregions of their paths.

[41] See also [Gibson and Pooley, 2006] where it is argued, against Gilmore and in favor of three-dimensionalism, that there is no reason to expect that there is a general principle telling us where any object is located within its path.

How important is the requirement that the slices of S_{3D} Models include no more than one location per object? Taking our cue from Gilmore's corner slice example, might we not weaken Condition (5.ii) of S_{3D} models so that it requires only that, if x has any location in slice s, then x has a *maximal* location in slice s? This modified condition would allow the object x in the corner slice example to have both $\mathbf{PATH}(x) \cap s_1$ and $\mathbf{PATH}(x) \cap s_2$ as its locations, since both of these regions (as well as any other location x might have in s_2) are included in $\mathbf{PATH}(x) \cap s_2$. The corresponding slice-relative parthood relation would hold between objects x and y at slice s if and only if i) both x and y have locations in s and ii) x's maximal location in s is included in y's maximal location in s. However, unlike $\mathbf{P_{S\text{-}3D}}$, this modified slice-indexed parthood relation need not satisfy the supplementation principle (SM3).[42] More generally, as far as I can tell, *any* weakening of (5.ii) results in a slice-relative parthood relation that does not satisfy all of **SM***'s axioms and is thus significantly weaker than $\mathbf{P_{S\text{-}3D}}$. This sort of weakness does not necessarily disqualify a relation from serving as a parthood relation. But, as we saw in Section 3, it can make trouble for the kind of work philosophers have tried to do with mereological relations.

I close this section by briefly sketching a different sort of variation on S_{3D} Models and $\mathbf{P_{S\text{-}3D}}$ parthood. Like S_{3D} Models, S_{4D} Models assume a distinguished set of slices and require that no object has more than one location in any slice. However, unlike S_{3D} Models, S_{4D} Models do not require that every location is included in some slice. Thus, even if slices are three-dimensional subregions of spacetime, some of the objects in S_{4D} Models may have four-dimensional locations. The parthood relation $\mathbf{P_{S\text{-}4D}}$ is a spacetime counterpart of the time-indexed parthood relation that four-dimensionalists have introduced in terms of temporal parts (see, e.g., [Sider, 2001, Ch 3]).[43] Most four-dimensionalists assume that objects are *not* multiply located and that there is a binary parthood relation along the lines of $\mathbf{P_{UL}}$ satisfying the axioms of classical mereology over the object domain. But, as we noted in our examination of $\mathbf{P_{UL}}$, this sort of binary parthood relation does not preserve ordinary assumptions about parthood (e.g., that JA's head is part of JA). The four-dimensionalist's time-indexed parthood relation is introduced as a secondary parthood relation which is supposed to match ordinary temporalized parthood better than the binary parthood relation. If slices are spacetime regions that roughly correspond to time instants, then $\mathbf{P_{S\text{-}4D}}$ should also fit ordinary assumptions better than the binary relation $\mathbf{P_{UL}}$.

A Slice$_{4D}$ Model (S_{4D} Model) is an ordered quintuple <**ST**, **R, OB, L**, **S**> where <**ST**, **R, OB, L**> is an L Model (satisfying conditions 1-4 on L Models) and where, in addition, the following condition is satisfied:

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[42] To see this, suppose that for some slice s, x is located at both r_{x1} and r_{x2} where $r_{x1} \subset r_{x2} \subseteq s$ and r_{x2} is x's maximal location in s. Suppose further that i) no object is located within any proper subregion of r_{x1} and ii) there is some object y such that r_{x2}/r_{x1} is y's maximal location in s. Then, for the proposed parthood relation, x would exist at s and would not be a part of y at s, but would have no part at s that fails to overlap y at s.

[43] See also [Balashov, 2008] for a slightly different spacetime adaptation of the four-dimensionalist's time-relative parthood relation.

5*. **S** (the set of slices) is a subset of **R** such that i) if $r \in \mathbf{R}$ and there is some $< x, r^*> \in \mathbf{L}$ such that $r \subseteq r^*$, then there is some $s \in \mathbf{S}$ with $r \cap s \neq \varnothing$; ii) for any $x \in \mathbf{OB}$ and any $s \in \mathbf{S}$, if $\mathbf{PATH}(x) \cap s \neq \varnothing$, then there is some $z \in \mathbf{OB}$ such that $<z, \mathbf{PATH}(x) \cap s> \in \mathbf{L}$; iii) for any $<x, r> \in \mathbf{L}$ and any $s \in \mathbf{S}$, if $r \subseteq s$, then $r = \mathbf{PATH}(x) \cap s$.

