# Applied Cryptography and Computer Security
# CSE 664 Spring 2020

## Lecture 9: Hash Functions

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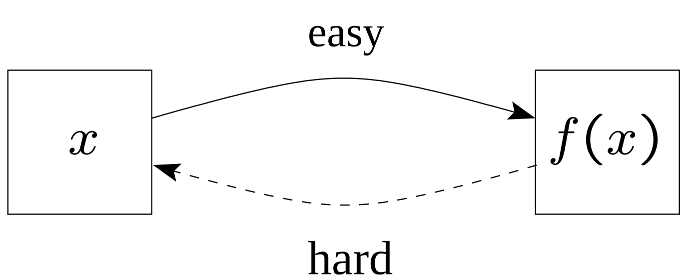
# Lecture Outline

- So far we learned about
  - theoretical tools
  - practical algorithms
- In this lecture we learn about another practical tool of great importance in cryptography
  - hash functions
  - HMAC
  - other uses of hash functions

# Quick Detour: One-Way Functions

- A one-way function is easy to compute, but is hard to invert

- More formally, if $f$ is one-way, then it is easy to compute $f(x)$ from $x$, but given $f(x)$ it is infeasible to find $x$




- Example: breaking a glass

- One-way functions are a very powerful tool

- It is not known whether they exist

# Hash Functions

- A hash function $h$ at minimum should satisfy the following properties:
  - compression: $h$ maps an input $x$ of an arbitrary length to a (short) fixed-length output $h(x)$
  - ease of computation: given $h$ and $x$, $h(x)$ is easy to compute
- Hash functions have many uses including hash tables
- We are interested in cryptographic hash function that must satisfy certain security properties
- Informally, what we are looking for in a hash function $h$ is:
  - given $h(x)$, it is hard to compute $x$
  - it is hard to find $x$ and $x'$ such that $h(x) = h(x')$

# Hash Functions

- Cryptographic hash functions are often used as a real-life substitute for ideal one-way functions
- But they have other important uses as well:
  - data integrity
  - message authentication
  - password hashing and one-time passwords
  - in digital signatures
  - timestamping
  - and others

# Hash Functions

- More formally, let $h : X \to Y$ be a cryptographic hash function
- $h$ must satisfy the following security properties:
  - Preimage resistance (one-way): given $h$ and $y \in Y$, it is difficult to find $x \in X$ such that $h(x) = y$
  - Second preimage resistance (weak collision resistance): given $h$ and $x \in X$, it is difficult to find $x' \in X$ such that $x' \neq x$ and $h(x') = h(x)$
  - Collision resistance (strong collision resistance): given $h$, it is difficult to find $x, x' \in X$ such that $x' \neq x$ and $h(x') = h(x)$

# Hash Functions

- Normally the input domain is all strings $\{0,1\}^*$ and the output is $\{0,1\}^{\ell(n)}$ for security parameter $n$
- Collision resilience formally: collision finding experiment $\mathsf{Hash\text{-}coll}_{\mathcal{A},h}(n)$:
  1. adversary $\mathcal{A}$ is given $h$ and outputs $x$, $x'$
  2. output 1 ($\mathcal{A}$ wins) if $x \neq x'$ and $h(x) = h(x')$, and 0 otherwise
- Definition: A function $h$ is collision resistant if any PPT adversary $\mathcal{A}$ can't win the game with more than a negligible probability, i.e.:

$$\Pr[\mathsf{Hash\text{-}coll}_{\mathcal{A},h}(n) = 1] \leq \mathsf{negl}(n)$$

# Hash Functions

- A good cryptographic hash function (satisfying the definition) will have:
  - non-correlation: input bits and output bits should not be correlated (and it is desirable that every input bit affects every output bit)
  - near-collision resistance: it should be hard to find any two inputs $x$ and $x'$ such that $h(x)$ and $h(x')$ differ only in a small number of bits
  - partial-preimage resistance or local one-wayness: it should be as difficult to recover any substring as to recover the entire input
    - and even if part of the input is known, it should difficult to find the remainder

