Math 141V Quiz 1

Name: Solution Key

September 8th, 2020

You must show all of your work and reasoning to receive full credit.

1. [6] Evaluate the indefinite integral.

$$\int \frac{x^9}{x^{10} - 144}\, \mathrm{d}x$$

Solution:

$$\begin{aligned} u &= x^{10} - 144 \\ \mathrm{d}u &= 10x^9\, \mathrm{d}x \end{aligned} \tag{1}$$

$$\frac{1}{10}\int \frac{1}{u}\, \mathrm{d}u = \frac{1}{10} \ln|u| + C = \boxed{\tfrac{1}{10} \ln\left|x^{10} - 144\right| + C} \tag{2}$$

2. [7] Evaluate the definite integral.

$$\int_1^e \ln x\, \mathrm{d}x$$

Solution:

$$\begin{aligned} u &= \ln x \qquad \mathrm{d}v = \mathrm{d}x \\ \mathrm{d}u &= \tfrac{1}{x}\, \mathrm{d}x \qquad v = x \end{aligned} \tag{3}$$

$$x \ln x \Big|_1^e - \int_1^e 1\, \mathrm{d}x = x \ln x \Big|_1^e - x \Big|_1^e = (e - 0) - (e - 1) = \boxed{1} \tag{4}$$

3. [7] Evaluate the indefinite integral.

$$\int \left(\pi x - \frac{\pi}{4}\right) \cos\left(\frac{\pi}{6}x\right)\, \mathrm{d}x$$

Solution:

$$\begin{array}{ll} u = \pi x - \frac{\pi}{4} & \mathrm{d}v = \cos\left(\frac{\pi}{6}x\right)\, \mathrm{d}x \\ \mathrm{d}u = \pi\, \mathrm{d}x & v = \frac{6}{\pi}\sin\left(\frac{\pi}{6}x\right) \end{array} \tag{5}$$

$$\left(\pi x - \frac{\pi}{4}\right)\frac{6}{\pi}\sin\left(\frac{\pi}{6}x\right) - \int 6\sin\left(\frac{\pi}{6}x\right)\, \mathrm{d}x \tag{6}$$

$$= \boxed{6\left(x - \tfrac{1}{4}\right)\sin\left(\tfrac{\pi}{6}x\right) + \tfrac{36}{\pi}\cos\left(\tfrac{\pi}{6}x\right) + C} \tag{7}$$

Math 141V Quiz 1 **Name:**

September 11th, 2020

This is a make-up quiz. If you have not received my permission to take this make-up quiz, then your submission will not be accepted.

You must show all of your work and reasoning to receive full credit.

1. [6] Evaluate the indefinite integral.

$$\int \frac{x^{19}}{x^{20}-64}\,\mathrm{d}x$$

Solution:

$$\begin{aligned} u &= x^{20}-64 \\ \mathrm{d}u &= 20x^{19}\,\mathrm{d}x \end{aligned} \tag{8}$$

$$\frac{1}{20}\int \frac{1}{u}\,\mathrm{d}u = \frac{1}{20}\ln|u| + C = \boxed{\tfrac{1}{20}\ln\left|x^{20}-64\right| + C} \tag{9}$$

2. [7] Evaluate the definite integral.

$$\int_0^1 \tan^{-1}x\,\mathrm{d}x$$

Solution:

$$\begin{aligned} u &= \tan^{-1}x \quad & \mathrm{d}v &= \mathrm{d}x \\ \mathrm{d}u &= \tfrac{1}{1+x^2}\,\mathrm{d}x \quad & v &= x \end{aligned} \tag{10}$$

$$x\tan^{-1}x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x = \left(\frac{\pi}{4}-0\right) - \int_0^1 \frac{x}{1+x^2}\,\mathrm{d}x \tag{11}$$

$$\begin{aligned} u &= 1+x^2 \quad & x=0 &\Rightarrow u=1 \\ \mathrm{d}u &= 2x\,\mathrm{d}x \quad & x=1 &\Rightarrow u=2 \end{aligned} \tag{12}$$

$$\frac{\pi}{4} - \frac{1}{2}\int_1^2 \frac{1}{u}\,\mathrm{d}u = \frac{\pi}{4} - \frac{1}{2}\ln|u|\Big|_1^2 = \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1) = \boxed{\tfrac{\pi}{4} - \tfrac{1}{2}\ln 2} \tag{13}$$

3. [7] Evaluate the indefinite integral.

$$\int (x+1)\, e^{9x}\, \mathrm{d}x$$

Solution:

$$\begin{aligned} u &= x+1 \quad & \mathrm{d}v &= e^{9x}\,\mathrm{d}x \\ \mathrm{d}u &= \mathrm{d}x \quad & v &= \tfrac{1}{9}e^{9x} \end{aligned} \tag{14}$$

$$\frac{1}{9}(x+1)\, e^{9x} - \frac{1}{9}\int e^{9x}\,\mathrm{d}x = \boxed{\tfrac{1}{9}(x+1)\, e^{9x} - \tfrac{1}{81}e^{9x} + C} \tag{15}$$