Condition (5*.i) requires that, taken together, slices cover every subregion of every location. Notice that (5*.i) is automatically satisfied if the slices cover all of spacetime (i.e., if $\cup\mathbf{S} = \mathbf{ST}$). (5*.i) replaces the stronger requirement in S_{3D} Models that every location is included in some slice. Condition (5*.ii) requires that there is some object located at any region that is the intersection of a slice and an object's path. If slices correspond to times, the object which is located at the intersection of *x*'s path with a slice is, I will presume, a temporal part of *x*.[44] Condition (5*.iii) requires that if object *x* has any location within slice *s*, that location must be the intersection of *s* and *x*'s path. It is an immediate consequence of (5*.iii) that no object has more than one location within any slice. But notice that for an arbitrary object $x \in \mathbf{OB}$, none of (5*.i-iii) require that *x* has a location within any slice.

The slice-relative parthood relation $\mathbf{P_{S\text{-}4D}}$ is defined over S_{4D} Models as follows:

$$<x, y, s> \in \mathbf{P_{S\text{-}4D}} \text{ if and only if } s \in \mathbf{S} \text{ and } \varnothing \neq \mathbf{PATH}(x) \cap s \subseteq \mathbf{PATH}(y) \cap s\,.$$

In other words, *x* is part of *y* at slice *s* if and only if the intersection of *x*'s path with *s* is non-empty and is included in the intersection of *y*'s path with *s*.

The supplementary *exists at* ($\mathbf{E_{S\text{-}4D}}$) and *overlaps* ($\mathbf{O_{S\text{-}4D}}$) relations are defined over S_{4D} Models as follows:

$$<x, s> \in \mathbf{E_{S\text{-}4D}} \text{ if and only if } s \in \mathbf{S} \text{ and } \varnothing \neq \mathbf{PATH}(x) \cap s;$$
$$<x, y, s> \in \mathbf{O_{S\text{-}4D}} \text{ if and only if } s \in \mathbf{S} \text{ and } \mathbf{PATH}(x) \cap \mathbf{PATH}(y) \cap s \neq \varnothing.$$

Object *x* exists at slice *s* just in case *x*'s path overlaps *s*. Object *x* overlaps object *y* at slice *s* just in case *x*'s path overlaps *y*'s path within *s*.

When the ternary predicate **P** is interpreted over S_{4D} Models as $\mathbf{P_{S\text{-}4D}}$, the defined predicates **E** and **O** are interpreted as, respectively, $\mathbf{E_{S\text{-}4D}}$ and $\mathbf{O_{S\text{-}4D}}$ and all of **SM***'s axioms are satisfied.[45] Notice, though, that **SM**'s antisymmetry axiom (SM4) is not satisfied. Even when we retain the original requirement that no more than one object is exactly located at any region, distinct objects may still have paths that cross a slice at

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[44] Whether this is so or not depends on exactly how temporal parts are defined. I do not wish to digress from the discussion of region-relative parthood to in order to compare different ways of introducing temporal parts. But, given an appropriate set of *time*-slices, I cannot see that we would run into any trouble in introducing temporal parts either in terms of location, as is done in [Heller, 1984], or in terms of a binary parthood relation as is done in [Sider, 2001].

[45] To see that $\mathbf{P_{S\text{-}4D}}$ satisfies (SM3) over all S_{4D} Models, suppose $\mathbf{PATH}(x) \cap s \neq \varnothing$ and $\mathbf{PATH}(x) \cap s \not\subseteq \mathbf{PATH}(y) \cap s$. Case 1: $\mathbf{PATH}(y) \cap s = \varnothing$. Then $\mathbf{PATH}(x) \cap \mathbf{PATH}(y) \cap s = \varnothing$ and $\varnothing \neq \mathbf{PATH}(x) \cap s \subseteq \mathbf{PATH}(x) \cap s$. Case 2: $\mathbf{PATH}(y) \cap s \neq \varnothing$. By (5*.ii), both $\mathbf{PATH}(x) \cap s$ and $\mathbf{PATH}(y) \cap s$ are locations. By (4.iii), since $(\mathbf{PATH}(x) \cap s)/ (\mathbf{PATH}(y) \cap s) \neq \varnothing$, there is some $<z, r_z> \in \mathbf{L}$ such that $r_z \subseteq (\mathbf{PATH}(x) \cap s)/ (\mathbf{PATH}(y) \cap s)$. Since $r_z \subseteq s$, by (5*.iii), $r_z = \mathbf{PATH}(z) \cap s$. Thus, $\varnothing \neq \mathbf{PATH}(z) \cap s \subseteq \mathbf{PATH}(x) \cap s$ and $\mathbf{PATH}(z) \cap \mathbf{PATH}(y) \cap s = \varnothing$.

