Math 141V Quiz 3 **Name: Solution Key**

September 22nd, 2020

You must show all of your work and reasoning to receive full credit.

1. [10] Evaluate the indefinite integral.

$$\int \frac{5x^2 + x + 3}{x^3 + x}\,\mathrm{d}x$$

Solution: This is most easily done by integration by parts:

$$\frac{5x^2 + x + 3}{x\left(x^2 + 1\right)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1} \tag{1}$$

$$5x^2 + x + 3 = A\left(x^2 + 1\right) + \left(Bx + C\right)x \tag{2}$$

$$5x^2 + x + 3 = Ax^2 + A + Bx^2 + Cx \tag{3}$$

$$5x^2 + x + 3 = \left(A + B\right)x^2 + Cx + A. \tag{4}$$

This gives the simultaneous equations

$$\begin{aligned} A + B &= 5 \\ C &= 1 \\ A &= 3 \end{aligned} \quad . \tag{5}$$

The integral now becomes

$$\begin{aligned} \int \frac{3}{x} + \frac{2x + 1}{x^2 + 1}\,\mathrm{d}x &= \int \frac{3}{x}\,\mathrm{d}x + \int \frac{2x}{x^2 + 1}\,\mathrm{d}x + \int \frac{1}{x^2 + 1}\,\mathrm{d}x \\ &= \boxed{3\ln|x| + \ln\left(x^2 + 1\right) + \tan^{-1}x + C}. \end{aligned} \tag{6}$$

2. [10] Evaluate the definite integral.

$$\int_0^1 e^{\sqrt{x}}\,\mathrm{d}x$$

Solution: We begin with u-substitution:

$$\begin{aligned} u &= \sqrt{x} \qquad & x = 0 \Rightarrow u = 0 \\ \mathrm{d}u &= \tfrac{1}{2\sqrt{x}}\,\mathrm{d}x \qquad & x = 1 \Rightarrow u = 1 \end{aligned} \tag{7}$$

The integral now becomes

$$\int_0^1 2ue^u\,\mathrm{d}u \tag{8}$$

This can be done by integration by parts (as u is already being used, we will use w and $\mathrm{d}v$):

$$\begin{aligned} w &= u \qquad & \mathrm{d}v = e^u\,\mathrm{d}u \\ \mathrm{d}w &= \mathrm{d}u \qquad & v = e^u \end{aligned} \tag{9}$$

$$2\left(ue^u\Big|_0^1 - \int_0^1 e^u\,\mathrm{d}u\right) = 2\left(ue^u - e^u\Big|_0^1\right) = 2\,(e-e) - 2\,(0-1) = \boxed{2}. \tag{10}$$

Math 141V Quiz 3

Name: Solution Key

September 24nd, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

1. [10] Evaluate the indefinite integral.

$$\int \frac{2x^2 - 2x + 1}{x^3 - 2x^2 + x}\, \mathrm{d}x$$

Solution: We begin with a partial fraction decomposition:

$$\frac{2x^2 - 2x + 1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \tag{11}$$

$$2x^2 - 2x + 1 = A(x-1)^2 + Bx\,(x-1) + Cx \tag{12}$$

$$2x^2 - 2x + 1 = A\left(x^2 - 2x + 1\right) + B\left(x^2 - x\right) + Cx \tag{13}$$

$$2x^2 - 2x + 1 = (A + B)\,x^2 + (-2A - B + C)\,x + A \tag{14}$$

$$\begin{aligned} A + B &= 2 \\ -2A - B + C &= -2 \\ A &= 1 \end{aligned} \tag{15}$$

This indicates that $A = 1$, $B = 1$, and $C = 1$. Our integral now becomes

$$\begin{aligned} &\int \frac{1}{x} + \frac{1}{x-1} + \frac{1}{(x-1)^2}\, \mathrm{d}x \\ &\qquad = \int \frac{1}{x}\, \mathrm{d}x + \int \frac{1}{x-1}\, \mathrm{d}x + \int (x-1)^{-2}\, \mathrm{d}x \\ &\qquad = \boxed{\ln|x| + \ln|x-1| - \tfrac{1}{x-1} + C}. \end{aligned} \tag{16}$$

2. [10] Evaluate the definite integral.

$$\int_1^4 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$$

We begin with a u-substitution:

$$\begin{array}{ll} u = 1+\sqrt{x} & x = 1 \Rightarrow u = 2 \\ \mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x & x = 4 \Rightarrow u = 3 \end{array}. \tag{17}$$

The integral now becomes

$$\begin{aligned} \int_2^3 \frac{1}{u} 2\left(u-1\right)\,\mathrm{d}u &= 2\int_2^3 1 - \frac{1}{u}\,\mathrm{d}u \\ &= 2\left(u - \ln|u| \Big|_2^3\right) = \boxed{2\left(1+\ln 2 - \ln 3\right)}. \end{aligned} \tag{18}$$