Summation Relations and Portions of Stuff

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1. Portions of stuff

We have in front of us a full glass of water. Consider the water in this glass. We shall have a lot to say about the structural properties of this particular portion of water so it will help to give it a name—Walter.[1] Like other portions of stuff and unlike familiar integral objects (people, tables, glasses), Walter can survive quite a lot of scattering. For instance, Walter survives being distributed into 100 smaller glasses. Similarly, a portion of gold may be distributed into 100 rings and a portion of furniture may be distributed to 100 households. Compared in this respect to its glass, Walter is strikingly robust—the glass does not even survive being split in two. But there is after all a limit to how much scattering a portion of stuff can take. As an extreme example, if every water molecule were split and all hydrogen atoms grouped together on one side of the universe and all oxygen atoms grouped on the other side of the universe, there would no longer be any water at all.[2] Walter cannot survive *this* sort of scattering of its parts.

At the very least, though, we take it as uncontroversial that the following principle holds:

Scattering Principle for Portions of Stuff: Let x be a portion of stuff. Let Sub_x be any collection of subportions[3] of x which at a given time comprise all[4] of x. Necessarily, whenever the members of Sub_x are all present, x is also present and the members of Sub_x comprise all of x. (In particular, if the members of Sub_x are distributed arbitrarily far from one another then, as long as they all survive relocation, x survives as a scattered entity.)

Notice that the Scattering Principle allows for very many different ways of dividing Walter along subportion lines and scattering Walter to the four corners of the universe. The water currently in the top half of the glass and the water currently in the bottom half of the glass now comprise all of Walter and will continue to do so even if one of these subportions is sent to Venus and the other to Mars. But Walter is also comprised of the

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[1] Here and throughout this paper, we use 'portion of water' as a synonym for 'portion of H_2O' and not, e.g., to pick out the sort of liquid mass you expect to have delivered to your table when you ask your waitperson for a glass of water (where, presumably, you do not expect to receive a glass of ice). Note, however, that our use of 'water' differs from that of [Barnett 2000] where it is argued that 'water' and 'H_2O' do not function as rigid designators for the same kind of stuff.

[2] [Zimmerman 1995, 64] makes this point.

[3] By a 'collection', we mean a non-empty set of individuals. By a 'subportion' of the portion of stuff x, we mean a portion of stuff *of the same kind as* x which is part of x. For example, any portion of water that is part of Walter (e.g. the water currently in the bottom half of our glass) is a subportion of Walter. Notice, however, that very many of the portions of stuff which are part of Walter are *not* subportions of Walter. For example, many portions of hydrogen are parts of Walter but are not subportions of Walter since they are not portions of water.

[4] By 'comprise all of x', we mean that the subportions sum to x at the time in question in the sense of the summation relation ***SUM*** introduced in our formal theory in Section 3. Briefly, the subportions in Sub_x comprise all of x at time t if and only if every member of Sub_x is part of x at t and every part of x at t overlaps a member of Sub_x at t. In Walter's case, this requirement guarantees that every part of every molecule in Walter overlaps at least one of the subportions under consideration.

water currently in the top one hundredth of the glass, the water currently in the second hundredth of the glass, and so on. And Walter survives even if these one hundred subportions are distributed unharmed to one hundred different planets.

The Scattering Principle embodies a sufficient condition for the survival of a portion of stuff— as long as an appropriately extensive collection of *x*'s subportions survives, then so does *x*.[5] Philosophers have generally assumed that the survival of its subportions is also a necessary condition for the survival of the portion of stuff.[6] We will see in the next section persuasive reasons, defended in [Barnett 2004], for thinking otherwise—that portions of certain kinds of stuffs can indeed gain and lose subportions. Nonetheless, we concede (as does Barnett) that very many familiar portions of stuff cannot gain or lose subportions. In particular, we take it that Walter can neither gain nor lose subportions.[7] As we have just remarked, by removing half (or one third, or one hundredth,...) of the water from the glass we merely transform Walter into a scattered entity but do not cause him to lose any parts. And by, say, dumping Walter into a larger bucket of water, we do not add parts to Walter, but merely disperse Walter within another portion of water.

We also assume, as do [Burge 1977], [Zimmerman 1995], and [Barnett 2004], that portions of water cannot lose or gain molecules.[8] This, we admit, is rather strict by commonsense standards. In ordinary contexts—where we cannot track the movements of molecules—we generally talk as though the same portion of water might remain in our glass even after a period of time during which some molecules of the original water must have evaporated. But we agree here with [Zimmerman 1995] in urging a distinction between loose and strict standards of identity for portions of stuff. Strict standards, linking each portion of stuff to a fixed set of constituents, are needed to make sense of how portions of stuff remain identical over time through a potentially drastic change in their physical configurations (one moment Walter lies placidly in its glass; minutes later Walter is scattered throughout the world in vapor clouds). Unlike bicycles, glasses, or cats, whose identities are plausibly tied to a stable form, no fixed form characterizes a (non-minimal) portion of stuff throughout its lifetime. This is why a 'ship of Theseus' paradox cannot even get off the ground for portions of stuff, as it can for integral objects. (Imagine taking each of Walter's molecules and replacing them one by one within the glass with different water molecules. At the end of this process, there is no temptation to think that we still have the same water in the glass. Walter is definitely somewhere else—wherever its molecules have been moved.) Some level of mereological constancy is the only sort of identity criteria that makes sense for portions of stuff. There is no question here of any competing claims for tracing identity through form or function as we do when

[5] It goes without saying that the survival of *just any* collection of Walter's subportions does not guarantee Walter's survival. For example, if the water in the top half of the glass is annihilated then Walter is also destroyed even though the portions of water in the bottom half of the glass survive.

[6] See, e.g., [Thomson 1998], [Burge 1977], [Zimmerman 1995], and [Simons 1987].

[7] [Barnett 2004] claims that portions of H_2O cannot gain or lose subportions. But, in keeping with the position of [Barnett 2000] (see note 1 above), he leaves open the possibility that 'water' designates a kind of stuff whose portions can gain and lose subportions.

[8] In fact, Burge, Zimmerman, and Barnett all hold that a single H_2O molecule *is* a (minimal) portion of H_2O. On this view, the fact that a portion of H_2O cannot gain or lose molecules is just a special case of its inability to gain or lose subportions.

we assume that a ship remains the same through drastic changes in its parts as long as it continues to operate as a ship.

So portions of stuff, unlike familiar integral objects (which may be identified through their functions or configurations), require a constant basis of parts. The mereological constancy principle for portions of stuff that we adopt in this paper—and which is also assumed in [Burge 1977], [Zimmerman 1995], and [Barnett 2004]—is the following.