exactly the same place. More precisely, for $x \neq y$, there may be some slice s such that $\varnothing \neq$ **PATH**(x) $\cap$ s = **PATH**(y) $\cap$ s. In this case, x and y would each be $\mathbf{P_{S\text{-}4D}}$-parts of one another at s and the object located at **PATH**(x) $\cap$ s would be a shared $\mathbf{P_{UL}}$-part of x and y. But this fits the standard four-dimensionalist treatment of temporalized parthood, which allows that objects like a statue and the lump of clay from which it is formed may each be part of the other at time t in the sense that they share a temporal part at t (see, e.g., [Heller, 1984] or [Sider, 2001, Ch 5]).

We noted above that, lacking a three-dimensionalist account of location in relativistic spacetimes, it is not obvious that $\mathbf{P_{S\text{-}3D}}$ works out for slices that could be considered *time*-slices (at least, not if the slices over which $\mathbf{P_{S\text{-}3D}}$ ranges are required to satisfy Conditions (5.i) and (5.ii) of S_{3D} Models). Given four-dimensionalism and a relativistic spacetime, is the case for $\mathbf{P_{S\text{-}4D}}$ parthood equally inconclusive? Not if the four-dimensionalist assumes (a) that each object has a unique location in spacetime and (b) that any non-empty intersection of a location and a maximal achronal region is a location. (Note that (b) is entailed by the stronger, but not unreasonable, assumption that every subregion of a location is a location.) Suppose that (a) and (b) hold and let MAX be the collection of all maximal achronal regions in the actual spacetime, ST. Since every point in ST is included in some maximal achronal region, $\cup$MAX = ST and MAX satisfies condition (5*.i) on S_{4D} slice sets. It follows from assumption (b) that MAX also satisfies condition (5*.ii). To see that MAX satisfies the final condition, (5*.iii), suppose that object x is located within the maximal achronal region s. Then since x is uniquely located, **PATH**(x) $\subseteq$ s and x's location within s is just **PATH**(x) $\cap$ s = **PATH**(x).[46]

5. Other Region-Relative Parthood Relations

In the previous section, I proposed two general strategies for introducing ternary parthood relations that relativize parthood to special regions (slices). But some philosophers have made use of parthood-at-a-region relations whose third terms are not restricted to the members of a distinguished subclass of regions. The best-developed example of this approach is found in [Hudson, 2001, Ch. 2]. The main task of this section is to examine a version of Hudson's relation in the context of L Models. I will also briefly consider the rather different region-relative parthood relation of [Crisp and Smith, 2005].

Hudson holds that ordinary objects such as people, chairs, and tables are located at multiple overlapping four-dimensional regions. However, Hudson allows that there are three-dimensional objects (instantaneous temporal parts of ordinary objects) that have multiple locations within fixed achronal regions.[47] Thus, neither of the slice-relative parthood relations considered in the previous section works out for Hudson's ontology if we take slices to be something along the lines of maximal achronal regions. Hudson's

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[46] Given unique location, we obtain analogous results with the weaker assumption that, for some subset MAX* of MAX, $\cup$MAX* = ST and any non-empty intersection of an object's path with a member of MAX* is a location. In the context of Special Relativity, the slice set MAX* might be restricted to frame-relative time-slices as is suggested in [Balashov, 2008, p. 28-37].

[47] In fact, when Hudson first provisionally attempts to develop his position in a three-dimensionalist ontology, he assumes that an ordinary object may occupy distinct *spatial* regions at a fixed time [Hudson, 2001, p. 52-53].

own region-relative parthood relation is, as we shall see shortly, quite different from either $\mathbf{P_{S\text{-}3D}}$ or $\mathbf{P_{S\text{-}4D}}$.

