# Math 142V final exam practice test 1 solutions

*You must show all of your work and reasoning to receive full credit.*

[10] **1.1**

[7.2] Textbook (Q.15) $\int_0^{\pi/6} \sqrt{1+\cos(2x)}\, dx$

$\Rightarrow \sqrt{1+\cos 2x} = \sqrt{1+1-2\sin^2 x} = \sqrt{2-2\sin^2 x} = \sqrt{2(1-\sin^2 x)} = \sqrt{2\cos^2 x} = \sqrt{2}\cos x$

$\therefore \int_0^{\pi/6} \sqrt{2}\cos x\, dx = \sqrt{2}\left[\sin x\right]_0^{\pi/6} = \sqrt{2}\left(\sin(\pi/6) - \sin(0)\right) = \frac{1}{\sqrt{2}}$

[10] **1.2.**

Solution: $\frac{1}{x} \le \frac{\cos^2(x)+1}{x}$

$\int_1^\infty \frac{1}{x}\, dx = \int_1^\infty$

$= \ln(\infty) - \ln(1)$

$\boxed{= \infty}$

The integral is divergent.

Since, denominator of $\frac{1}{x}$ has power $\le 1$, the integral is divergent

So,
By the comparison test $\int_1^\infty \frac{\cos^2(x)+1}{x}\, dx$ is $\boxed{\text{divergent}}$

[15] **1.3.**

$x = \tan(\theta)$

$dx = \sec^2(\theta)\, d\theta$

$$\int \frac{\sqrt{1+\tan^2(\theta)}}{\tan(\theta)} \cdot \sec^2(\theta)\, d\theta \rightarrow \sqrt{1+\tan^2\theta} = \sec(\theta)$$

$$\int \frac{(\sec(\theta))\sec^2(\theta)}{\tan(\theta)}\, d\theta$$

$$\int \frac{\sec(\theta)\,(1+\tan^2(\theta))}{\tan\theta}$$

$$\int \frac{\sec(\theta)}{\tan(\theta)} + \frac{\sec(\theta)\tan^{\cancel{2}}(\theta)}{\cancel{\tan}(\theta)} \rightarrow \frac{\frac{1}{\cos}}{\frac{\sin}{\cos}} = \frac{1}{\sin} = \csc(\theta)$$

$$\int \csc(\theta) + \sec(\theta)\tan(\theta)$$

$$\ln|\csc(\theta) - \cot(\theta)| + \sec\theta$$


$$\ln\left|\frac{\text{hyp}}{\text{opp}} - \frac{\text{adj}}{\text{opp}}\right| + \frac{\text{hyp}}{\text{adj}}$$

Answer: $\boxed{= \ln\left|\frac{\sqrt{x^2+1}}{x} - \frac{1}{x}\right| + \sqrt{x^2+1} + C}$

[10] **1.4.** Use partial fraction decomposition. Answer:

$$\ln|x-1| - \frac{1}{2}\ln\left(x^2+9\right) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C \qquad (1)$$

[10] **1.5.**

$\Rightarrow$ rewriting $a_n = \dfrac{\cancel{(2n-1)!}\,(6n^2)}{(2n+1)(2n)\cancel{(2n-1)!}} = \dfrac{6n^2}{4n^2+2n}$

$\therefore \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{6n^2}{4n^2+2n} = \lim\limits_{n\to\infty} \dfrac{6}{4+2/n} = \dfrac{6}{4} = 3/2$

[15] **1.6.**

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}5^{n+1}x^{n+1}}{\sqrt[3]{n+1}}\cdot\frac{\sqrt[3]{n}}{(-1)^n5^nx^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{5x(n)^{\frac{1}{3}}}{(n+1)^{\frac{1}{3}}}\right| = |5x|\lim_{n\to\infty}\frac{n^{\frac{1}{3}}}{(n+1)^{\frac{1}{3}}}$$

$$= |5x|\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{\frac{1}{3}}$$

$$= |5x|\lim_{n\to\infty}(1+n)^{\frac{1}{3}} = \infty > 1$$

Radius of convergence**= 0**

Interval of convergence**=** $\{\mathbf{0}\}$

[10] **1.7.**

$$f(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \frac{a}{1-r}$$

$a=1$

$r=-x$

$$\sum_{n=0}^{\infty} ar^n = \sum_{n=0}^{\infty} 1(-x)^n = \boxed{\sum_{n=0}^{\infty} (-1)^n (x)^n}$$