# Hash Function

- A cryptographic hash function can be keyed
  - it takes a secret key as its another parameter
  - that secret key defines the function's behavior
    - i.e., each new key makes it a new hash function
- Formally, a hash family is defined by algorithms $(\mathsf{Gen}, \mathsf{H})$
  - key generation algorithm $\mathsf{Gen}$, on input security parameter $1^n$, outputs key $k$
  - hashing algorithm $\mathsf{H}$, on input a key $k$ and string $x \in \{0,1\}^*$, outputs a string $y \in \{0,1\}^{\ell(n)}$
- The key $k$ can be public or private

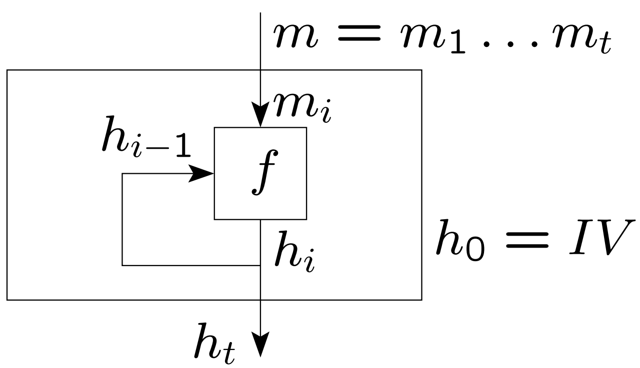
# Hash Functions

- Commonly used hash function algorithms:
  - MD5
  - SHA-1
  - SHA-2 family (SHA-256, SHA-384, and others)
- Normally hash function algorithms are iterated
  - they use a compression function
  - the input is partitioned into blocks
  - a compression function is used on the current block $m_i$ and the previous output $h_{i-1}$ to compute

$$h_i = f(m_i, h_{i-1})$$

## Hash Functions

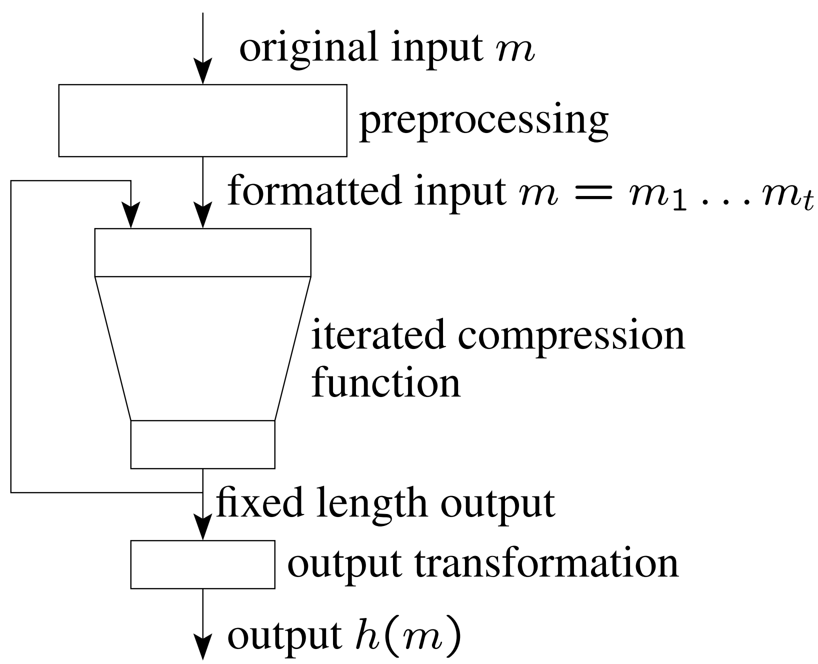
- Most unkeyed hash functions use a compression function $f$
  - $f$ takes a fixed length $\ell$-bit input and outputs an intermediate result of length $n$ ($\ell > n$)
- Most unkeyed hash functions use chaining
  - output of the current block depends on all previous blocks
  - let the input be $m = m_1 m_2 \ldots m_t$
  - set $h_0 = IV$; $h_i = f(m_i, h_{i-1})$; and $h(m) = h_t$



# Hash Functions

- Often, before the iterated compression function is called a preprocessing step is used