As was the case for the slice-relative parthood relations, we will want to introduce Hudson's parthood relation on a class of L Models that satisfies special criteria. For Hudson Models (H Models), we do not need a class of slices as in S_{3D} and S_{4D} Models. But for the Hudson relation to behave nicely, we do need to strengthen the original restrictions on the location relation. As I indicate in the notes below, the stronger restrictions are assumptions that Hudson endorses in [2001].[48]

H Models are L Models in which the location relation satisfies the following additional restrictions:

> 4. v) if $<x, r> \in \mathbf{L}$ and $\varnothing \neq r^* \subseteq r$, there is some $y \in \mathbf{OB}$ such that $<y, r^*> \in \mathbf{L}$; vi) if $<x, r> \in \mathbf{L}$ and $r^* \subset r$, $<x, r^*> \notin \mathbf{L}$. [49]

(4.v) stipulates that every subregion of a location is a location, while (4.vi) prohibits an object from being located at two regions, one of which is a proper subregion of the other.[50]

The Hudson parthood relation $\mathbf{P_H}$ is defined on H Models as follows:

$<x, y, r> \in \mathbf{P_H}$ if and only if for some $r_x, r_y \in \mathbf{R}$, $<x, r_x>, <y, r_y> \in \mathbf{L}$ and $r_x \subseteq r \subseteq r_y$.[51]

Object x is part of object y at region r just in case x and y have locations, r_x and r_y, that 'flank' r in the sense that r includes r_x and is included in r_y.

When we plug $\mathbf{P_H}$ into the **SM** definitions ($DS_{\mathbf{E}}$) and ($DS_{\mathbf{O}}$), we get *exists at* and *overlaps* relations that are quite different from their counterparts in S_{3D} and S_{4D} Models. The *exists at* relation defined in terms of $\mathbf{P_H}$ via ($DS_{\mathbf{E}}$) turns out to be just the location relation $\mathbf{L}$. This is because $<x, x, r> \in \mathbf{P_H}$ if and only if $<x, r_1>, <x, r_2> \in \mathbf{L}$ and $r_1 \subseteq r \subseteq r_2$. But it follows from condition (4.vi) that $<x, r_1>, <x, r_2> \in \mathbf{L}$ and $r_1 \subseteq r \subseteq r_2$ if and only if $r_1 = r = r_2$ and $<x, r> \in \mathbf{L}$. (Notice how different this interpretation of $\mathbf{E}$ is from its interpretation over S_{3D} Models. An object x in an S_{3D} Model exists at slice s just in

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[48] This is not to say, however, that the H Models capture *all* of Hudson's assumptions about location. Most notably, nothing in the H Models requires that any object is located at a four-dimensional region or that ordinary objects are located at multiple overlapping regions. I do not attempt to capture the former assumption because mereology has nothing to say about dimension. I do not attempt to capture the latter assumption because I cannot see that it makes for an interesting difference in the logical properties of the parthood relation.

[49] Notice that (4.v) entails (4.iii) and (4.iv). Thus, taken together, (4.1)-(4.vi) are redundant.

[50] I take Hudson's principles (SD) and (SDP) [2001, p. 65] as evidence that he endorses (4.v). (Note that in these principles, and throughout [2001, Ch. 2], Hudson uses the term 'exactly occupies' as I use 'is located at'. His binary relation EO corresponds to my **L**.) As evidence that Hudson endorses (4.vi), see Hudson's assumption that if an object x were located at regions r^* and r where $r^* \subset r$, then x would be of *proper* part of itself at r [2001, p.68-69]. But this is impossible, since it would imply that $x \neq x$.

[51] Though Hudson lists several principles governing his parthood relation, he never gives an intended model theoretic interpretation for it. However, he does explain the relation in terms of some examples (especially at [2001, p. 66-70]). In proposing this model theoretic treatment of Hudson's relation, I am guided primarily by his explanation of how the relation is supposed to apply in his examples.

case x has a location that is *included in s*—x need not have a location that is identical to s. Over S_{4D} Models, **E** has an even broader interpretation. An object x in an S_{4D} Model exists at slice s just in case x has a location that *overlaps s*—again, x need not have any location that is identical to s.)