Constant Basis Principle for Portions of Stuff: For any portion of stuff *x* there is some collection Bas_x (a *basis* of *x*) such that, necessarily: i) whenever *x* is present, *x* is comprised of the members of Bas_x and ii) every subportion of *x* is comprised of a subcollection of Bas_x whenever it is present.[9]

We will see in the next section that the Constant Basis Principle does not require that all portions of stuff retain the same subportions throughout their lives. For now, notice that Walter satisfies the Constant Basis Principle. Let WMol be the collection consisting of every water molecule in Walter. Necessarily, i) whenever Walter is present every molecule in WMol is part of Walter and, taken together, the members of WMol comprise all of Walter; and ii) every subportion of Walter is comprised of a subcollection of WMol. WMol is therefore a basis for Walter. But of course WMol is not the only basis for Walter. Let WSub be the collection consisting of all of Walter's subportions. Since Walter can neither lose nor gain subportions and since, trivially, each of Walter's subportions is comprised of one of Walter's subportions (itself!), WSub is also a basis for Walter. [10]

Taken together, the Scattering Principle and the Constant Basis Principle tell us that portions of stuff generally have a very different mereological structure than do ordinary integral objects. On the one hand, Walter must be present whenever its proper subportions are all present, no matter what relations hold between the subportions themselves. On the other hand, no portion of stuff can lose a single member of its basis. In *this* respect, Walter is much less robust than its glass. The glass, like most integral objects, can lose whole chunks of molecules and noticeable portions of its underlying material (if, e.g., it is chipped).

Impressed by these distinctive features of portions of stuff, philosophers have proposed that all portions of stuff are sums (or aggregates) of their subportions, where this is understood to imply that a portion of stuff can neither gain nor lose any subportion. Philosophers who have advocated (or at least seriously considered) some version of the 'sum view' of stuffs include [Bunt 1985], [Burge 1972, 1977], [Cartwright 1965, 1975], [Cochiarella 1976], [Grandy 1975], [Moravcsik 1970], [Sharvy 1980, 1983], [Quine 1960], [Roeper 1983], [Simons 1987], and [Zimmerman 1995].

In his paper "Some Stuffs are not Sums of Stuffs", David Barnett argues that the sum view is false—portions of certain kinds of stuff can gain and lose subportions and, thus,

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[9] Note that without condition (ii), the Constant Basis Principle would be trivially satisfied by any individual. If *o* is any object and $\{o\}$ is the one-member collection whose only member is *o*, then, necessarily, whenever *o* is present, the member of $\{o\}$ comprises all of *o*.

[10] Notice that if what is described below as the 'sum view' of stuffs were true, it would follow that for *any* portion of stuff *x*, the collection, Sub_x, of *x*'s subportions is a basis for *x*.

are not sums of their subportions. Barnett concludes that portions of stuff belong to at least two distinct ontological categories. Those, like Walter, which conform to the sum view are sums. But portions of stuff which can gain and lose subportions are not sums (of their subportions or of anything else) but are rather what [Fine 1999] calls 'rigid embodiments'.

The purpose of this paper to show how, while accepting Barnett's counter-example to the sum view, the important kernel of truth in the sum view can be preserved, and the view as a whole made more precise, by distinguishing between different kinds of summation relations. Throughout his paper (and, in particular, in his claim that some portions of stuff are not sums of their subportions), Barnett assumes a quite strong summation relation—one that holds between an object *x* and a collection *C* of objects only if *x* cannot be present without being comprised of the members of *C* and the members of *C* cannot all be present without comprising *x*. We will introduce significantly weaker summation relations and show how they capture different senses in which *all* portions of stuff are sums of, on the one hand, their subportions and, on the other hand, their basis members. Our characterization of stuffs focuses on the common structural features of portions of stuff—their conformity to the Scattering Principle and the Constant Basis Principle—which are somewhat obscured in Barnett's two category characterization of stuffs. We will also argue that it is not so clear that portions of stuff really do belong to two separate ontological categories in the way that Barnett supposes.

The remainder of this paper proceeds as follows. In the next section, we discuss Barnett's objection to the sum view. In Section 3, we develop a modal temporal mereology in which we introduce several different summation relations and show how these can be used to characterize portions of stuff in way that does justice to Barnett's counter-example as well as to the Scattering Principle and the Constant Basis Principle. In the final section, we consider Barnett's claim that certain portions of stuff are "not sums at all" and argue that it is in need of more support.

2. Structured stuffs and unstructured stuffs

Barnett's counter-example to the sum view trades on a distinction between two different ways in which basis elements may form a portion of stuff. In the case of Walter and other portions of what we call 'unstructured stuffs'[11], it is sufficient for the members of a basis to merely be present for the portion of stuff they form to also be present. Walter is present if and only if each of its molecules (or, alternatively, each of its subportions) is present. The water molecules do not need to stand in any special relation to one another for Walter to be present. They *could* sit nicely together in a glass as they do now, but

[11] What we are calling 'unstructured stuffs', Barnett calls 'discrete stuffs' and what we call 'structured stuffs', Barnett calls 'nondiscrete stuffs'. Barnett's terminology stems from an additional distinction he makes between the two categories of stuff—minimal portions of stuff kinds belonging to the first category are always discrete (i.e. do not share parts) while minimal portions of stuff kinds belonging to the second category can share parts. However, Barnett does not explain how the discrete/nondiscrete distinction is connected to the sum/rigid embodiment distinction. In particular, he gives no reason for thinking that *every* stuff kind whose portions are characterized in terms of a certain embodiment relation must have overlapping minimal portions. (We do, however, agree with Barnett that all *familiar* examples of structured stuffs do seem to have overlapping least portions, if they have least portions at all.) Since the discrete/nondiscrete issue is not relevant to our primary focus in this paper, we prefer to ignore it.

Walter would still be in the world even if the members of WMol were dispersed in the atmosphere or absorbed into various organisms' bodies.

But for other kinds of stuff—what we call 'structured stuffs'—the members of any basis must stand in a certain kind of relation to one another (what we will call a 'stuff-forming' relation) for the corresponding portion of stuff to be present. Next to Walter is a full glass of lemonade. Call the portion of lemonade in this glass 'Clem'. Like Walter, Clem cannot survive the loss or gain of a single molecule. (Again, we recognize that a looser standard of portion identification is often employed in ordinary contexts but, for the reasons stated in the previous section, we hold that this looser standard does not stand up to philosophical scrutiny.) Thus ClMol, the collection of Clem's molecules, is a basis for Clem—whenever Clem is present, Clem is comprised of the members of ClMol and every subportion of Clem (i.e., every portion of lemonade which is part of Clem) is comprised of a subcollection of ClMol. But, unlike Walter, the mere presence of the molecules in its basis is not a sufficient condition for Clem's presence. Indeed, under normal circumstances we would expect that a portion of lemonade's constituent molecules had been around for quite some time before being mixed together to form the lemonade. When we merely had a bottle of water, a package of sugar, and some lemons, we did not yet have any lemonade. Other examples of what Barnett takes to be structured stuffs are milk, crude oil, graphite, quartz, and wood [2004, 95].

