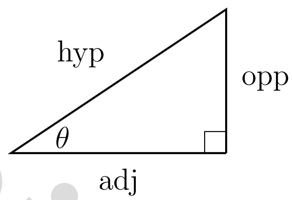
MTH 142 Final Exam Reference Page 1 of 2

University at Buffalo

Right Triangle Trigonometry



1) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

2) $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

3) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

4) $\csc\theta = \dfrac{1}{\sin\theta}$

5) $\sec\theta = \dfrac{1}{\cos\theta}$

6) $\cot\theta = \dfrac{1}{\tan\theta}$

Pythagorean Trig Identities

1) $\sin^2\theta + \cos^2\theta = 1$

2) $\tan^2\theta + 1 = \sec^2\theta$

3) $1 + \cot^2\theta = \csc^2\theta$

Half-Angle Formulas

1) $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$

2) $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$

Double-Angle Formula

1) $\sin 2\theta = 2\sin\theta\cos\theta$

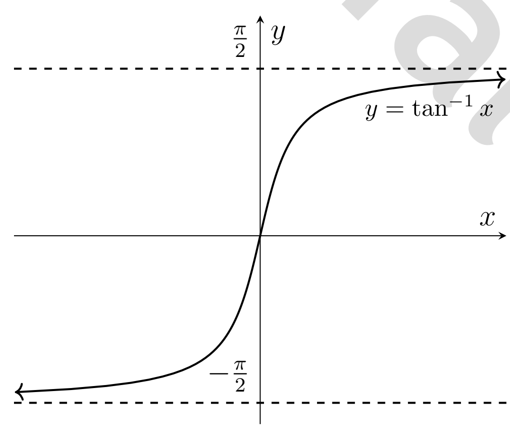
Expression	**Substitution**
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,\ -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,\ -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,\ 0 \le \theta < \dfrac{\pi}{2}$ or $\pi \le \theta < \dfrac{3\pi}{2}$

Cartesian coordinates (x, y) and Polar coordinates (r, θ)

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$x^2 + y^2 = r^2, \quad \tan\theta = \frac{y}{x}$$

Plot of $y = \tan^{-1} x$



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Geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \dots$$

If $|r| < 1$, the series is convergent. If $|r| \geq 1$, the series is divergent.

The Test For Divergence

If $\lim_{n\to\infty} a_n$ does not exist or $\lim_{n\to\infty} a_n \neq 0$ then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

The Integral Test

Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent.

The Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms.

(i) If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent.

(ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.

The Limit Comparison Test

Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = c$$

where c is a finite number and $c > 0$, then either both series converge or both series diverge.

The Alternating Series Test

If the alternating series

$$\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots \quad (b_n > 0)$$

satisfies

(i) $b_{n+1} \leq b_n$ for all n

(ii) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

The Ratio Test

(i) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(ii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, the Ratio Test is inconclusive.

The Root Test

(i) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, the Root Test is inconclusive.

Taylor series of the function f at a

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \qquad |x-a| < R$$