Math 141V Quiz 7 **Name:**

October 27th, 2020

You must show all of your work and reasoning to receive full credit.

1. [10] Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$$

Solution: We note that this is an alternating series, since $\sin\left(\frac{\pi}{n}\right) \geq 0$ for all $n \geq 1$. We define $b_n = \sin\left(\frac{\pi}{n}\right)$, the sequence of absolute values of the terms. This sequence is **eventually** decreasing, since for $n > 1$, $\sin\left(\frac{\pi}{n+1}\right) < \sin\left(\frac{\pi}{n}\right)$. Further,

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \sin\left(\frac{\pi}{n}\right) = \sin 0 = 0. \qquad (1)$$

Therefore, by the alternating series test, the series is $\boxed{\text{convergent}}$.

2. [10] Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{n5^{2n}}{10^{n+1}}$$

Solution: We use the ratio test:

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)\,5^{2(n+1)}}{10^{n+2}}\frac{10^{n+1}}{n5^{2n}}\right| = \lim_{n\to\infty}\frac{n+1}{n}\frac{5^2}{10} = \frac{5}{2}. \tag{2}$$

Since this limit is greater than 1, the ratio test indicates that the series is $\boxed{\text{divergent}}$.

Math 141V Quiz 7 **Name:**

October 29th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

1. [10] Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$$

Solution: We note that

$$\lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right) = \cos 0 = 1. \tag{3}$$

Therefore, the limit $\lim_{n\to\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$ does not exist. By the test for divergence, this indicates that the series is $\boxed{\text{divergent}}$.

2. [10] Use any test to determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{5^{2n}}{n10^{n+1}}$$

Solution: We use the ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{5^{2(n+1)}}{(n+1)\,10^{n+2}} \frac{n10^{n+1}}{5^{2n}} \right| = \lim_{n\to\infty} \frac{n}{n+1} \frac{5^2}{10} = \frac{5}{2}. \tag{4}$$

As this is greater than 1, the ratio test indicates that the series is $\boxed{\text{divergent}}$.