Before continuing to Barnett's counter-example to the sum view, two points are in order concerning this distinction between structured and unstructured stuffs. First, not every philosopher accepts it. Burge holds that Clem (or any other portion of lemonade) is an aggregate of its molecules in *exactly the same way* as Walter is an aggregate of its molecules [Burge 1977, 110]. On Burge's view, just as Walter is in the world for as long as each member of WMol is present, so Clem is in the world for as long as each member of ClMol is present. It is an immediate consequence of Burge's view that Clem need not always be a portion of lemonade. Rather, on Burge's view Clem has been in the world for quite some time as a scattered aggregate of water, sugar, and acid molecules before entering its temporary 'lemonade phase'. We do not adopt Burge's uniform treatment of (portions of) stuff because it takes us further from commonsense assumptions than we think it necessary to go. We see no reasons (and Burge gives none) for rejecting the ordinary assumption that new portions of stuff are created when the ingredients of a mixture are combined. When we mix together sugar, water, and lemon juice we make something new—some lemonade. We do not merely alter an aggregate of scattered molecules that has been sitting in our kitchen for weeks. Also, Burge's treatment of mixtures would seem to push us further in the direction of mereological universalism—requiring at least that scattered pluralities of random molecules comprise something—than we are willing to go.

Second, it may seem to some readers that Walter is, just as much as Clem, a portion of structured stuff. After all, we have no H_2O molecules until oxygen atoms stand in a certain relation to hydrogen atoms. Would not Walter, then, be the portion of structured stuff that is produced when its hydrogen atoms and its oxygen atoms are combined in this particular way? Against this proposal, we submit that no collection of hydrogen atoms and oxygen atoms is a basis for Walter or any other portion of water. Why? As a portion of H_2O, Walter is delineated by its water molecules— necessarily, Walter is present exactly where and exactly when the members of WMol are present. But it is possible, at least in principle, for any one of these molecules to lose and gain atoms. If a member of WMol were to survive the loss of an atom (if, say, it were to exchange a hydrogen atom with a molecule that is not a member of WMol), then Walter would survive the loss of an atom. Thus, the collection of hydrogen and oxygen atoms in Walter is not a basis for Walter since it is not necessary that Walter is always comprised of the same atoms.

Barnett agrees with the sum view that all portions of unstructured stuffs are sums of their subportions. But he holds that portions of structured stuffs are *not* sums of their subportions. To make his case, Barnett focuses on an example of a portion of crude oil. We have a portion of crude oil only when different types of hydrocarbon molecules are mixed together in a certain way, just as we have a portion of lemonade only when sugar, water, and lemon juice are mixed together in a certain way. Let Crude be the portion of crude oil occupying a given barrel and let CrdMol be the collection of Crude's molecules. Crude survives as long as, and only as long as, each member of CrdMol forms oil together with other members of CrdMol. For example, Crude would not survive a scattering that distributed each of its molecules to a separate barrel, any more than Clem would survive if its sugar, water, and lemon juice were returned to their separate containers. Nor would Crude survive "if just one of its molecules were plucked from the barrel and sent by itself to Venus, even if that molecule happened to join on Venus with some others to form a different portion of oil" [Barnett 2004, 91]. The stray molecule might stand in an oil-forming relation with hydrocarbon molecules on Venus, but would no longer stand in the appropriate relation to other members of CrdMol.

But note that members of CrdMol may over the course of Crude's life form small subportions of Crude together with different members of CrdMol. If Crude were whipped, shaken, or even just jolted around a bit while remaining in its barrel, then some member of CrdMol would end up in another part of the barrel, isolated from its original cluster of molecules and standing in the oil-forming relation with molecules in the other part of the barrel. But Crude survives this sort of internal rearrangement of its molecules as long as each of Crude's molecules continues to form oil with other of Crude's molecules.

It is the possibility of a molecular reorganization within Crude that leaves room for the loss and gain of subportions. (More generally, it is the possibility that the stuff forming relation R_K associated with structured stuff kind K holds throughout a time interval T on basis collection B while holding on and off during T on subcollections of B that allows portions of K to lose and gain subportions.) For, just as Crude cannot survive the unmixing of its molecules, no proper subportion of Crude can survive the unmixing of *its* molecules. Since any proper subportion, Crude*, of Crude is constituted by a proper subcollection, Crd*Mol, of CrdMol, it is possible for one of Crude*'s molecules to become isolated from all members of Crd*Mol while remaining in the oil-forming relation with members of CrdMol. In this case, Crude* would no longer be present when Crude is present. Crude would then have both lost a subportion (Crude*) and gained whatever new portions of oil are formed when members of Crd*Mol end up in the oil-forming relation with new members of CrdMol.

Barnett illustrates this kind of scenario graphically as follows.

> Suppose we suck the top two thirds of Crude into a very long and narrow straw, leaving $Crude_3$ [the subportion of Crude occupying the bottom third of the barrel] at the bottom of the barrel, though still contiguous with the oil that fills the straw. And suppose that we pull a single molecule from the bottom of the barrel through, and to the far end of, the oil-filled straw. Intuitively, $Crude_3$ no longer exists, for one of its molecules no longer stands to any of its others in an oil-forming relation. ...Crude, on the other hand, survives; for each of its molecules retains an oil-forming relation with some of the others throughout the entire procedure [2004, 91].

Barnett goes on to point out that less exotic cases of molecule displacement, and consequent loss and gain of subportions, must occur regularly in the life of ordinary portions of crude oil as a result of normal molecule motion. Thus, portions of crude oil (as well as portions of lemonade, portions of milk, and so on) are under normal conditions constantly gaining and losing subportions. (It follows that Clem, Crude, and other portions of structured stuff do *not* have collections of proper subportions as bases. The basis of a portion of stuff *x* consists of parts which *necessarily* comprise *x* whenever *x* is present. But portions of structured stuff generally *can* gain and lose subportions, even if, in special circumstances, some portions of structured stuff happen to maintain the same subportions throughout their lives.)

Where does this leave us with the sum view? Barnett's conclusion is that portions of unstructured stuff and portions of structured stuff belong to disjoint ontological categories. Whereas any portion of unstructured stuff is a sum (of its subportions), a portion of structured stuff is not a sum at all but is rather what Kit Fine calls a 'rigid embodiment' [Barnett 2004, 93]. Very roughly, a *rigid embodiment* is the object that results when a specific relation holds among the members of a fixed collection of objects. For example, according to Fine, a bouquet is the rigid embodiment of the *being bunched* relation by a handful of flowers [Fine 1999, 65].

