

Ordered pairs

Given two mathematical objects a and b , we want to define an “ordered pair,” (a, b) , so that $(a, b) = (x, y)$ if and only if $a = x$ and $b = y$. We need to do this in terms of sets. During class today, Professor Sikora asked for a possible definition of the object (a, b) . There was one response to this question that was a rather good idea. At the time, neither Professor Sikora nor myself could see a problem with it. However, in this document, we will show that the proposed definition is **not** sufficient.

For clarity’s sake, we will first make a distinction between the proposal and the actual definition.

Definition 0.1 *Let a and b be mathematical objects. The Sikora bracket of a and b is the set $[a, b] = \{a, \{b\}\}$.*

This is the proposed definition of ordered pair. Now, consider the situation that $a = \{5\}$ and $b = 1$. We notice that

$$[\{5\}, 1] = \{\{5\}, \{1\}\} = \{\{1\}, \{5\}\} = [\{1\}, 5]. \quad (1)$$

This shows that the Sikora bracket is not an adequate definition of ordered pair, since this would mean that $\{5\} = \{1\}$ and $1 = 5$, which is absurd.

The moral of the story: mathematical concepts (such as definitions) are phenomenally delicate, and require great carefulness to construct. (And, sometimes some jerk will come along and tell you that your idea doesn’t work.)