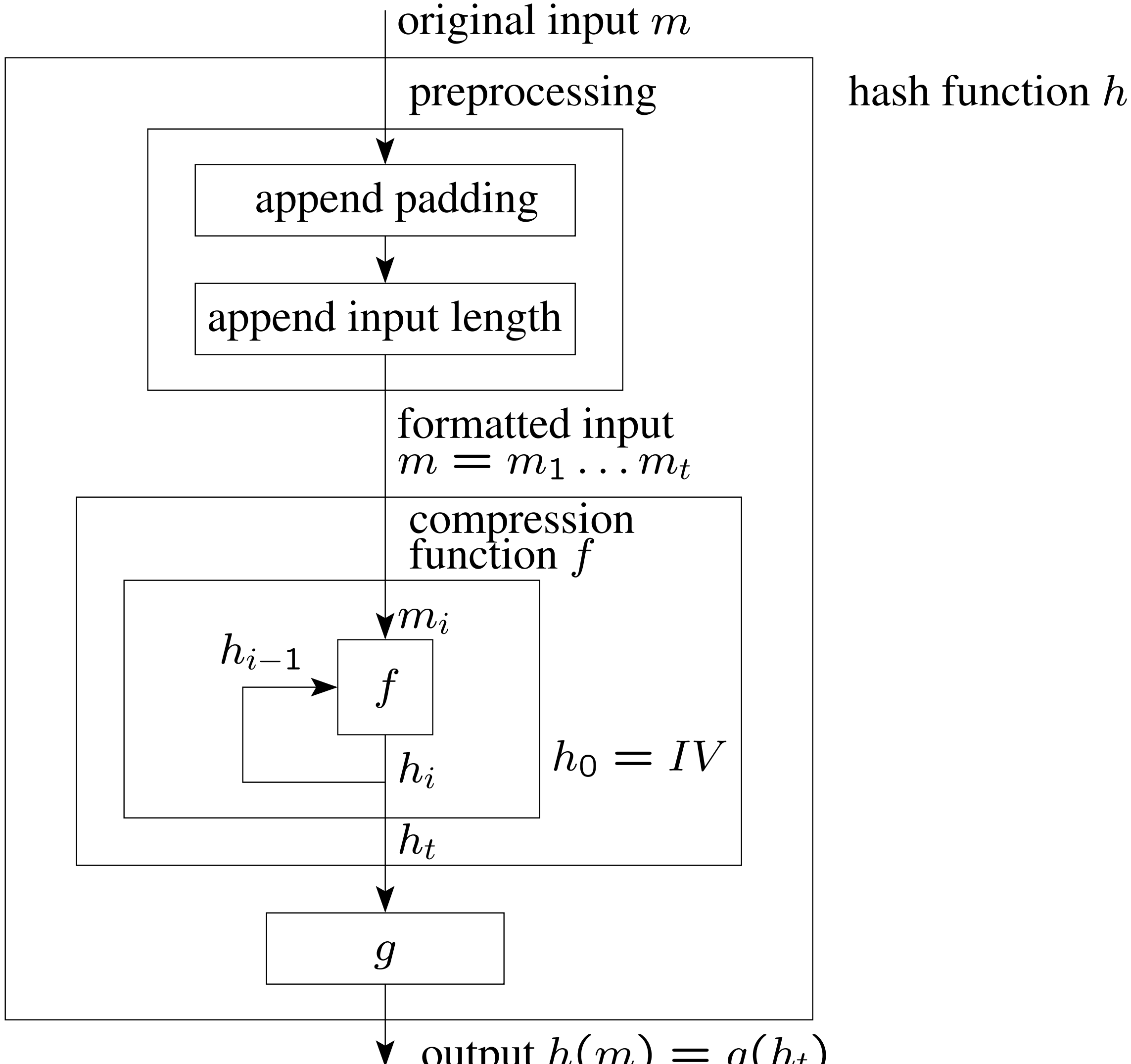
- Also, after the compression function, output transformation can be applied



# Hash Functions

- The preprocessing step typically includes:
  - padding the message (i.e., appending extra bits) to obtain a bitlength multiple of the blocklength $\ell$
  - appending the length of the unpadded input
    - this prevents collisions and thus improves security
- The output transformation $g$ is optional
  - it can map the $n$-bit output $h_t$ to a result of another length
  - often $g(h_t) = h_t$