We have no quarrel with Barnett's rigid embodiment analysis of structured stuffs, since we endorse the view that a portion of structured stuff is present only when a certain relation holds among its basis members and this seems to be all that is needed for it to count as a rigid embodiment. But we think that, left on its own, Barnett's two category theory of stuffs obscures the important structural features that sharply distinguish all portions of stuff from integral objects. Like Walter, and unlike their glasses, Clem can survive quite a lot of scattering. Walter and Clem can each be divided into 100 glasses. In both cases, the robustness of the portion of stuff in the face of dispersal is explained by its standing in a certain relation to its subportions. (True. Walter stands in an even stronger relation to *its* subportions—a relation which prohibits subportion loss—but only the relation described in the Scattering Principle is needed to explain how it is that they both survive being distributed into 100 glasses.) Also unlike their glasses, neither Walter nor Clem can lose a molecule. This feature of the portions of stuff is explained by their both standing in the same relation to their molecular bases. (Here again, a stronger relation also holds in Walter's case—one that requires Walter to be present whenever every molecule in WMol is present—but the weaker relation is all that is needed to prohibit molecule loss.)

Careful consideration of time and modality is crucial in an account of the mereological features of stuff. What distinguishes Walter and Clem from their glasses is not the way in which they are composed of their parts at a fixed time but rather the restrictions on how they might be linked to certain of their parts (their subportions and basis members) over time. In the next section, we distinguish between different summation relations in a modal temporal mereology. We will see that some, but not all, of these summation relations hold between all portions of stuff and their subportions. (There *is*, we claim, an important sense in which all portions of stuff are sums of their subportions.) A different summation relation holds between all portions of stuff and their bases. These summation relations may be used to restate the Scattering Principle and the Constant Basis Principle and to further develop the distinction between structured and unstructured stuffs.

3. Model Temporal Mereology

Most familiar mereologies treat parthood as an atemporal non-modal relation. (See, for example, [Whitehead 1929], [Leonard and Goodman 1940], and [Tarski 1956].) The modal temporal mereology, **MTM**, presented in this section is in part inspired by the formal system CT ('continuants and times') developed in [Simons 1987, 177-187]. Other similar proposals for linking parthood to time are found in [Thomson 1983], [Thomson 1998], and [Sider 2001]. Unlike CT, which uses free logic, **MTM** is developed in standard predicate logic. Also, besides adding modal operators, **MTM** introduces a different range of summation relations than does CT. Our summation relations are chosen because they are particularly useful for highlighting differences between structured stuffs and unstructured stuffs.

The domain of **MTM** is divided into three disjoint sorts of entities: objects (over which the variables w, x, y, z range), time instants (over which the variable t ranges), and collections of objects (over which the variables A, B, C range). Objects are understood to include *all* material individuals—portions of stuff such as Walter, Clem, and Crude, as well as integral objects such as tables, cats, and glasses. Collections of objects are non-empty sets of objects. We use the term $\{x_1,..., x_n\}$ to denote the collection whose members are x_1, ..., and x_n.

MTM uses two primitive non-logical predicates. One is the binary predicate ε which takes an object term as its first argument and a collection term as its second argument. We will assume that **MTM** includes axioms guaranteeing that there are arbitrary collections of objects and that ε is interpreted as the (time-independent) relation holding between object x and collection A if and only if x is a member of A.[12] Our primary interest in this paper is in **MTM**'s mereological primitive—the ternary predicate $\boldsymbol{P}$ which takes two object terms and one time instant term as arguments and where $\boldsymbol{P}_t\, xy$ is intended as:

$$x \text{ is part of } y \text{ at instant } t.$$

[12] A quite weak (but still sufficient for our purposes) first-order axiomatization of ε is presented in [Bittner and Donnelly 2007]. But there is also no reason why **MTM** as a whole could not be conjoined with a stronger set theory as long as collection variables are restricted to non-empty sets of objects.

The vocabulary of **MTM** also includes the sentential operator, □, interpreted as the necessity operator governed by standard S5 axioms. The possibility operator, ◊, is defined as usual in terms of □. For the semantics of **MTM**, we assume a collection, Ω, of possible worlds, all of which have the same object, collection, and time domains. We require that the interpretation of the collection membership predicate, ε, is invariant across worlds, but the parthood predicate, ***P***, may have different interpretations in different worlds as long as these interpretations satisfy the mereological axioms (A1)-(A4) stated below. All truth-value assignments for sentences of **MTM** are relative to a world with the necessity operator interpreted as usual as follows:

$$\models_\alpha \Box\, \Psi \text{ if and only if for all } \omega \in \Omega \models_\omega \Psi.$$

Additional mereological predicates are defined in terms of ***P***, ε, and the modal operators. Some that will be brought up in the discussion of summation relations are:

Object(/Time) predicates

$\boldsymbol{PP}_t\, xy$ (x is a proper part of y at t) ≡
$\boldsymbol{P}_t xy \;\&\; y \neq x$ (x is part of y at t and y is not identical to x)

$\boldsymbol{O}_t xy$ (x overlaps y at t) ≡
$\exists z\, (\boldsymbol{P}_t zx \;\&\; \boldsymbol{P}_t zy)$ (there is some object z that is part of both x and y at t)

$\boldsymbol{PR}\, xt$ (x is present at t) ≡ $\boldsymbol{P}_t xx$ (x is part of itself at t)[13]

$\boldsymbol{CP}\, xy$ (x is a constant part of y) ≡ $\exists \mathrm{t}\, \boldsymbol{PR}\, yt \;\&\; \forall \mathrm{t}\, (\boldsymbol{PR}\, yt \rightarrow \boldsymbol{P}_t xy)$ (y is present at some time and, whenever y is present, x is part of y)

$\boldsymbol{EP}\, xy$ (x is an essential part of y) ≡ $\Box\, \forall t\, (\boldsymbol{PR}\, yt \rightarrow \boldsymbol{P}_t xy)$ (necessarily, whenever y is present, x is a part of y)

Collection(/Time) predicates

$\boldsymbol{FP}\, At$ (collection A is fully present at t) ≡
$\forall x\, (x \;\varepsilon\; A \rightarrow \boldsymbol{PR}\, xt)$ (every member of A is present at t)

$A \subseteq B$ (A is a subcollection of B) ≡
$\forall x\, (x \;\varepsilon\; A \rightarrow x \;\varepsilon\; B)$ (any member of A is also a member of B)

MTM has four mereological axioms.

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[13] Simons treats his counterpart of our presence predicate as a separate primitive in CT. But since he requires that i) x is part of y at t only if both x and y are present at t (a version of our (A1)) and ii) x is part itself whenever it is present, it turns out that in CT, x's being present at t is equivalent to x's being part of itself at t. Thus, Simons might have defined his presence predicate as we do here without any change in the theorems of CT.

