Math 142V Quiz 11

Name: Solution Key

December 8th, 2020

You must show all of your work and reasoning to receive full credit.

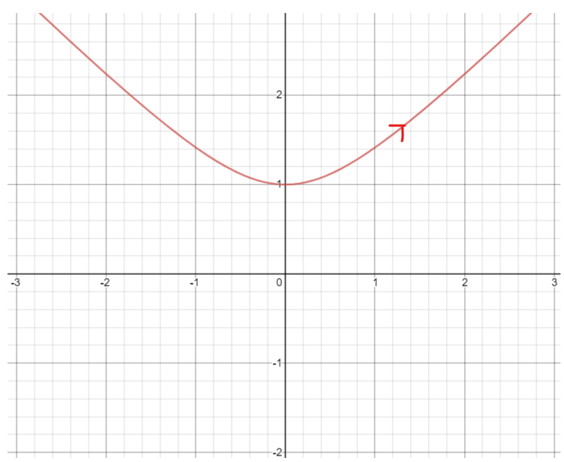
1. [10] Eliminate the parameter to find a Cartesian equation for the curve, and sketch the curve, indicating with an arrow the direction in which the curve is traced as the parameter increases.

$$\begin{matrix} x(t) = \tan t \\ y(t) = \sec t \end{matrix}, \quad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}$$

Solution: We note that $\sec^2 t - \tan^2 t = 1$, and so the curve satisfies the Cartesian equation

$$\boxed{y^2 - x^2 = 1}. \tag{1}$$

This is a hyperbola with the y-axis as its transverse axis:



(The graph should not contain the lower branch of this hyperbola, since $y(t) > 0$ for all t satisfying $-\frac{\pi}{2} < t < \frac{\pi}{2}$.) □

2. [10] Find an equation of the tangent line to the curve at the given point.

$$\begin{matrix} x\,(t) = t^2 \\ y\,(t) = t^3 \end{matrix}, \quad (1, -1)$$

Solution: First, we find the derivatives with respect to t:

$$\tfrac{\mathrm{d}x}{\mathrm{d}t} = 2t \quad \tfrac{\mathrm{d}y}{\mathrm{d}t} = 3t^2. \tag{2}$$

Therefore, if $t \neq 0$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3t^2}{2t} = \frac{3}{2}t. \tag{3}$$

From here, we need to determine which value of t corresponds to the point $(1, -1)$. To do this, we need only solve the following system of equations:

$$\begin{matrix} 1 = x(t) = t^2 \\ -1 = y(t) = t^3 \end{matrix}. \tag{4}$$

The only solution is $t = -1$. Therefore, the slope of the tangent line is

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=-1} = \frac{3}{2}(-1) = -\frac{3}{2}. \tag{5}$$

Using the fact that $(1, -1)$ is a point on the tangent line, we can write the point-slope form of the equation of the tangent line as

$$\boxed{y + 1 = -\tfrac{3}{2}(x - 1)}. \tag{6}$$

□

Math 142V Quiz 11

Name: Solution Key

December 9th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

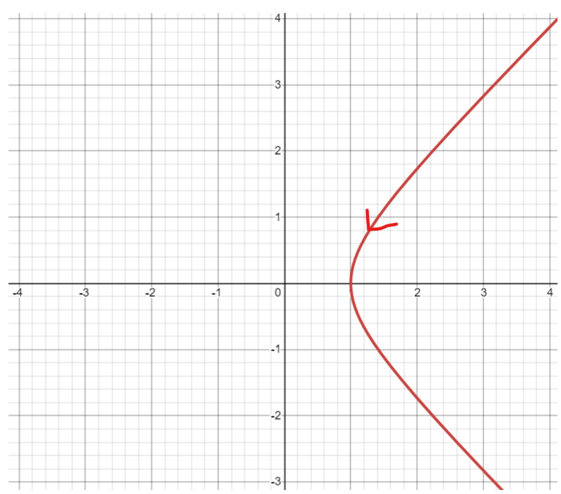
1. [10] Eliminate the parameter to find a Cartesian equation for the curve, and sketch the curve, indicating with an arrow the direction in which the curve is traced as the parameter increases.

$$\begin{matrix} x(t) = \csc t \\ y(t) = \cot t \end{matrix}, \quad 0 < t < \pi$$