# Hash Functions: Detailed View



# Hash Functions

- Merkle-Damgard construction
  - we are given a compression function $f : \{0,1\}^{\ell+n} \to \{0,1\}^n$
  - divide the input $m$ into $t$ blocks $m_1 m_2 ... m_t$ of size $\ell$ padding the last block with 0s if necessary
  - define an extra final block $m_{t+1}$ to hold the right justified binary representation of original $m$'s length
  - set $h_0 = 0^n$ and compute $h_i = f(h_{i-1} || m_i)$ for $i = 1, ..., t+1$
  - output $h(m) = h_{t+1}$
- Theorem: If $f$ is (fixed-length) collision resistant hash function, this construction is collision resistant

# Hash Functions

- Cascading hash functions
  - we are given two hash functions $h_1$ and $h_2$
  - if either $h_1$ or $h_2$ is collision resistant, $h(x) = h_1(x)||h_2(x)$ is a collision resistant hash function
  - if $h_1$ and $h_2$ are independent, have to find a collision in both simultaneously
  - hopefully this would require the product of the effort to attack them individually
  - this is a simple yet powerful way to increase strength using available functions

# Attacks on Hash Functions

- Attacks on the bitsize of a hash
  - assume we are given a message $m$ and its hash $h(m)$
  - we want to find another message $m'$ with the same hash
  - a naive approach for finding a collision is to pick a random $m'$ and check whether $h(m) = h(m')$
  - this can result in very little effort, but for well-distributed hashes the probability of a match is $2^{-n}$
  - however, if we have control over $m$ as well, the effort greatly reduces
  - colliding pairs of messages $m$ and $m'$ where $h(m) = h(m')$ can be done in $2^{n/2}$ time

# Birthday Attack

- Birthday attack is one of cryptographic applications of birthday paradox
- Birthday paradox:
  - we are given a group of people
  - what is the minimum group size required to find two people who who share the same birthday with probability at least $1/2$?
- General problem statement:
  - we are given a random variable that is an integer with uniform distribution between 1 and $n$
  - given a selection of $k$ instances ($k < n$) of the variable, what is the probability $\mathsf{Pr}(n, k)$ that there is at least one duplicate?

# Birthday Paradox

- Calculating $\Pr(365, k)$
  - if we pick $k$ random days out of 365, what is the probability that there are no collisions?
  - the number of possibilities with no collision:
    $365 \times 364 \times \cdots \times (365 - k + 1) = 365!/(365 - k)!$
  - the total number of possibilities: $365^k$
  - thus, we obtain

$$\Pr(365, k) = 1 - \frac{365!}{(365 - k)!365^k}$$

  - if $k = 23$, $\Pr(365, 23) = 0.5073$

# Birthday Paradox

- In general:

$$\begin{aligned}\Pr(n,k) &= 1-\frac{n!}{(n-k)!n^k} = 1-\frac{n(n-1)\cdots(n-k+1)}{n^k}\\ &= 1-\frac{n}{n}\cdot\frac{n-1}{n}\cdot\frac{n-2}{n}\cdots\frac{n-(k-1)}{n}\\ &= 1-\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right)\end{aligned}$$

  - if $x$ is a small real number, then $1-x \approx e^{-x}$
  - using it in our equations, we obtain:

$$\Pr(n,k) \approx 1-e^{-\frac{1}{n}}\cdot e^{-\frac{2}{n}}\cdots e^{-\frac{k-1}{n}} = 1-e^{-\frac{k(k-1)}{2n}}$$

# Birthday Paradox

- Say, we want $\Pr(n,k) > 0.5$. What $k$ is needed?

$$\frac{1}{2} = 1 - e^{-\frac{k(k-1)}{2n}} \;\Rightarrow\; e^{-\frac{k(k-1)}{2n}} = \frac{1}{2} \;\Rightarrow$$

$$-\frac{k(k-1)}{2n} = \ln(1/2) \;\Rightarrow\; \frac{k(k-1)}{2n} = \ln 2$$

- For large $k$, $k(k-1) \approx k^2$, thus we obtain:

$$\frac{k^2}{2n} \approx \ln 2 \;\Rightarrow\; k^2 \approx (\ln 2)2n \;\Rightarrow$$

$$k \approx \sqrt{(2\ln 2)n} = 1.18\sqrt{n} \approx \sqrt{n}$$

## Security of Hash Functions

- This directly applies to hash functions:
  - for a hash function that produces $n$-bit output, there are $2^n$ possible output values
  - but about $\sqrt{2^n} = 2^{n/2}$ tries are needed to find a collision with a good probability
- Choosing output length
  - to achieve 128-bit security, we need 256-bit output values
- As applied to hash functions, birthday paradox is used in Yuval's birthday attack