(A1) $\boldsymbol{P}_t\, xy \rightarrow \boldsymbol{PR}\, xt\ \&\ \boldsymbol{PR}\, yt$
(A2) $\boldsymbol{P}_t\, xy\ \&\ \boldsymbol{P}_t\, yz \rightarrow \boldsymbol{P}_t\, xz$
(A3) $\boldsymbol{PR}\, xt\ \ \&\ \sim \boldsymbol{P}_t\, xy \rightarrow \exists z\, (\boldsymbol{P}_t\, zx\ \&\ \sim \boldsymbol{O}_t\, zy)$
(A4) $\lozenge\ \exists t\, \boldsymbol{PR}\, xt$

Axiom (A1) requires that x is a part of y only at times at which both objects are present. For example, Cleopatra has no parts now and is not now a part of any object. Axiom (A2) requires that, when restricted to a fixed time, parthood is transitive. (A3) requires that at any time t at which x is present, x is not part of y at t only if x has at t a part that does not overlap y at t. For example, if our cat is not now part of his torso then he must have some part (e.g. his ears or his nose) which does not share any parts with his torso. (A4) requires that every object is present at some time in some possible world. (A4) leaves open the possibility that, say, a given actual portion of lemonade might never have been present (if its sugar, water, and lemon juice had never been mixed together).

MTM is in several ways a rather weak mereology. This is intentional. Whenever possible, we have avoided commitment to controversial assumptions so that it will be clear that the summation relations introduced below are compatible with a variety of ontological positions.[14] For example, we leave open the possibility that distinct objects may spatially coincide in the sense of overlapping exactly the same objects at the same time. This is a possibility that Barnett also leaves room for [2004, 97].[15] But Barnett seems to think that his analysis of stuff in terms of sums and rigid embodiments commits him to universalism for both summation and rigid embodiment—i.e., principles requiring that any objects have a sum or that for any relation R holding among objects x, y, z,...there is an object which is the rigid embodiment of R in x, y, z,... . We prefer not to make these assumptions and instead leave open the intuitively plausible possibility that there are some objects (say, our cat and the Empire State Building) which are not now and never have been parts of the same object. Also, **MTM** does not *require*, but merely leaves open the possibility, that objects exist in some possible worlds but not in others. **MTM** is compatible with the assumption that every object exists in every possible world. It is also compatible with the assumption that no object exists in more than one possible world.

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[14] We do not have room in this paper to address perdurantism (i.e. the theory that ordinary objects are temporally extended and persist by having different temporal parts at different times) separately, but it is not hard to see how **MTM** and the summation relations introduced below can be reformulated in perdurantist terms. To do this, we need only embed **MTM** in a modal mereology that includes, in addition to the perdurantists' atemporal parthood predicate, a counterpart of our *present at* predicate (in this context treated as a separate primitive and not as a defined predicate). A time-indexed parthood predicate corresponding to our $\boldsymbol{P}$ could then be defined along the lines suggested in [Sider, 2001, 52-62] and used to formulate counterparts of **MTM**'s axioms and definitions.

[15] In fact, Barnett seems to take it as a consequence of his two category theory of stuffs that some portions of stuff will spatially coincide with other portions of stuff. The idea is that at each moment of its life Crude spatially coincides with a portion of *unstructured* stuff which also has CrdMol as a basis. Though we agree that Barnett's two category account of stuffs would *seem* to bring possibilities for coincidence along with it, we do not here make the positive assumption that distinct objects (or, in particular, distinct portions of stuff) do in fact coincide.

For introducing summation predicates, we find it simpler to represent summation relations as holding (at a time) between an object and a collection rather than between a variable number of objects. This eliminates the need for variable-arity predicates and lets us avoid excessive repetition of ellipses in our explications of formulas using these predicates. Our initial summation predicate, ***SUM***, links an object *x* to collections whose members comprise *x* at a given moment.

$\boldsymbol{SUM}_t\, xA$ (*x* is at *t* a sum of the collection *A*) ≡

$$\forall y\,(y \in A \rightarrow \boldsymbol{P}_t\, yx)\ \&\ \forall z\,(\boldsymbol{P}_t\, zx \rightarrow \exists y\,(y \in A\ \&\ \boldsymbol{O}_t\, zy))$$

(every member of *A* is part of *x* at *t* and every part of *x* at *t* overlaps some member of *A* at *t*)[16]

For example, every member of WMol (the collection of Walter's molecules) is now a part of Walter and any part of Walter (in particular, each of Walter's subportions) currently shares a part with at least one member of WMol. Thus, Walter is now a sum (***SUM***) of WMol. Walter is also currently a sum of the collection of its proper subportions—each of these smaller portions of water is now part of Walter and anything that is now part of Walter currently overlaps one of these subportions.

On its own, ***SUM*** is not so helpful for characterizing structured stuffs and unstructured stuffs or for distinguishing portions of both kinds of stuff from integral objects. When looking at just an instantaneous snapshot of the actual world, Clem's relation to its subportions is exactly analogous to Walter's relation to its subportions and Clem's relation to its molecules is exactly analogous to Walter's relation to its molecules. Also, each of the two glasses is in this sense now a sum of its current molecules and of its current portions of glass.

There are many different ways of introducing cross-temporal summation relations.[17] Two summation relations that are especially useful for our purposes are:

$\boldsymbol{BSUM}\, xA$ (*x* is a bound sum of *A*) ≡

$$\exists t\ \boldsymbol{FP}\, At\ \&\ \forall t\,(\boldsymbol{FP}\, At \rightarrow \boldsymbol{SUM}_t\, xA)$$

(*A* is fully-present at some time and whenever *A* is fully present, *x* is a sum of *A*)

$\boldsymbol{CSUM}\, xA$ (*x* is a constant sum of *A*) ≡

$$\exists t\ \boldsymbol{PR}\, xt\ \&\ \forall t\,(\boldsymbol{PR}\, xt \rightarrow \boldsymbol{SUM}_t\, xA)$$

(*x* is present at some time and whenever *x* is present, *x* is a sum of *A*)

Let **T** be any fixed moment during Clem's life and let $\mathrm{ClSub}_{\mathbf{T}}$ be any collection of portions of lemonade to which Clem stands in the ***SUM*** relation at **T.** For example, if **T** is now, then $\mathrm{ClSub}_{\mathbf{T}}$ may be the two-member collection consisting of the portion of lemonade in the top half of the glass and the portion of lemonade in the bottom half of the glass. Alternatively, $\mathrm{ClSub}_{\mathbf{T}}$ may be the hundred-member collection consisting of the

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[16] ***SUM*** is intended to pick out the same relation as does Sider's (similarly defined) 'fusion-at-a-time' predicate [Sider 2001, 58].