The *overlaps* relation for $\mathbf{P_H}$ is the relation $\mathbf{O_H}$ such that

$\langle x, y, r\rangle \in \mathbf{O_H}$ if and only if there are $\langle x, r_x\rangle, \langle y, r_y\rangle \in \mathbf{L}$ with $\varnothing \neq r \subseteq r_x, r_y$.

Objects x and y $\mathbf{O_H}$-overlap at region r just in case both x and y have locations that include r. (By contrast, $\mathbf{O_{S\text{-}3D}}$ holds between x and y at slice s just in case x and y have overlapping locations that are *included in s*.) To see that $\mathbf{O_H}$ is the result of plugging $\mathbf{P_H}$ into ($DS_{\mathbf{O}}$), suppose $\langle z, x, r\rangle \in \mathbf{P_H}$ and $\langle z, y, r\rangle \in \mathbf{P_H}$. Then there are regions r_{z1}, r_{z2}, r_x, and r_y such that $\langle z, r_{z1}\rangle, \langle z, r_{z2}\rangle, \langle x, r_x\rangle, \langle y, r_y\rangle \in \mathbf{L}$, $r_{z1} \subseteq r \subseteq r_x$, and $r_{z2} \subseteq r \subseteq r_y$. It follows that $r \subseteq r_x, r_y$. Conversely, suppose that $\langle x, r_x\rangle, \langle y, r_y\rangle \in \mathbf{L}$ and there is some region r which is included in both r_x and r_y. By (4.v), some object z is located at r. Since $r \subseteq r \subseteq r_x$, and $r \subseteq r \subseteq r_y$, $\langle z, x, r\rangle \in \mathbf{P_H}$ and $\langle z, y, r\rangle \in \mathbf{P_H}$.

How do the logical properties of $\mathbf{P_H}$ compare to those of the slice-relative relations $\mathbf{P_{3D\text{-}S}}$ and $\mathbf{P_{4D\text{-}S}}$? $\mathbf{P_H}$ satisfies **SM**'s (SM0), (SM2), (SM3), and (SM4) over all H Models.[52] But $\mathbf{P_H}$ does *not* satisfy the existence axiom (SM1) on any H Model which includes objects located at extended regions. To see why, suppose that y is an object in an H Model, that y is located at region r_y, and that region r_y has a proper subregion r_x. By condition (4.v), some object x is located at r_x. Since $\langle x, r_x\rangle, \langle y, r_y\rangle \in \mathbf{L}$ and $r_x \subseteq r_y \subseteq r_y$, $\langle x, y, r_y\rangle \in \mathbf{P_H}$. But it follows from Condition (4.vi) that, since x is located at r_x, x is *not* located at r_y. Thus, $\langle x, r_x\rangle \notin \mathbf{L}$. (Similarly, $\langle x, y, r_x\rangle \in \mathbf{P_H}$ and $\langle y, r_x\rangle \notin \mathbf{L}$.) Since **E** is interpreted over H Models as **L**, $\mathbf{P_H}$ does not satisfy (SM1) on this model.

In its failure to satisfy (SM1), $\mathbf{P_H}$ is slightly weaker than the slice-relative parthood relations of the previous section. But the most remarkable difference between $\mathbf{P_H}$ and the slice-relative relations is that $\mathbf{P_H}$ satisfies some rather strong mereological principles that neither $\mathbf{P_{S\text{-}3D}}$ nor $\mathbf{P_{S\text{-}4D}}$ satisfies. For example, $\mathbf{P_H}$ satisfies the following principle on all H Models.

(HM1) **P**xyr & **P**wzr → **P**xzr

It is an immediate consequence of (HM1) that if y and z each has a part at r, then y and z have *exactly the same parts* at r. It is easy enough to see that neither $\mathbf{P_{S\text{-}3D}}$ nor $\mathbf{P_{S\text{-}4D}}$ generally satisfies (HM1). If x and y are objects in either an S_{3D} Model or an S_{4D} Model and x and y are located at disjoint subregions of slice s, then both x and y have parts at s, but x and y do not share any parts at s.