R.O.C. : $r = -x$ ; $|-x| < 1$

I.O.C. $: |-x| < 1$

$= |x| < 1 \Rightarrow -1 < x < 1$

$\boxed{\text{R.O.C} = 1}$

$\boxed{\text{I.O.C} = (-1, 1)}$

$-1 \quad 0 \quad 1$

[10] **1.8.**

$$\Rightarrow \quad y = r\sin\theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

$$x = r\cos\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

ewrite:

$$r^2 = 16\cos^2\theta + \cos^2\theta + \sin^2\theta - 8\cos\theta = 16\cos^2\theta - 8\cos\theta + 1 = (4\cos^2\theta - 1)^2$$

$$\therefore \quad r = 4\cos^2\theta - 1$$

$$\frac{dr}{d\theta} = 8\cos\theta(-\sin\theta) = -4\sin(2\theta)$$

$$\therefore \quad \frac{dy}{dx} = \frac{-4\sin(2\theta) + (4\cos^2\theta - 1)\cos\theta}{-4\sin(2\theta) - (4\cos^2\theta - 1)\sin\theta}$$

at $\theta = \pi/2 \Rightarrow \frac{dy}{dx} = \frac{0+0}{0+1} = 0$

$\therefore$ the slope is a horizontal line at $\theta = \pi/2$.

[10] **1.9.**

Inside: $r_1 = 1-\sin\theta$

Outside: $r_2 = 1$

Intersection:

$1 = 1-\sin\theta$

$\sin\theta = 0$

$\theta = \pi, 2\pi$

integration:

$r_1^2 - r_2^2 = (1-\sin\theta)^2 - 1^2$

$= 1 - 2\sin\theta + \sin^2\theta - 1$

$= 1/2(1-\cos(2\theta)) - 2\sin\theta$

$= 1/2 - 1/2\cos(2\theta) - 2\sin\theta$

Area between polar curves:

$$A = \frac{1}{2}\int_a^b (r_1^2 - r_2^2)\,d\theta$$

$$= 1/2\int_\pi^{2\pi}\left(\frac{1}{2} - \frac{1}{2}\cos(2\theta) - 2\sin\theta\right)d\theta$$

$$= \frac{1}{2}\left(\frac{1}{2}\theta - \frac{1}{2}\cdot\frac{1}{2}\sin(2\theta) + 2\cos\theta\right)\Big|_\pi^{2\pi}$$

$$= \frac{1}{2}\left(\pi - \frac{1}{4}\sin(4\pi) + 2\cos(2\pi) - \left(\frac{\pi}{2} - \frac{1}{4}\sin(2\pi) + 2\cos(\pi)\right)\right)$$

$$= \frac{1}{2}\left(\pi - 0 + 2 - \frac{\pi}{2} + 0 + 2\right)$$

$$\boxed{= 2 + \frac{\pi}{4}}$$

# Math 142V final exam practice test 2 solutions

*You must show all of your work and reasoning to receive full credit.*

[10] **2.1.**

u= 3x　　　　　　dv=cos(2x) dx

du=3 dx　　　　　　v=1/2 sin(2x)

$$\int_0^{\frac{\pi}{2}} 3x\cos(2x)dx = \frac{3}{2}x\sin(2x)\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{3}{2}\sin(2x)\,dx$$

$$= \frac{3}{2}\left(\frac{\pi}{2}\right)\sin(\pi) - \frac{3}{2}(0)\sin(0) + \frac{3}{4}\cos(2x)\Big|_0^{\frac{\pi}{2}}$$

$$= 0 + \left(\left(\frac{3}{4}\right)\cos(\pi) - \frac{3}{4}\cos(0)\right)$$

$$= 0 + \left(\frac{3}{4}(-1) - \frac{3}{4}(1)\right)$$

**=-1.5**

[10] **2.2.**

7.5 Strategy for Integration

Evaluate The definite Integral:

$$\int_1^\infty e^{-x}\,dx$$

$u = -x$

$du = -dx$

$dx = -du$

$$-\int e^u\,du$$

$$-e^u + C = -e^{-x} + C$$

$$\int_1^\infty e^{-x}\,dx = \lim_{b\to\infty}\left[\int_1^b e^{-x}\,dx\right]$$

$$\lim_{b\to\infty}\left[-e^{-x}\right]_1^b$$

$$\lim_{b\to\infty}\left(-e^{-b} + e^{-1}\right)$$

$$\lim_{b\to\infty}\left(\frac{1}{e} - \frac{1}{e^b}\right) = \frac{1}{e}$$

[15] **2.3.**

Evaluate $\int \tan x \sec x \, dx$

identity: $\tan^2 x + 1 = \sec^2 x$

$\frac{d}{dx} \tan = \sec^2$

$\int \tan^6 x \sec^4 x \, dx$

split $\sec^4$ into $\sec^2 x \cdot \sec^2 x$

$= \int \tan^6 x \sec^2 x \underbrace{\sec^2 x \, dx}_{(du)}$

$= \int \tan^6 x (\tan^2 x + 1) \sec^2 x \, dx$

$= \int \tan^8 x + \tan^6 x \sec^2 x \, dx$

$\to$ u-sub, $u = \tan x$, $du = \sec^2 x \, dx$

$\int (u^8 + u^6) \, du$

ans.

$\to \frac{u^9}{9} + \frac{u^7}{7} + C \Rightarrow \boxed{\frac{1}{9} \tan^9 x + \frac{1}{7} \tan^7 x + C}$

[10] **2.4.** Use partial fraction decomposition. Answer:

$$\frac{1}{4}\left(\ln|t+1| - \frac{1}{t+1} - \ln|t-1| - \frac{1}{t-1}\right) + C \tag{2}$$

[10] **2.5.**

$$\sum_{n=1}^{\infty}\left|\frac{\cos(8n)}{3+4^n}\right| = \sum_{n=1}^{\infty}\frac{|\cos 8n|}{3+4^n} \le \frac{1}{3+4^n}$$

$$\frac{1}{3+4^n} < \frac{1}{4^n} = \left(\frac{1}{4}\right)^n$$

$\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$ convergent geometric series

By comparison test $\sum_{n=1}^{\infty}\left|\frac{\cos(8n)}{3+4^n}\right|$ is convergent

This means $\sum_{n=1}^{\infty}\frac{\cos(8n)}{3+4^n}$ is $\boxed{\text{absolutely convergent.}}$

[15] **2.6.**

Textbook Question 15

15. $\sum_{n=0}^{\infty}\frac{(x-2)^n}{n^2+1}$ ratio test $= \lim_{n\to\infty}\left|\frac{\frac{(x-2)^{n+1}}{(n+1)^2+1}}{\frac{(x-2)^n}{n^2+1}}\right| =$

$\lim_{n\to\infty}\frac{|x-2|^{n+1}}{(n+1)^2+1}\cdot\frac{n^2+1}{|x-2|^n} = \lim_{n\to\infty}|x-2|\cdot\frac{n^2+1}{(n+1)^2+1} =$

Divide by $n^2$

$\lim_{n\to\infty}|x-2|\cdot\frac{\frac{n^2+1}{n^2}}{\frac{(n+1)^2+1}{n^2}} =$

$\lim_{n\to\infty}|x-2|\cdot\frac{1+\frac{1}{n^2}}{(1+\frac{1}{n})^2+\frac{1}{n^2}} = |x-2|\frac{1+0}{(1+0)^2+0} = |x-2|$

$|x-2|<1$ Convergent

$-1 < x-2 < 1$

$-1+2 < x < 1+2$

$1 < x < 3$

Radius of Convergence $\boxed{=1}$

Test endpoints

$x=1$ $\sum\frac{(-1)^n}{n^2+1}$ is convergent by AST

$x=3$ $\sum\frac{1}{n^2+1}$ convergent by comparision with $\sum\frac{1}{n^2}$ a convergent p-series when $p>1$

