Contents

1 Exam 01

1.1 Summary

Exam 01 is based on Chapters 1, 2, 4, 5, and 6.

In Chapter 1, we talked about how to solve systems of linear equations using Gauss-Jordan elimination. This technique is crucial for most of the computational problems in this course. Gaussian elimination is the process of finding a row echelon form of the matrix. Jordan elimination is the process of taking a row echelon form and finding the reduced row echelon form.

Chapter 2 focused on vectors in Euclidean space. We discussed addition and scalar multiplication of vectors, and linear combinations. In particular, our Quiz 01 was about determining whether a given vector was a linear combination of some other vectors. We also briefly mentioned length, and the dot product.

Chapter 4 was about matrices. We defined addition of matrices, scalar multiplication of matrices, and matrix products. We also defined the identity matrix, which serves as a multiplicative identity for matrices. This brought with it the topic of inverse matrices. We also defined the transpose of a matrix.

Chapter 5 was mainly about vector subspaces of $\mathbb{R}^n$. Every span of a sequence of vectors is a vector subspace, and conversely, every vector subspace is the span of some sequence of vectors. Now, among all the different sequences of vectors that could span a vector subspace, there are some that are "nice" in the sense of being linearly independent, while still corresponding to the entire vector subspace. These sequences are called bases of a vector subspace. We also introduced the columnspace and nullspace of a matrix, as examples of vector subspaces of $\mathbb{R}^n$.

Chapter 6 was about linear transformations of Euclidean spaces. These are functions between Euclidean spaces that "preserve" addition and scalar multiplication. We also mentioned that every linear transformation corresponds to a matrix, and we discussed how to find the matrix representation of a linear transformation.

1.2 Practice problems

Section 1.3: Exercises 1.3.1-4, 1.3.6

Section 1.4: Exercises 1.4.5-13, 1.4.16-25, 1.4.27, 1.4.28

Section 1.5: Exercises 1.5.1-6

Section 1.7: Exercise 1.7.2

Section 2.1: Exercises 2.1.2, 2.1.3

Section 2.4: Exercises 2.4.1-3

Section 2.6: Exercises 2.6.1, 2.6.7

Section 4.3: Exercise 4.3.2

Section 4.4: Exercises 4.4.1, 4.4.2, 4.4.4-7, 4.4.9-12, 4.4.14

Section 4.5: Exercises 4.5.1-8, 4.5.18

Section 4.7: Exercises 4.7.1-3

Section 5.1: Exercises 5.1.1-5

Section 5.2: Exercises 5.2.3-11

Section 5.3: Exercises 5.3.1, 5.3.2

Section 5.4: Exercises 5.4.1-5, 5.4.8, 5.4.14, 5.4.15

Section 5.5: Exercises 5.5.1, 5.5.2

Section 6.1: Exercises 6.1.1, 6.1.2, 6.1.5

Section 6.2: Exercise 6.2.5

1.3 Test content

The directions of each problem are given here:

1. [15] Describe the set of solutions of the following linear system.

[linear system]

2. [15] Express the vector

$$\overrightarrow{u} = [\text{ vector }]$$

as a linear combination of the vectors

$$\overrightarrow{v_1} = [\text{ vector }], \quad \overrightarrow{v_2} = [\text{ vector }], \quad \overrightarrow{v_3} = [\text{ vector }],$$

or show that no such expression is possible.

3. [15] Find the inverse of the following matrix, or show that the matrix is not invertible.

$$A = [\text{ matrix }]$$

4. [15] Find a basis for the nullspace of the following matrix.

$$A = [\text{ matrix }]$$

5. [20] For each of the following sets, write "Y" if the set is a vector subspace of [some Euclidean space] and "N" if it is not. (It is not necessary to show your work or reasoning for this problem.)

[Four different sets]

6. [20] For each of the following functions, write "Y" if the function is a linear transformation and "N" if it is not. (It is not necessary to show your work or reasoning for this problem.)