To see that $\mathbf{P_H}$ does indeed satisfy (HM1) over all H Models, suppose that $\langle x, y, r\rangle$, $\langle w, z, r\rangle \in \mathbf{P_H}$. Then for some $r_x, r_y, r_w, r_z \in \mathbf{R}$, $\langle x, r_x\rangle, \langle y, r_y\rangle, \langle w, r_w\rangle, \langle z, r_z\rangle \in \mathbf{L}$, r_x

[52] $\mathbf{P_H}$ serves as an important example for philosophers who think that specifying a relation requires little more than listing some of that relation's logical properties. $\mathbf{P_H}$ and $\mathbf{P_{3D\text{-}S}}$ both satisfy the standard ternary mereological principles (SM0), (SM2), (SM3) and (SM4), but are clearly very different proposals for a region-relative parthood relation.

$\subseteq r \subseteq r_y$, and $r_w \subseteq r \subseteq r_z$. It follows immediately that $r_x \subseteq r \subseteq r_z$ and, consequently, $<x, z, r> \in$ $\mathbf{P_H}$.

I use the name '**HM**' for the sorted first-order theory axiomatized by the following three formulas, where the predicates **E** and **O** are defined as in ($D_{\mathbf{E}}$) and ($D_{\mathbf{O}}$).[53]

(HM0) $\exists r\, \mathbf{E}xr$ (every object exists at some region)

(HM1) $\mathbf{P}xyr\ \&\ \mathbf{P}wzr \rightarrow \mathbf{P}xzr$ (if *x* is part of *y* at *r* and *w* is part of *z* at *r*, then *x* is part of *z* at *r*)

(HM2) $\mathbf{P}xyr\ \&\ \mathbf{P}yxr \rightarrow x = y$ (if *x* is part of *y* at *r* and *y* is part of *x* at *r*, then *x* and *y* are identical)

$\mathbf{P_H}$ satisfies **HM**'s three axioms over all H Models. Theorems of **HM** include counterparts of **SM**'s (SM2) and (SM3). In addition, each of following formulas is a theorem of **HM** which is satisfied by neither $\mathbf{P_{S\text{-}3D}}$ nor $\mathbf{P_{S\text{-}4D}}$.

(HT1) $\mathbf{E}xr\ \&\ \mathbf{E}yr \rightarrow x = y$ (if *x* exists at *r* and *y* exists at *r*, then *x* and *y* are identical)
(HT2) $\mathbf{O}xyr\ \&\ \mathbf{O}wzr \rightarrow \mathbf{O}yzr$ (if *x* overlaps *y* at *r* and *w* overlaps *z* at *r*, then *y* overlaps *z* at *r*)
(HT3) $\mathbf{O}xyr\ \&\ \mathbf{O}yzr \rightarrow \mathbf{O}xzr$ (if *x* overlaps *y* at *r* and *y* overlaps *z* at *r*, then *x* overlaps *z* at *r*)
(HT4) $\mathbf{E}xr\ \&\ \mathbf{O}xyr \rightarrow \mathbf{P}xyr$ (if *x* exists at *r* and *x* overlaps *y* at *r*, then *x* is part of *y* at *r*)
(HT5) $\mathbf{P}xyr\ \&\ \mathbf{P}zwr \rightarrow \mathbf{O}ywr$ (if *x* is part of *y* at *r* and *z* is part of *w* at *r*, then *y* overlaps *w* at *r*)

How suitable is $\mathbf{P_H}$ for philosophical tasks? On the one hand, $\mathbf{P_H}$ certainly does satisfy (ternary versions of) principles that philosophers commonly invoke to describe parthood relations. In particular, $\mathbf{P_H}$ satisfies **SM**'s transitivity axiom, (SM2), and **SM**'s supplementation principle, (SM3). But, on the other hand, there is a sense in which $\mathbf{P_H}$'s strong non-classical properties trivialize the familiar properties. For example, with only the weak assumption that *y* and *z* both have parts at *r*, (HM1) lets us transfer all of *y*'s parts at *r* to *z* (and vice versa). For $\mathbf{P_H}$, we do not need the extra assumption that *y* is part of *z* at *r* which is required by the antecedent of the transitivity principle (SM2). Also, it follows from (HT4) that if *x* exists at *r*, then *x* overlaps *y* at *r* if and only if *x* is part of *y* at *r*. Thus, if *x* exists at *r* and does not stand in $\mathbf{P_H}$ to *y* at *r*, then *x* itself does not $\mathbf{O_H}$-overlap *y* at *r*—we do not need to invoke the supplementation principle to support the weaker conclusion that *x* has some $\mathbf{P_H}$-part at *r* which fails to $\mathbf{O_H}$-overlap *y* at *r*.