[17] See [Simons 1987, 177-187] for other interesting possibilities.

portion of lemonade in the top one hundredth of the glass, the portion of lemonade in the second one hundredth of the glass, and so on. Or, $\text{ClSub}_{\mathbf{T}}$ may be the collection consisting of every one of Clem's current subportions. In any case, the Scattering Principle tells us that Clem is a *bound sum* of $\text{ClSub}_{\mathbf{T}}$ – whenever every member of $\text{ClSub}_{\mathbf{T}}$ is present (even if the members of $\text{ClSub}_{\mathbf{T}}$ are distributed randomly throughout the universe), Clem is also present and is comprised of the members of $\text{ClSub}_{\mathbf{T}}$.

Of course the same holds for Walter and its subportions. For any moment, **T**, during Walter's life and any collection, $\text{WSub}_{\mathbf{T}}$, of portions of water that sum to Walter at **T**, Walter is a bound sum of $\text{WSub}_{\mathbf{T}}$. Again, this is just a reformulation of the Scattering Principle in terms of the vocabulary of **MTM**. But a stronger relation also holds between Walter and its subportions since, as a portion of water (an unstructured stuff), Walter is necessarily comprised of exactly the same subportions whenever it is present. Thus, Walter is also a constant sum of $\text{WSub}_{\mathbf{T}}$. In particular, Walter is a constant and bound sum of the collection, WSub, consisting of *all* of Walter's subportions. But since Clem, unlike Walter, is constantly gaining and losing subportions, we cannot assume that Clem is a constant sum of $\text{ClSub}_{\mathbf{T}}$.[18]

The Constant Basis Principle tells us that for any portion of stuff, *x*, there is some collection, Bas_x, such that *x* is a constant sum of Bas_x and every subportion of *x* is a constant sum of a subcollection of Bas_x. Both Walter and Clem stand in the constant sum relation to their molecular bases—they are each comprised of the same molecules whenever they are present. Moreover, every portion of water that is part of Walter is a constant sum of a subcollection of WMol and every portion of lemonade that is part of Clem is a constant sum of a subcollection of ClMol. But Walter is also a bound sum of its molecular basis since the members of WMol automatically comprise Walter whenever they are all present. By contrast, Clem is *not* a bound sum of its molecular basis, since its sugar, water, and acid molecules were all present before they were mixed together to form Clem.

To better characterize the distinction between portions of stuff and integral objects, as well as the distinction between structured stuffs and unstructured stuffs, it is useful to also introduce modal summation relations. It *could* be the case that the portions of material (glass) of which Walter's glass is currently comprised happen to sum to the glass whenever they are all present. In other words, the glass might in actuality be a bound sum of certain portions of glass, just as Walter is a bound sum of certain portions of water and Clem is a bound sum of certain portions of lemonade. But what is distinctive of portions of stuff is that they are *necessarily* bound sums of their subportions. Whereas the glass may be destroyed (by, say, splitting it in two) in a way that preserves every portion of glass constituting it, it is not possible to destroy Clem in a way that preserves all of its proper subportions (or even just a few appropriately extensive subportions—e.g., the lemonade currently in the top half of the glass and the lemonade currently in the bottom half of the glass). Similarly, it *could* be the case that Walter's glass is in fact a constant sum of its molecules. This might be so if, say, the glass is present for only a short time and is handled very carefully until it is suddenly destroyed. What is distinctive of portions of stuff is that they are *necessarily* constant sums of their bases. Whereas

[18] Unless $\text{ClSub}_{\mathbf{T}}$ happens to be the one member collection consisting of just Clem. But this is an uninteresting case.

Walter's glass *could* (even if it in fact does not) lose millions of molecules, it is impossible for Walter, Clem, and other portions of stuff with molecular bases to lose a single molecule.

The following modal summation relations let us express these distinctive features of portions of stuff.

ECSUM xA (x is an essential constant sum of A) ≡

$\Box\, \forall t\, (\boldsymbol{PR}\ xt \rightarrow \boldsymbol{SUM}_t\ xA)$ (necessarily, whenever x is present, x is a sum of A)

EBSUM xA (x is an essential bound sum of A) ≡

$\Diamond\, \exists \mathrm{t}\ \boldsymbol{FP}\ At\ \&\ \Box\, \forall t\, (\boldsymbol{FP}\ At \rightarrow \boldsymbol{SUM}_t\ xA)$

(possibly, A is fully-present at some time and, necessarily, whenever A is fully present, x is a sum of A)

The Constant Basis Principle says that every portion of stuff is an essential constant sum of at least one collection, its basis (or, if it has more than one basis, its bases). In particular, Walter is an essential constant sum of WMol and Clem is an essential constant sum of CMol. The Scattering Principle says that, for any portion of stuff x and any collection Sub_{xt} of subportions that sum to x at time t, x is an essential bound sum of Sub_{xt}.

So much is common to all portions of stuff. The model summation relations also distinguish between structured stuffs and unstructured stuffs. While a very short-lived portion of lemonade might happen to maintain the same subportions throughout its life, it is possible (and more likely) that the lemonade loses and gains subportions as it is mixed or moved around in its container. By contrast, it is distinctive of portions of unstructured stuff that they *necessarily* maintain the same subportions throughout their lives. Portions of unstructured stuffs are both essential bound sums and essential constant sums of their subportions, while portions of structured stuffs are merely essential bound sums of (appropriately extensive) collections of their subportions. Also, a portion of unstructured stuff must be present at exactly those times at which its basis is fully present, since it is characteristic of unstructured stuff that no stuff-forming relation need hold among basis members. Thus, portions of unstructured stuffs are both essential constant sums and essential bound sums of their bases. But portions of structured stuff are generally not essential bound sums of their bases—if it is possible for the basis members to be present without standing in the appropriate stuff-forming relation (as are Clem's molecules before they are mixed together), then it is possible for the basis members to be present without summing to the corresponding portion of structured stuff.

We have just seen an important sense in which Clem, Crude, and other portions of structured stuffs are (essential bound) sums of certain collections of their subportions. However, this should not be taken as a refutation of Barnett's claim that portions of structured stuffs are not sums of their subportions, since Barnett clearly has in mind a much stronger summation relation than our ***EBSUM***. Barnett stipulates that sums can neither lose nor gain summands, but essential bound sums need not have their summands as essential parts. What Barnett seems to have in mind is a combination of ***EBSUM*** and ***ECSUM***—a strong sum relation that holds between an individual x and a collection A if and only if, necessarily, x is present at exactly those times at which A is fully-present and A sums to x at each of these times. It is likely that this is also the sort of strong summation relation which proponents of the sum view have had in mind.[19] In any case, Barnett's crude oil example does present a problem for the sum view since this view at least assumes that, unlike ***EBSUM***, the sort of summation relation holding between portions and subportions prohibits subportion loss.

