Math 142V Quiz 8 **Name:**

November 3rd, 2020

You must show all of your work and reasoning to receive full credit.

1. [10] Test the series for convergence or divergence.

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt[100]{\ln n}}$$

Solution: We note that, for $n > e$,

$$\frac{1}{\sqrt[100]{\ln n}} > \frac{1}{\ln n} > \frac{1}{n}. \tag{1}$$

Since $\sum_{n=2}^{\infty} \frac{1}{n}$ is one term less than the harmonic series (which is a p-series), it is divergent. By the comparison test, this indicates that $\sum_{n=2}^{\infty} \frac{1}{\sqrt[100]{\ln n}}$ is also $\boxed{\text{divergent}}$.

2. [10] Determine the radius of convergence and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{2^n}$.

Solution: We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{(n+1)\,(x+3)^{n+1}}{2^{n+1}} \frac{2^n}{n(x+3)^n} \right| = \lim_{n\to\infty} \frac{n+1}{2n}|x+3| = \frac{1}{2}|x+3|. \quad (2)$$

In order for the ratio test to guarantee absolute convergence, this must be less than 1: $\frac{1}{2}|x+3| < 1$. In other words, convergence is guaranteed for $|x+3| < 2$ and divergence is guaranteed for $|x+3| > 2$, so the radius of convergence is $\boxed{2}$.

The center of convergence is located at $a = -3$, so convergence is guaranteed if $-5 < x < -1$. We now check the endpoints of the interval. If $x = -5$:

$$\sum_{n=0}^{\infty} \frac{n(-5+3)^n}{2^n} = \sum_{n=0}^{\infty} (-1)^n n. \quad (3)$$

The limit $\lim_{n\to\infty} (-1)^n n$ does not exist, so the test for divergence indicates that divergence occurs at $x = -5$. As for $x = -1$:

$$\sum_{n=0}^{\infty} \frac{n(-1+3)^n}{2^n} = \sum_{n=0}^{\infty} n. \quad (4)$$

The limit $\lim_{n\to\infty} n = \infty$, so the test for divergence indicates that divergence occurs at $x = -1$. Therefore, the interval of convergence is $\boxed{(-5, -1)}$.

Math 142V Quiz 8 **Name:**

November 5th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

1. [10] Test the series for convergence or divergence.

$$\sum_{n=2}^{\infty} \frac{1}{n \sin^2 n}$$

Solution: Since $0 < \sin^2 n \leq 1$ for all integer n,

$$\frac{1}{n \sin^2 n} > \frac{1}{n}. \tag{5}$$

The series $\sum_{n=2}^{\infty} \frac{1}{n}$ is one term less than the harmonic series (which is a p-series), and so is divergent. By the comparison test, this indicates that $\sum_{n=2}^{\infty} \frac{1}{n \sin^2 n}$ is also $\boxed{\text{divergent}}$.

2. [10] Determine the radius of convergence and the interval of convergence of the power series $\sum\limits_{n=0}^{\infty} \frac{(x+100)^n}{5^n(n+1)}$.

Solution: We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{(x+100)^{n+1}}{5^{n+1}(n+2)} \frac{5^n(n+1)}{(x+100)^n} \right| = \lim_{n\to\infty} \frac{n+1}{n+2}\frac{1}{5}|x+100| = \frac{1}{5}|x+100|. \tag{6}$$

The ratio test guarantees (absolute) convergence if $\frac{1}{5}|x+100| < 1$ and divergence if $\frac{1}{5}|x+100| > 1$. Equivalently, the ratio test guarantees convergence if $|x+100| < 5$ and divergence if $|x+100| > 5$, so the radius of convergence is $\boxed{5}$.

The center of convergence is located at $a = -100$, so the ratio test guarantees absolute convergence if $-105 < x < -95$. We now check the endpoints of the interval. If $x = -105$:

$$\sum_{n=0}^{\infty} \frac{(-105+100)^n}{5^n(n+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}. \tag{7}$$

The alternating series test indicates that this series is convergent, and so convergence occurs at $x = -105$. As for $x = -95$:

$$\sum_{n=0}^{\infty} \frac{(-95+100)^n}{5^n(n+1)} = \sum_{n=0}^{\infty} \frac{1}{n+1}. \tag{8}$$

We do a limit comparison between this and the harmonic series:

$$\lim_{n\to\infty} \frac{\left(\frac{1}{n+1}\right)}{\left(\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{n}{n+1} = 1. \tag{9}$$

As this is positive and finite, the limit comparison test indicates that $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ and $\sum\limits_{n=1}^{\infty} \frac{1}{n+1}$ have the same behavior. Since the harmonic series is divergent (by the p-series test), $\sum\limits_{n=0}^{\infty} \frac{1}{n+1}$ is also divergent, and so divergence occurs at $x = -95$. Therefore, the interval of convergence is $\boxed{[-105, -95)}$.