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[53] In his own presentation of the logical properties of his parthood relation, [Hudson, 2001] uses a region inclusion predicate. I think that this is definitely the right approach to take in a comprehensive treatment of Hudson's relation. (If nothing else, we can only describe the distinctive 'flanking' of objects' locations around the regions at which they stand in parthood relations if we can say something about region inclusion.) However, since my interest here is only in general differences between Hudson's relation and the slice-relative parthood relations, I will not go into so much detail in my formal analysis of Hudson's relation.

I think that in its strength $\mathbf{P_H}$ does not, after all, fit traditional philosophical thinking about parthood relations—why would philosophers have bothered with a transitivity principle for parthood if so strong a principle as (HM1) were on offer? Although philosophers do not generally explicitly deny (HM1), the assumption must have been all along that parthood is not *that* strong. Note also that (HT3) tells us that the overlap relation $\mathbf{O_H}$ satisfies a ternary transitivity principle. Again, although most philosophers do not explicitly deny that overlap is transitive, transitivity is usually explicitly attributed only to parthood and not to the overlap relation.

But, unlike the very weak binary parthood relation $\mathbf{P_{OC}}$ of Section 3, $\mathbf{P_H}$ does have useful formal properties and can serve as a basis for interesting relations among objects. It is only important that care be taken in recognizing that these relations may not behave as expected. We have already seen some surprising properties of $\mathbf{P_H}$ and $\mathbf{O_H}$. It is also worth noting that Hudson adopts a more complicated definition of summation than the standard definition used in Section 4 above (see [2001, p. 65] for Hudson's definition). Hudson's own summation relations works out nicely. But it is easy to verify that, plugging $\mathbf{P_H}$ and $\mathbf{O_H}$ into the standard definition of summation, we end up with an ugly relation that makes any object the 'sum' of any *one* of its atomic $\mathbf{P_H}$-parts at the (point-sized) region where the atomic part is located.

Even if we can come to terms with its unexpected logical properties, $\mathbf{P_H}$ is clearly at odds with ordinary thinking about parthood. $\mathbf{P_H}$ holds between x and y at r only if the region r is *included in* one of y's locations. Thus, if y is an ordinary object like a cat or a bicycle, the indexing region r must have a quite constricted spatiotemporal extent—it must be small enough to 'fit inside' of y. But we do not in ordinary discourse link parthood to anything like these small spacetime regions. (There is no time that corresponds in a natural way to any spatiotemporal region that fits inside my cat.) Moreover, we ordinarily assume that objects with widely separated locations—objects, like the Empire State Building and the Taj Mahal, which never occupy overlapping regions and never share parts—may have parts at the same time. By contrast, objects x and y can have $\mathbf{P_H}$-parts at the same region only if x and y have overlapping locations. In these respects, $\mathbf{P_H}$'s region-relativization of parthood takes a form quite different from the ordinary time-relativization of parthood.

In addition, under Hudson's assumptions concerning the dimension of objects' locations in spacetime, $\mathbf{P_H}$ would not preserve many of what we take to be the most obvious parthood linkages among objects. As stated earlier, Hudson claims that ordinary objects are located at multiple *temporally-extended* regions. Presumably then an object like Jane Austen is located at multiple regions, all of which extend from some time in 1775 to some time on July 18, 1817. And Jane's head (as well as Jane's hands, Jane's legs, and so on) occupy multiple regions, all of which extend somewhat past July 18, 1817. In this four-dimensionalist picture, there is no region r which includes one of Jane's head's locations and is included in one of JA's locations. In other words, there is no region r at which Jane's head is a $\mathbf{P_H}$-part of JA. Similarly, there is no region at which Jane's hands, legs, liver, and so on, are $\mathbf{P_H}$-parts of JA. This is the same sort of divergence from ordinary parthood ascriptions which we have already noted in the binary relation $\mathbf{P_{UL}}$.

Another region-relative parthood relation plays a central role in [Crisp and Smith, 2005]. Like $\mathbf{P_H}$ (and unlike $\mathbf{P_{S\text{-}3D}}$ and $\mathbf{P_{S\text{-}4D}}$), Crisp and Smith's ternary parthood relation—call it '$\mathbf{P_{CS}}$'—does not limit its third argument to the regions of any special collection such as a slice set. Although Crisp and Smith do not fill in many details concerning $\mathbf{P_{CS}}$, their explicit assumptions make it clear that this parthood relation is different from $\mathbf{P_H}$. It is only in this difference that I am interested here.