# Birthday Attack

- We have a legitimate message $m_1$ and a fraudulent message $m_2$
- We want to find $m_1'$ and $m_2'$ resulting from minor modifications of $m_1$ and $m_2$ with $h(m_1') = h(m_2')$
  - then a signature on the hash of $m_1'$ is a valid signature on $m_2'$'s hash
- Birthday attack:
  - find $n/2$ places to tweak $m_1$
  - generate $2^{n/2}$ minor modifications $m_1'$ of $m_1$
  - hash each modified message and store message-hash pairs (searchable by the hash value)
  - generate minor modifications $m_2'$ of $m_2$ computing $h(m_2')$ for each and checking for matches with any $m_1'$ above until a match is found

# Birthday Attack

- Example:
  - message $m_1$ and its $2^{14}$ modifications:

$$\left\{\begin{array}{c}\text{This letter is}\\ \text{I am writing}\end{array}\right\} \text{to introduce} \left\{\begin{array}{c}\text{you to}\\ \text{to you}\end{array}\right\} \left\{\begin{array}{c}\text{Mr.}\\ -\end{array}\right\} \text{Alfred} \left\{\begin{array}{c}\text{P.}\\ -\end{array}\right\} \text{Barton,}$$

$$\text{the} \left\{\begin{array}{c}\text{new}\\ \text{newly appointed}\end{array}\right\} \left\{\begin{array}{c}\text{chief}\\ \text{senior}\end{array}\right\} \text{jewelry buyer for} \left\{\begin{array}{c}\text{our}\\ \text{the}\end{array}\right\} \text{Northern}$$

$$\left\{\begin{array}{c}\text{European}\\ \text{Europe}\end{array}\right\} \left\{\begin{array}{c}\text{area}\\ \text{division}\end{array}\right\}\text{. He} \left\{\begin{array}{c}\text{will take}\\ \text{has taken}\end{array}\right\} \text{over} \left\{\begin{array}{c}\text{the}\\ -\end{array}\right\}$$

$$\text{responsibility for} \left\{\begin{array}{c}\text{all}\\ \text{the whole of}\end{array}\right\} \text{our interests in}$$

$$\left\{\begin{array}{c}\text{watches and jewellery}\\ \text{jewellery and watches}\end{array}\right\} \text{in the} \left\{\begin{array}{c}\text{area}\\ \text{region}\end{array}\right\}.$$

- No generic attacks of effort less than $2^n$ are known for other security properties (pre-image and second pre-image resistance)

# Random Oracle Model

- The Random Oracle Model (ROM) models an "ideal" hash function
- This ideal function is such that
  - the only efficient way to determine the value of $h(x)$ is to actually evaluate the function on $x$
  - the output is truly random and cannot be predicted even if other values $h(x')$, $h(x'')$, etc. are known
- Every time the ideal hash function is used, you consult an "oracle"
  - you send $x$ to the oracle and obtain $h(x)$ back
- This model was introduced by Bellare and Rogaway in 1993

# Random Oracle Model

- The rationale for using the random oracle model is
  - collision or preimage resistance of a hash function is not always sufficient to prove security
  - constructions that use hash functions can be more efficient than constructions without them
  - if we use an ideal function, we can prove construction with hash functions secure
- Is this model secure?
  - generally it is secure, but there are counterexamples
  - avoid this model if alternatives exist

# Hash Function Algorithms

- Families of customized hash functions
  - MD2, MD4, MD5 (MD = message digest)
    - a family of cryptographic hash functions designed by Ron Rivest
    - all have 128-bit output
    - MD2 was perceived as slower and less secure than MD4 and MD5
    - MD4 is specified as internet standard in RFC 1320
    - MD5 was designed as a strengthened version of MD4 before weaknesses in MD4 were found
    - MD5 is specified as internet standard RFC 1321
  - SHA-0, SHA-1
  - SHA-2 family