Solution: We note that $\csc^2 t - \cot^2 t = 1$, and so the curve satisfies the Cartesian equation

$$\boxed{x^2 - y^2 = 1}. \tag{7}$$

This is a hyperbola with the x-axis as its transverse axis:



(The graph should not contain the left branch of this hyperbola, since $x(t) > 0$ for all t satisfying $0 < t < \pi$.) □

2. [10] Find an equation of the tangent line to the curve at the given point.

$$\begin{array}{l} x(t) = t^4 \\ y(t) = t^3 \end{array}, \quad (1, -1)$$

Solution: First, we find the derivatives with respect to t:

$$\tfrac{\mathrm{d}x}{\mathrm{d}t} = 4t^3 \quad \tfrac{\mathrm{d}y}{\mathrm{d}t} = 3t^2. \tag{8}$$

Therefore, if $t \neq 0$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3t^2}{4t^3} = \frac{3}{4t}. \tag{9}$$

From here, we need to determine which value of t corresponds to the point $(1, -1)$. To do this, we need only solve the following system of equations:

$$\begin{array}{r} 1 = x(t) = t^4 \\ -1 = y(t) = t^3 \end{array}. \tag{10}$$

The only solution is $t = -1$. Therefore, the slope of the tangent line is

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=-1} = \frac{3}{4(-1)} = -\frac{3}{4} \tag{11}$$

Using the fact that $(1, -1)$ is a point on the tangent line, we can write the point-slope form of the equation of the tangent line as

$$\boxed{y + 1 = -\tfrac{3}{4}(x - 1)}. \tag{12}$$

Math 142V Quiz 11

Name: Solution Key

December 14th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

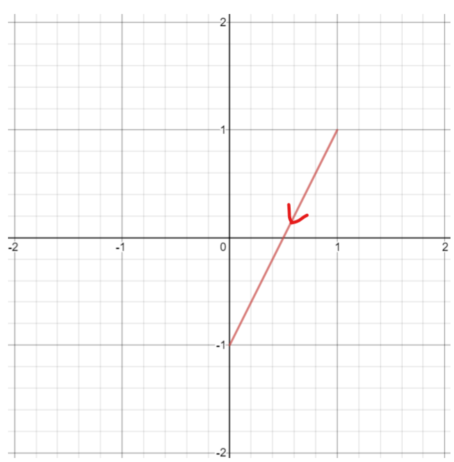
1. [10] Eliminate the parameter to find a Cartesian equation for the curve, and sketch the curve, indicating with an arrow the direction in which the curve is traced as the parameter increases.

$$\begin{matrix} x(t) = \cos^2 t \\ y(t) = \cos(2t) \end{matrix}, \quad 0 \leq t \leq \frac{\pi}{2}$$

Solution: We note that $\cos^2 t = \frac{1}{2}(1 + \cos(2t))$, and so the curve satisfies the Cartesian equation

$$\boxed{x = \frac{1}{2}(1 + y)}. \tag{13}$$

In other words, $y = 2x - 1$, which gives a line:



(For t satisfying $0 \leq t \leq \frac{\pi}{2}$, $0 \leq \cos^2 t \leq 1$, and so the graph should only contain the portion of the line with x-coordinates between 0 and 1.) □

2. [10] Find an equation of the tangent line to the curve at the given point.

$$\begin{array}{c} x\left(t\right)=t^2+1 \\ y\left(t\right)=t^3 \end{array}, \quad (2,-1)$$

Solution: First, we find the derivatives with respect to t:

$$\tfrac{\mathrm{d}x}{\mathrm{d}t}=2t \quad \tfrac{\mathrm{d}y}{\mathrm{d}t}=3t^2. \tag{14}$$

Therefore, if $t \neq 0$, then

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3t^2}{2t}=\frac{3}{2}t. \tag{15}$$

From here, we need to determine which value of t corresponds to the point $(2,-1)$. To do this, we need only solve the following system of equations:

$$\begin{array}{c} 2=x(t)=t^2+1 \\ -1=y(t)=t^3 \end{array}. \tag{16}$$

The only solution is $t=-1$. Therefore, the slope of the tangent line is

$$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=-1}=\frac{3}{2}\left(-1\right)=-\frac{3}{2}. \tag{17}$$

Using the fact that $(2,-1)$ is a point on the tangent line, we can write the point-slope form of the equation of the tangent line as

$$\boxed{y+1=-\tfrac{3}{2}\left(x-2\right)}. \tag{18}$$