Crisp and Smith stipulate that $\mathbf{P_{CS}}$ must satisfy three principles [2005, p. 332-333]. I shall consider only the second of these principles.[54] It is stated as follows:

(*) "...if *x* is wholly present at *R* and *x* is a part of *y* at *R*, then *x* is a part of *y* at every superregion and every subregion of *R*" [2005, p. 333].

I take it that by 'is wholly present at *R*', Crisp and Smith mean roughly what I do by 'is located at *R*'.[55] On this assumption $\mathbf{P_H}$, unlike $\mathbf{P_{CS}}$, does not generally satisfy (*). From condition (4.vi) on H Models, it follows that if *x* is a $\mathbf{P_H}$-part of itself at region *r*, then *x* is not a $\mathbf{P_H}$-part of itself at *any* proper subregion or proper superregion of *r*. Let <**ST**, **R**, **OB**, **L**> be any H Model in which **ST** includes more than one spacetime point. In any such model, every region has a proper subregion or a proper superregion. It follows that for any object $x \in \mathbf{OB}$, there is some region *r* at which *x* is wholly-present (i.e., located) and a $\mathbf{P_H}$-part of itself, but where there is at least one subregion or superregion of *r* at which *x* is *not* at $\mathbf{P_H}$-part of itself. Thus, (*) fails for $\mathbf{P_H}$ on all H Models in which there are at least two spacetime points.

6. Concluding Remarks

In this paper, I have used mathematical models to represent different types of binary and ternary parthood relations. I have shown that these model theoretic relations have significantly different logical properties and that some of these relations can capture ordinary assumptions about parthood better than others. My primary conclusions are as follows.

- Philosophers have proposed several different region-relative parthood relations for domains of multiply-located objects. These relations have significantly different logical properties and rely on different kinds of assumptions about how objects are located in spacetime.
- On a relativistic account of spacetime, it is not obvious whether there is any relation that both fits ordinary thinking about parthood *and* can play the central role in an analysis of relations among objects that some philosophers have tried to assign parthood. Given multi-location, I think that $\mathbf{P_{S\text{-}3D}}$ is the best shot at a relation that can satisfy both criteria. But, as we have seen, it is not obvious that there is an appropriate slice set for the $\mathbf{P_{S\text{-}3D}}$ relation.

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[54] For the record, the other two assumptions are

-- in case $\mathbf{P_{CS}}$ is "analyzable in terms of parthood *simpliciter*, the analysis is given by... :

x is part of *y* at $R =_{def}$ (i) *x* is a part *simpliciter* of *y*, and ii) *x* overlaps *r*" [2005, p. 332];

-- "...if *y* is wholly present at *R* and *x* is part of *y* at *R*, then *x* is wholly present at a subregion of *R*" [2005, p. 333].

[55] This assumption has been confirmed by Crisp in email correspondence.

Of course I do not think that the sort of model theoretic investigation pursued in this paper can cover all issues relevant to parthood and location. There may be factors involved in parthood relations (e.g., relations of functional interdependence) that cannot be represented in a natural or illuminating way in terms of mathematical models. Also, there are important aspects of spatiotemporal location (the dimension of a location, the continuity of an object's path) that can only be represented in more complex models than those used in this paper.

But I do think that model theoretic representations are a good starting point for a discussion of parthood relations, especially if we leave open the possibility of multiply located objects. As we have seen, over multiply located domains, the only plausible *binary* parthood relations are either extremely weak or fail to preserve important common sense intuitions. A ternary region-relative parthood relation may be a more appropriate choice for these domains but different types of region-relative parthood relations are possible. A model theoretic representation can help determine which relation is intended, what logical properties this relation is supposed to have, and what general assumptions are made about the interaction between location and parthood. Too little is said about these important issues in philosophical work that makes use of region-relative (or time-relative) parthood relations. As a result, the reader is sometimes given little indication of how he is to understand the author's claims about region-relative parthood.[56]

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[56] I am grateful for the helpful comments of Yuri Balashov, Thomas Crisp, Cody Gilmore, Hud Hudson, and Dean Zimmerman on the work presented in this paper.

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