Our analysis is intended as a reformulation of the sum view in terms of the weaker summation relations of **MTM**. With these weaker relations, we can pin down the important features of portions of stuff in a way that avoids difficulties raised by Barnett's counter-example. In addition, because **MTM** is a deductive theory, we can use it to elaborate properties of portions of stuff that may not have been obvious at first glance. For example, both of the following are theorems of **MTM**.

(T1) $\boldsymbol{ECSUM}\, xA \ \ \&\ z \in A \ \&\ \boldsymbol{EP}\, wz \rightarrow \boldsymbol{EP}\, wx$
(T2) $\boldsymbol{EBSUM}\, xA \ \ \&\ \boldsymbol{ECSUM}\, yB \ \ \&\ A \subseteq B \rightarrow \boldsymbol{EP}\, xy$

(T1) tells us that if x is an essential constant sum of A, then any essential part of any member of A is also an essential part of x. For example, suppose that the nucleus of a cell is an essential part of the cell. Let LTissue be the tissue of which our cat's liver is currently comprised. We take LTissue to be a portion of (structured) stuff having a fixed collection of cells as a basis. Given these assumptions, (T1) tells us that LTissue has as essential parts not only the cells of its basis, but also the nuclei of those cells.

(T2) tells us that if y is an essential constant sum of B and x is an essential bound sum of a subcollection of B, then x is an essential part of y. (T2) is especially interesting because it establishes a necessary connection between portions of structured stuffs and the portions of unstructured stuffs with which they share basis members. For example, Clem is an essential constant sum of the collection of molecules, ClMol. Though Clem and its lemonade parts are portions of structured stuff, there are also many portions of unstructured stuffs (water, sugar, citric acid) which are parts of Clem. These portions of unstructured stuffs are essential bound sums (as well as essential constant sums) of subcollections of ClMol. According to (T2), each of these portions is an essential part of Clem. Thus, though Clem might lose or gain large subportions of lemonade, Clem can neither lose nor gain any portion of water, sugar, or acid.

5. Sums and Summation Relations

Barnett uses the term 'sum' both as the name of a relation and as the name for a class of objects. As we have seen, his initial claim is that some portions of stuff are not sums of their subportions—i.e. that a summation relation fails to hold between certain portions of stuff and their subportions. But he also describes a 'sum' as 'one possible type of whole' [2004, ftnt 3] and ends up with the thesis that "there are at least two fundamental categories of stuff: sums and rigid embodiments" [2004, 92].

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[19] In fact, though, this is not as clear as one would like, since the modal and temporal aspects of the relation between a sum and its summands are often not specified.

So far in this paper we have considered only summation relations and have not introduced predicates for distinguishing between objects that are sums and objects that are not sums. It is not hard to see how multiple unary sum predicates might be introduced into **MTM**. For each binary summation predicate S, we could define a corresponding unary sum predicate $S*$ as follows:

$S* x$ (x is an S-sum) ≡ $\exists A\ S\,xA$ (x stands in the S summation relation to some collection)

$S*$ would then pick out all objects which are S-sums of some collection. For example, ***CSUM**** x holds if and only if x is a constant sum of some collection, ***BSUM**** x holds if and only if x is a bound sum of some collection, and so on.

However, none of the resulting sum predicates, ***CSUM****, ***BSUM****, ***ECSUM****, or ***EBSUM****, distinguishes an interesting subcategory of individuals, much less of portions of stuff. It follows from the axioms of **MTM** that *every* object is a ***ECSUM**** sum and a ***EBSUM**** sum. And every existent object is a ***CSUM**** sum and a ***BSUM**** sum. This is because, for any object x, necessarily, whenever x is present x is a sum (***SUM***) of the singleton collection $\{x\}$. Thus, for example, Clem stands in each of the **MTM** sum relations to the singleton collection {Clem} just as Walter stands in each of these sum relations to {Walter}.

We might try to work around this problem by explicitly eliminating the trivial case of $\{x\}$ summing to x. On this somewhat stricter approach, we would say that x is an S-sum only if x stands in the S summation relation to a collection other than $\{x\}$. When we add this extra restriction, we can no longer prove that every object is a sum in each of the four **MTM** senses. However, each of the following is a theorem of **MTM**.

(T3) $\boldsymbol{CP}\ zy \rightarrow \boldsymbol{CSUM}\ x\{z, x\}$
(T4) $\boldsymbol{CP}\ zy \rightarrow \boldsymbol{BSUM}\ x\{z, x\}$
(T5) $\boldsymbol{EP}\ zy \rightarrow \boldsymbol{ECSUM}\ x\{z, x\}$
(T6) $\boldsymbol{EP}\ zy \rightarrow \boldsymbol{EBSUM}\ x\{z, x\}$

It follows from (T5) – (T6) that any object x with an essential proper part is both an essential constant sum and an essential bound sum of a collection other than $\{x\}$. Since all non-minimal portions of stuff have essential proper parts (their basis members), all non-minimal portions of (structured or unstructured) stuff are non-trivial sums in each of the four **MTM** senses. For example, Clem is an essential constant and bound sum of $\{m_C$, Clem$\}$, where m_C is any molecule in ClMol, and Walter is an essential constant and bound sum of $\{m_W$, Walter$\}$, where m_W is any molecule in WMol. Thus these sum predicates, though an improvement on the trivial predicates initially proposed, are definitely not strong enough to distinguish portions of unstructured stuff from portions of structured stuff.

We can do much better by first introducing stronger summation relations—relations which require that all summands are *proper parts*, not just parts, of their sum. We will say that x is a *proper sum* of A at t if and only if all members of A are proper parts of x at t and any part of x at t overlaps some member of A at t.

$\boldsymbol{P\text{-}SUM}\ xA \equiv \forall y\ (y \in A \rightarrow \boldsymbol{PP}_t\ yx)\ \&\ \forall z\ (\boldsymbol{P}_t\ zx \rightarrow \exists y\ (y \in A\ \&\ \boldsymbol{O}_t\ zy))$

Corresponding proper (essential) constant sum and proper (essential) bound sum relations (***P-CSUM***, ***P-ECSUM***, ***P-BSUM***, ***P-EBSUM***) are defined using ***P-SUM*** in the place of ***SUM***. The following is, by our lights, just about the strongest unary sum predicate that might be useful in this context.[20]

SSM *x* (*x* is a strong sum) ≡ ∃*A* (***P-ECSUM*** *xA* & ***P-EBSUM*** *xA*) (for some collection *A*, *x* is both a proper essential constant sum and a proper essential bound sum of *A*)

x is a strong sum if and only if, for some collection *A*, necessarily, *x* is present at exactly those times at which *A* is fully present and, at each of these times, the members of *A* are proper parts of *x* and *A* sums to *x*. Notice that if ***P-ECSUM*** *xA* holds, then every member of *A* is an essential proper part of *x*. Thus, when determining whether an object is a sum in the sense of ***SSM***, we can restrict our attention to the object itself and collections of its essential proper parts.