Interval of Convergence $\boxed{=[1,3]}$

[10] **2.7.**

$$f(x) = \frac{x^2}{x^4+16} = \frac{x^2}{16}\left(\frac{1}{1+x^4/16}\right)$$

$$= \frac{x^2}{16}\left(\frac{1}{1-(-\frac{x}{2})^4}\right)$$

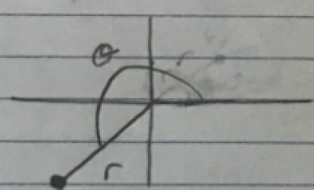
$$= \frac{x^2}{16}\sum_{n=0}^{\infty}\left[-\left(\frac{x}{2}\right)^4\right]^n \quad < 1$$

$$\boxed{\begin{array}{l}\text{Series converges when } |x| < 2 \\ ROC = 2 \\ I = (-2, 2)\end{array}}$$

$$\left|\frac{x}{2}\right| < 1$$
$$|x| < 2$$

[10] **2.8.**

$(-2, -2\sqrt{3})_{\text{cartesian}} = (4, 4\pi/3)_{\text{polar}}$




$-2 = r\cos\theta$
$-2\sqrt{3} = r\sin\theta$
$(-2)^2 + (-2\sqrt{3})^2 = r^2$
$4 + 4(3) = r^2$
$16 = r^2$
$4 = r$ or $r = -4$

$-2 = 4\cos\theta$
$-2\sqrt{3} = 4\sin\theta$
$\cos\theta = -1/2$
$\sin\theta = -\frac{\sqrt{3}}{2}$

$\boxed{\begin{array}{l}\theta = 4\pi/3 \\ \theta = 4\pi/3\end{array}}$ $\begin{array}{l}\text{or } \theta = 2\pi/3 \\ \text{or } \theta = 5\pi/3\end{array}$

[10] **2.9.** Use the integral formula for the area within a polar curve from $0$ to $\pi$. Answer: $\pi$.

# Math 142V final exam practice test 3 solutions

*You must show all of your work and reasoning to receive full credit.*

[10] **3.1.**

10. $\int \ln\sqrt{x} = \int \frac{\ln(x)}{2}\,dx = \frac{1}{2}\int \ln(x)\,dx =$

$\int fg' = fg - \int f'g = \int \ln(x)x = \ln(x)x - \int 1\,dx =$

$x\ln(x) - \int 1\,dx \quad \int 1\,dx = x \quad x\ln(x) - x$

$\frac{1}{2}\int \ln x\,dx = \frac{x\ln x}{2} - \frac{x}{2} = \boxed{\frac{x(\ln x - 1)}{2} + C}$

[10] **3.2.**

$\int_1^2 \frac{4x}{\sqrt[3]{x^2-4}}\,dx = \lim_{t\to 2} \int_1^t \frac{4x}{\sqrt[3]{x^2-4}}\,dx$

$4\int \frac{x}{\sqrt[3]{x^2-4}}\,dx \qquad u = x^2-4 \qquad du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$4\cdot\frac{1}{2}\int \frac{1}{\sqrt[3]{u}}\,du$

$4\cdot\frac{1}{2}\cdot\left[\frac{(x^2-4)^{-1/3+1}}{-1/3+1}\right]$

$4\cdot\frac{1}{2}\left[\frac{3(x^2-4)^{2/3}}{2}\right]$

$3(x^2-4)^{2/3} + C$

$= \lim_{t\to 2} \left(3(x^2-4)^{2/3}\right)\Big|_1^t$

$= \lim_{t\to 2} \left(3(t^2-4)^{2/3} - 3(-3)^{2/3}\right)$

$= -3(-3)^{2/3} = (-3)^{5/3}$

[15] **3.3.** Use trigonometric substitution with $x = 3\sin\theta$. Answer:

$$\frac{1}{2}\left(9\sin^{-1}\left(\frac{x}{3}\right) - x\sqrt{9-x^2}\right) \tag{3}$$

[10] **3.4.** Use partial fraction decomposition. Answer:

$$\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C \tag{4}$$

[10] **3.5.**

$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2}$ } continuous and "+" from $[1,\infty)$

$f(x) = \frac{\sqrt{n}+4}{n^2}$

$f'(x) = \frac{(n^2)(0.5n^{-.5}) - (\sqrt{n}+4)(2n)}{(n^2)^2}$

$\int_1^\infty \frac{\sqrt{x}+4}{x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{\sqrt{x}+4}{x^2}\,dx$