# Hash Function Algorithms

- MD4/MD5
  - for 128-bit hashes, collisions are expected in $2^{64}$ time
  - collisions have been found for MD4 in $2^{20}$ compression function computations (90s)
  - MD5 was widely used until relatively recently
  - attacks on MD5
    - Boer and Bosselaers found a pseudo collision (same message, two different IV's) in 1993
    - Dobbertin created collisions for MD5 compression function with a chosen IV in 1996
    - Wang et al. in 2004 found collisions for MD5 for any IV which are easy to find

# Hash Function Algorithms

- Secure Hash Algorithm (SHA)
  - SHA was designed by NIST and published in FIPS 180 in 1993
  - In 1995 a revision, known as SHA-1, was specified in FIPS 180-1
    - it is also specified in RFC 3174
  - SHA-0 and SHA-1 have 160 bit output and MD4-based design
  - In 2002 NIST produced a revision of the standard in FIPS 180-2
  - SHA-2 hash functions have length 256, 384, and 512 to be compatible with the increased security of AES
    - they are known as SHA-256, SHA-384, and SHA-512
  - Also, SHA-224 was added to compatibility with 3DES

# Hash Function Algorithms

- Comparison of SHA parameters

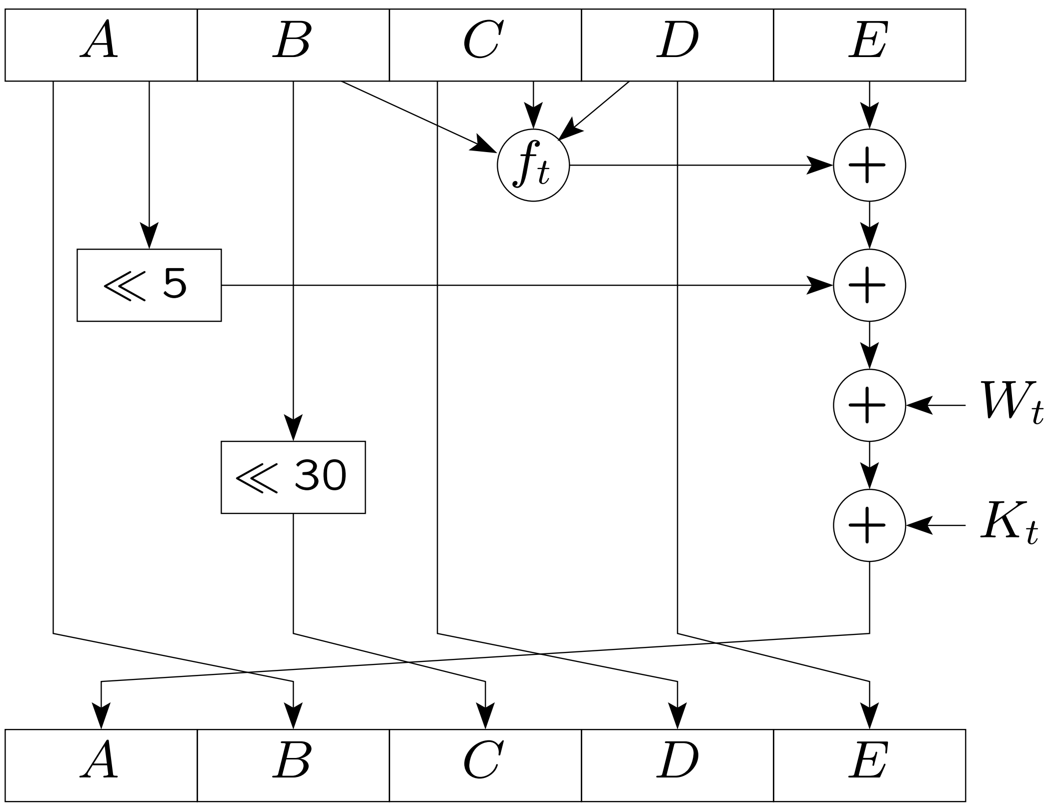
| | SHA-1 | SHA-256 | SHA-384 | SHA-512 |
|---|---|---|---|---|
| hash size | 160 | 256 | 384 | 512 |
| message size | $< 2^{64}$ | $< 2^{64}$ | $< 2^{128}$ | $< 2^{128}$ |
| block size | 512 | 512 | 1024 | 1024 |
| word size | 32 | 32 | 64 | 64 |
| number of steps | 80 | 64 | 80 | 80 |
| security (birthday attack) | 80 | 128 | 192 | 256 |

