

Things to know for Test 1

1. You should know the basic properties of the integers, including: the well-ordering principle, the division algorithm, Bézout's lemma, Euclid's lemma, and the fundamental theorem of arithmetic.

2. You should know how to compute things in modular arithmetic.

3. You should be able to determine whether a given set is closed under a given operation. You should know whether the operation is associative, or commutative. You should be able to produce examples of the aforementioned concepts.

4. You should know the definitions of inverse element and identity element.

5. You should know how to construct the Cayley table for a given group.

6. You should be able to determine when a group is Abelian.

7. You should be able to prove basic propositions about groups by working with their elements.

8. You should know the classification of subgroups of \mathbb{Z}/n , and cyclic groups in general.

9. You should know how to find the order of an element of a given group.

10. You should be able to prove or disprove that a given group is a subgroup of another group.

11. You should know the definitions and relevant facts about permutations and the symmetric group.

Things to know for Test 2

1. You should know how to prove various properties of group homomorphisms.
2. Given a subgroup H of a group G , you should know how to find all cosets of H in G .
3. You should know Lagrange's theorem, and how to use it.
4. You should know how to find all of the elements of a given order in a cyclic group.
5. By extension of 4, you should know how to find all of the elements of a given order in a direct sum of cyclic groups.
6. You should know how to recognize when a direct sum of cyclic groups is cyclic.
7. You should know how to prove or disprove that a given subgroup is normal or not.
8. You should know how to count the number of group homomorphisms between two cyclic groups.
9. You should know how to prove that two groups are non-isomorphic by exhibiting properties of one group that do not hold in the other.
10. You should know how to prove that a given ring is a subring of another ring.
11. You should be able to produce examples of zero divisors and units.
12. You should know how to prove properties of integral domains.

Things to know for Test 3

1. You should know how to prove that a given ring is an ideal.
2. You should know how to prove various properties of homomorphisms from \mathbb{Z}_m to \mathbb{Z}_n .
3. You should know how to prove various basic properties of polynomials over rings.
4. You should know how to use Eisenstein's criterion.
5. You should know how to reduce a polynomial modulo a prime in order to check for irreducibility.
6. You should be able to come up with multiple different factorizations for elements of rings that are not unique factorization domains.
7. You should know how to find the splitting field of a polynomial over \mathbb{Q} .
8. You should know how to find the degree of a simple extension, and you should know how to use the tower theorem.
9. Given a root of a polynomial that is irreducible over a given field, you should be able to find an extension field containing that root, and any other roots within said extension field.