Math 141V Quiz 5 **Name:**

October 13th, 2020

You must show all of your work and reasoning to receive full credit.

1. [10] Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{9}{10^n}$$

Solution: This is a geometric series:

$$\sum_{n=1}^{\infty} \frac{9}{10^n} = \sum_{n=1}^{\infty} \frac{9}{10}\left(\frac{1}{10}\right)^{n-1}. \tag{1}$$

Since $\left|\frac{1}{10}\right| < 1$, the characterization theorem for geometric series indicates that

$$\sum_{n=1}^{\infty} \frac{9}{10}\left(\frac{1}{10}\right)^{n-1} = \frac{\left(\frac{9}{10}\right)}{1-\frac{1}{10}} = \boxed{1}. \tag{2}$$

2. [10] Write the first five terms of the series, and then determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{1}{2}\right)^n}.$$

Solution: The first five terms are as follows:

$$\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{1}{2}\right)^n} = \frac{2}{3} + \frac{4}{5} + \frac{8}{9} + \frac{16}{17} + \frac{32}{33} + ... \tag{3}$$

Concerning convergence, we note that

$$\lim_{n\to\infty} \frac{1}{1+\left(\frac{1}{2}\right)^n} = \frac{1}{1+0} = 1 \neq 0. \tag{4}$$

By the test for divergence, this indicates that the series is $\boxed{\text{divergent}}$.

Math 141V Quiz 5 **Name:**

October 15th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

1. [10] Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{7}{10^n}$$

Solution: This is a geometric series:

$$\sum_{n=1}^{\infty} \frac{7}{10^n} = \sum_{n=1}^{\infty} \frac{7}{10}\left(\frac{1}{10}\right)^{n-1}. \tag{5}$$

Since $\left|\frac{1}{10}\right| < 1$, the characterization theorem for geometric series indicates that

$$\sum_{n=1}^{\infty} \frac{7}{10}\left(\frac{1}{10}\right)^{n-1} = \frac{\left(\frac{7}{10}\right)}{1-\frac{1}{10}} = \boxed{\frac{7}{9}}. \tag{6}$$

2. [10] Write the first five terms of the series, and then determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{1}{\pi}\right)^n}.$$

Solution: The first five terms are as follows:

$$\sum_{n=1}^{\infty} \frac{1}{1+\left(\frac{1}{\pi}\right)^n} = \frac{\pi}{\pi+1} + \frac{\pi^2}{\pi^2+1} + \frac{\pi^3}{\pi^3+1} + \frac{\pi^4}{\pi^4+1} + \frac{\pi^5}{\pi^5+1} + \ldots \tag{7}$$

Concerning convergence, we note that

$$\lim_{n\to\infty} \frac{1}{1+\left(\frac{1}{\pi}\right)^n} = \frac{1}{1+0} = 1 \neq 0. \tag{8}$$

By the test for divergence, this indicates that the series is $\boxed{\text{divergent}}$.