# Hash Function Algorithms

- SHA-1 algorithm
  - pad the input before processing
  - initialize the 5-word (160-bit) buffer with
    - $A = \texttt{67452301}$; $B = \texttt{EFCDAB89}$; $C = \texttt{98BADCFE}$
    - $D = \texttt{10325476}$; $E = \texttt{C3D2E1F0}$
  - message is processed in 16 32-bit words
    - expand 16 words into 80 words by XORing and shifting
    - use 4 rounds of 20 steps each on a message block and the buffer
  - the buffer is updated as ($t$ is the step number)
    $(A, B, C, D, E) =$
    $((E + f_t(B, C, D) + (A \lll 5) + W_t + K_t), A, (B \lll 30), C, D)$

# Hash Function Algorithms

- One step of SHA-1



# Hash Function Algorithms

- SHA-1 details
  - $t$ is the step number
  - $K_t$ is the a constant value derived from the sin function
  - $W_t$ is derived from the message block $m_i = W_0 W_1 \ldots W_{15}$ as
    - $W_t = W_t$ for $t = 0, \ldots, 15$
    - $W_t = (W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16}) \lll 1$ for $t = 16, \ldots, 79$
- The difference between SHA-0 and SHA-1 is that SHA-0 doesn't have 1-bit shift in the construction of $W_{16}, \ldots, W_{79}$

# Hash Function Algorithms

- Security of SHA
  - brute force attack is harder than in MD5 (160 bits vs. 128 bits)
  - SHA performs more complex transformations that MD5
    - it makes finding collisions more difficult
  - Joux and also Wang et al. found collisions in SHA-0 in 2004
    - collisions can be found in SHA-0 in $< 2^{40}$
  - in 2005 collisions have been found in 58-round "reduced" SHA-1 ($2^{33}$ work)
  - finding collisions in the full version of SHA-1 is estimated at $< 2^{69}$
  - several other results followed

# Hash Function Algorithms

- Search for SHA-3
  - Feb 2007: NIST announces requests for candidate algorithms for SHA-3 family
  - Oct 2008: 64 algorithms were received
  - Dec 2008: 51 first-round algorithms meeting minimum requirements were announced
  - Jul 2009: 14 second-round candidates were announced
  - Dec 2010: 5 finalists were selected
  - Oct 2012: the winner, Keccak, was announced
  - 2013: controversy about NIST-announced changes
  - Aug 2015: SHA-3 standard was released

# Hash Function Algorithms

- SHA-3 Requirements
  - digest sizes of 224, 256, 384, and 512 bits
  - support of maximum message length of at least $2^{64} - 1$ bits
  - must be implementable in a wide range of hardware and software platforms
  - other requirements
- Evaluation criteria (ordered)
  - security
  - cost and performance
  - algorithm and implementation characteristics

# SHA-3

- SHA-3 is specified in NIST's FIPS 202 standard
  - it is based on Keccak family of sponge functions
  - the sponge construction is a mode of operation that builds a function mapping variable-length input to variable-length output using a fixed-length permutation and a padding rule
  - Keccak instances call one of seven permutations with SHA-3 using the largest permutation Keccak-f[1600]
  - each permutation uses a round function with simple operations such as XOR, AND and NOT and rotations
  - the design is dictinct from other widely used techniques (SHA-2, AES, etc.)

# SHA-3

- In December 2016, NIST released Special Publication (SP) 800-185 with SHA-3 derived functions:
  - cSHAKE is a customizable variant of the SHAKE function used in Keccak and is a building block for all functions below
  - KMAC (= Keccak MAC) is a PRF and keyed hash function based on Keccak
    - it is faster than HMAC
  - TupleHash is a variable-length hash function designed to hash tuples of input strings without trivial collisions
  - ParallelHash is a variable-length hash function that can hash very long messages in parallel

# Summary

- Hash function design
  - iterated functions with chaining
  - Merkle-Damgard construction
- Attacks on hash functions
  - birthday attack applies to find collisions
  - finding preimage requires brute force search
- Customized hash functions
  - MD4/MD5
  - SHA-0, SHA-1, SHA-2
  - new SHA-3