Does this strong sense of sumhood work for the sort of distinction Barnett wants to make between unstructured and structured stuffs? Not exactly. Barnett and others hold that a single H_2O molecule is a minimal portion of H_2O, a single piece of gravel is a minimal portion of gravel, and so on (see note 8 above). We have no reason for thinking that all minimal portions of unstructured stuff are strong sums because there is no reason to think that minimal portions of unstructured stuffs generally have essential proper parts. For example, it is not obvious that a piece of gravel has any essential proper parts at all, much less a collection of essential proper parts that necessarily sums to it whenever it is fully present. Minimal portions of unstructured stuff are generally similar to portions of structured stuff in that both are present only when certain relations hold among their proper parts. If anything, minimal portions of unstructured stuff would seem to have looser ties to their proper parts than do portions of structured stuff— a piece of gravel can, unlike a portion of lemonade, gain and lose indefinitely many molecules. It seems that any sense of sumhood weak enough to cover pieces of gravel should also extend to portions of lemonade or crude oil.

For this reason, we will assume that it is only *non-minimal* portions of unstructured stuffs which are supposed to be sums in a sense that excludes portions of structured stuffs. It is easy to see that any non-minimal portion of unstructured stuff is indeed a strong sum since it is an essential bound sum and an essential constant sum of the collection of its *proper* subportions.

It is not quite so easy, however, to establish that no portion of structured stuff is a strong sum (or even to establish the weaker claim that, for every structured stuff kind *K*, there are portions of *K* that are not strong sums). In fact, whether or not this is the case depends on stronger ontological commitments than we have made so far in our analyses of portions of stuff. Let *y* be a portion of structured stuff. In the previous section, we saw that *y* must have at least the following essential proper parts: i) the members of *y*'s basis,

[20] A somewhat stronger, but still reasonable, sum predicate would require that *x* is an essential bound and constant sum of a collection whose members are pair-wise discrete (i.e., do not share parts). Here, we would use a sense of summation that is more in spirit of the composition relation of [van Inwagen 1990]. But we do not see that the added discreteness condition helps in this context and it raises modal issues (e.g. whether distinct molecules are necessarily discrete) which we do not wish to take up here.

ii) any essential bound sum of a subcollection of y's basis, and iii) any essential parts of basis members. Any object in i)-iii) must be present whenever y's basis is fully-present. Thus whenever y's basis is fully-present, any collection consisting of essential parts from i)-iii) is also fully-present. If we assume that y's basis can be fully-present at times when y's stuff-forming relation (i.e. the stuff-forming relation for y's stuff kind) fails to hold among basis members, then it is possible for any collection comprised of items from i)-iii) to be fully-present at a time when y is not present. It follows immediately that y is not an essential bound sum of any such collection.

So far, so good. But, though we have no immediate reason for thinking that portions of structured stuffs have essential proper parts other than those listed under i)-iii), we have not ruled out the possibility that they do. Indeed, if we were to accept Barnett's strong concluding suggestion that "whenever and where ever material objects stand in a relation" there is a rigid embodiment having that relation as its embodiment relation and those objects as its basis [2004, 97], then it seems that we must hold that portions of structured stuff have very many additional essential proper parts. For example, whenever the members of CrdMol stand in the oil-forming relation, any two members of CrdMol stand in the binary relation R_{Crude} that holds between two molecules when they are both parts of Crude. Each embodiment of R_{Crude} on a pair of members of CrdMol is an essential proper part of Crude, since, necessarily, whenever Crude is present the two molecules stand in R_{Crude}. Moreover, necessarily, whenever all of these two molecule embodiments are present, Crude is also present and they sum to Crude. Thus if we allow that there are such rigid embodiments, then Crude is after all a strong sum—it stands in the ***P-ECSUM*** and ***P-EBSUM*** relations to the collection of all embodiments of R_{Crude} on pairs of members of CrdMol.

We ourselves do not see any reason to accept the strong thesis that there are rigid embodiments wherever there are relations holding between objects. On the contrary, we tend to think that portions of unstructured stuff probably are not sums in the ***SSM*** sense. But it would take stronger assumptions than we have made in this paper to establish that this is so. The chief conclusion of this section is just that it is not nearly so obvious as it might at first appear that portions of unstructured stuffs qualify as sums in any interesting sense that does not also apply to portions of structured stuffs. If there is such a sense of sumhood, then careful formulation and substantial ontological commitments are needed to establish that it applies as intended.

A secondary point is that, whether or not Barnett's sum/rigid embodiment dichotomy works out for portions of stuff, the summation relations introduced in the previous section seem to be much more useful in our investigation of stuffs than are the sumhood properties discussed in this section. Portions of stuff are characterized primarily by the way they are linked over time to their subportions and bases. We can describe the mereological relations among these items without making the ontological commitments needed to determine exactly which essential parts portions of stuff have.

6. Conclusion

The primary purpose of this paper has been to re-examine the sum view of stuffs in the context of a mereology that explicitly links summation to time and modality. By clearly distinguishing between different summation relations, we have specified an important sense in which all portions of stuff are sums of their subportions. In addition, we have shown how the sum relations of our theory can be used to fill out the distinction between structured and unstructured stuffs which grounds Barnett's counter-example to the sum view.

Although we have focused only on portions of stuff in this paper, the sum relations introduced in **MTM** can be used more generally to describe the structure of material objects. For example, we might reasonably hold that a drinking glass is an essential constant sum of its upper and lower halves, but it is certainly not an essential bound sum of these parts since it is possible for the two halves to survive a separation that destroys the glass.

As stated earlier, most familiar mereologies are atemporal. For describing changes in parthood relations over time some sort of temporal apparatus needs to be incorporated into the mereology. Here, we have used a ternary parthood relation as is also done in [Thomsen 1983, 1998]. But temporalization can also be introduced into a mereology through time-indexed parthood relations as in [Simons 1987], or, for four-dimensionalists, through a relation that links temporally extended objects to their instantaneous temporal parts as in [Sider 2001]. We think that many philosophers have not appreciated the added complexity which results when temporal relations and modal operators are incorporated into a mereology. More attention should be paid to distinguishing between and exploiting the multiple possibilities for temporal and modal versions of familiar mereological relations. [21]

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[21] We are grateful for the very helpful comments of David Barnett and an anonymous reviewer. A version of the mereology **MTM** used in this paper is presented in greater detail in [Bittner and Donnelly 2007].

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