[Four different functions]

2 Exam 02

2.1 Summary

Exam 02 is based on Chapters 9, 10, 7, 8, and 11.

Chapter 9 was about extending the definitions and concepts that we learned about Euclidean spaces to more general settings. We defined the term “vector space” in a way that was general enough to include many interesting sets, such as vector spaces of polynomials, matrices, and general functions.

In Chapter 10, we discussed the properties of linear transformations in the context of general vector spaces. Particular emphasis was placed on finding matrix representations of linear transformations with respect to given bases on the domain and codomain. This led us back to the context of matrices over the real numbers.

Chapter 7 was about determinants. With some amount of effort, we defined the determinant of a general square matrix in a computationally-minded way. We also discussed some shortcuts that assist in computing the determinant of a square matrix. The main reason that we concern ourselves with determinants is that it gives a straightforward criterion for figuring out whether a square matrix is invertible; a square matrix is invertible if and only if its determinant is nonzero.

Once we knew how to compute the determinant of a square matrix, we were then able to move into Chapter 8, which used these results to find eigenvalues. Eigenvalues and eigenvectors serve many important purposes in applied math, but we were mostly interested in them because they are crucial to the notion of diagonalization. Diagonalization allows for simplified computations involving matrices and exponents.

In Chapter 11, we returned to the business of generalizing concepts from Euclidean space; in this chapter, we described the dot product as an operation known as a “real inner product.” This allows us to work with notions of magnitude and orthogonality in abstract vector spaces. We also returned to the process of diagonalization, introducing the more specific concept of “orthogonal diagonalization.”

2.2 Practice problems

Section 9.1: Exercises 9.1.1-4, 9.17

Section 9.2: Exercises 9.2.2-4, 9.26, 9.2.7

Section 9.3: Exercises 9.3.2-4, 9.3.8, 9.3.9, 9.3.11, 9.3.16

Section 9.4: Exercises 9.4.1-4, 9.4.8, 9.4.9

Section 10.1: Exercises 10.1.1, 10.1.2, 10.1.4, 10.1.5,

Section 10.3: Exercise 10.3.1

Section 10.4: Exercises 10.4.1-6,

Section 7.1: Exercises 7.1.1, 7.1.2

Section 7.2: Exercises 7.2.1-7

Section 7.3: Exercise 7.3.1

Section 7.4: Exercise 7.4.1

Section 7.5: Exercises 7.5.1, 7.5.3, 7.5.6, 7.5.7, 7.5.10, 7.5.13

Section 8.1: Exercises 8.1.1-5

Section 8.2: Exercises 8.2.1-7

Section 8.4: Exercises 8.4.5, 8.4.6

Section 8.9: Exercises 8.9.1, 8.9.2

Section 11.1: Exercises 11.1.2, 11.1.3

Section 11.2: Exercises 11.2.1, 11.2.2, 11.2.4, 11.2.5

Section 11.3: Exercises 11.3.1, 11.3.2,

Section 11.7: Exercise 11.7.1

2.3 Test content

The directions of each problem are given here:

1. [15] Determine whether the following sequence of vectors in [vector space] is linearly independent.

[sequence of vectors]

2. [15] Find the matrix representation of the linear transformation

[linear transformation]

with respect to the bases

$$B = [\text{ basis for domain }]$$
$$C = [\text{ basis for codomain }]$$

on [domain] and [codomain], respectively.

3. [15] Compute the determinant of the following matrix.

$$A = [\ 5 \times 5 \text{ matrix }]$$

4. [15] Determine whether the following matrix is diagonalizable.

$$A = [\text{ matrix }]$$

5. [20] Find an orthogonal basis for the following vector subspace of $\mathbb{R}^3$.

$$V = \operatorname{span}\left([\text{ two vectors in } \mathbb{R}^3\ ]\right)$$

6. [20] Compute an orthogonal diagonalization of the following matrix.

$$A = [\text{ symmetric matrix }]$$

(To clarify, you are being asked to produce an orthogonal matrix P and a diagonal matrix D such that $P^{-1}AP = D$.)