Math 142V Quiz 9

Name: Solution Key

November 12th, 2020

You must show all of your work and reasoning to receive full credit.

1. [10] Find a power series representation for the function and determine the radius of convergence.

$$f(x) = \frac{x}{2x^2+1}$$

Solution: We begin by re-writing the function as

$$f(x) = x\left(\frac{1}{2x^2+1}\right) = x\left(\frac{1}{1-(-2x^2)}\right). \tag{1}$$

Based on the characterization theorem for geometric series, if $|-2x^2| < 1$, then this becomes

$$f(x) = x\sum_{n=0}^{\infty}\left(-2x^2\right)^n = \boxed{\sum_{n=0}^{\infty}(-2)^n x^{2n+1}}. \tag{2}$$

The inequality $|-2x^2| < 1$ can be re-written as $x^2 < \frac{1}{2}$, or in other words, $|x| < \frac{1}{\sqrt{2}}$. Thus, the radius of convergence is $\boxed{\frac{1}{\sqrt{2}}}$. $\square$

2. [10] Find a power series representation for the function and determine the radius of convergence.

$$f(x) = \ln(1-x)$$

Solution: We note that

$$f'(x) = \frac{-1}{1-x}. \tag{3}$$

Based on the characterization theorem for geometric series, if $|x| < 1$, then this becomes

$$f'(x) = -\sum_{n=0}^{\infty} x^n. \tag{4}$$

Thus, the derivative has this power series representation with radius of convergence 1. Now we need only take the indefinite integral of both sides:

$$f(x) = \int -\sum_{n=0}^{\infty} x^n \, \mathrm{d}x = -\sum_{n=0}^{\infty} \int x^n \, \mathrm{d}x = -\left(\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}\right) + C. \tag{5}$$

In order to find the value of C, we take $f(0)$:

$$0 = \ln(1-0) = f(0) = -\left(\sum_{n=0}^{\infty} \frac{0^{n+1}}{n+1}\right) + C = 0 + C = C. \tag{6}$$

This reveals that

$$f(x) = \boxed{-\sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}}. \tag{7}$$

As this is a power series, the radius of convergence is the same as the radius of convergence of the derivative: $\boxed{1}$. □