$\lim_{t\to\infty}\int_1^t \frac{x^{1/2}}{x^2}\,dx + \int_1^t \frac{4}{x^2}\,dx$

$\lim_{t\to\infty}\int_1^t x^{-3/2}\,dx + 4\int_1^t x^{-2}\,dx$

$\lim_{t\to\infty} \frac{x^{-1/2}}{-1/2}\Big|_1^t + 4\left(\frac{x^{-1}}{-1}\Big|_1^t\right)$

Negative if $(\sqrt{n}+4)(2n) > (n^2)(0.5n^{-.5})$

so sequence eventually decreases

$\lim_{t\to\infty} \frac{-2}{x^{1/2}}\Big|_1^t + 4\left(-\frac{1}{x}\Big|_1^t\right)$

$\lim_{t\to\infty}\left(\frac{-2}{t^{1/2}} + \frac{+2}{1^{1/2}}\right) + 4\left(-\frac{1}{t} + \frac{1}{1}\right)$

$(0+2)+4(1)$

$6 \rightarrow$ Improper integral is convergent

By the integral test, the series is also convergent

[15] **3.6.**

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\lim_{n\to\infty} \left| \frac{\frac{x^{2n+1}}{(2(n+1))!}}{\frac{x^{2n}}{(2n)!}} \right| = \lim_{n\to\infty} \left| \frac{x^{2n+2}}{(2n+2)!} \cdot \frac{(2n)!}{x^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{x^{2n}} \cdot x^{2}}{(2n+2)(2n+1)\cancel{(2n)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{x^{2n}}} \right| = \lim_{n\to\infty} \left| \frac{1}{(2n+2)(2n+1)} \right| \cdot \left| x^2 \right|$$

$$= 0 \cdot x^2 = 0 \qquad \boxed{R = \infty,\ I(-\infty, \infty)}$$

[10] **3.7.**

$$= x^3\left(\frac{1}{-x^2+3}\right)$$

Of Convergence.

$$= x^3\left(\frac{1}{3}\right)\left(\frac{1}{-\frac{x^2}{3}+1}\right)$$

$$= \frac{x^3}{3}\left(\frac{1}{1-\frac{x^2}{3}}\right) \qquad r = \frac{x^2}{3}$$

$$f(x) = \frac{x^3}{3}\sum_{n=0}^{\infty}\left(\frac{x^2}{3}\right)^n \quad \text{as long as } \frac{x^2}{3} < 1$$

$$= f(x) = \frac{1}{3}(x^3)\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n (x^2)^n$$

$$f(x) = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n\left(\frac{1}{3}\right)(x^{2n})(x^3)$$

$$\boxed{f(x) = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n+1} x^{2n+3}} \Longleftarrow \text{Power series rep}$$

$$\frac{x^2}{3} < 1$$

$$= \frac{|x|^2}{3} < 1$$

$$= |x|^2 < 3$$

$$= |x| < \sqrt{3}$$

$$\boxed{= -\sqrt{3} < x < \sqrt{3}} \Longleftarrow \text{Interval Of Convergence}$$

[10] **3.8.**

$$\frac{dr}{d\theta} = -\sin 2\theta(2)$$

$$\frac{r(\cos\theta) + \frac{dr}{d\theta}\sin\theta}{-r(\sin\theta) + \frac{dr}{d\theta}\cos\theta} = \frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{(\cos 2\theta)(\cos(\frac{\pi}{4})) + (-\sin(2\theta)(2))(\sin(\frac{\pi}{4}))}{-(\cos 2\theta)(\sin(\frac{\pi}{4})) + (-\sin(2\theta)(2))(\cos(\frac{\pi}{4}))}$$

$$= \frac{(\cos 2(\frac{\pi}{4}))(\cos(\frac{\pi}{4})) + (-\sin(2(\frac{\pi}{4}))(2))(\sin(\frac{\pi}{4}))}{-(\cos 2(\frac{\pi}{4}))(\sin(\frac{\pi}{4})) + (-\sin(2(\frac{\pi}{4}))(2))(\cos(\frac{\pi}{4}))}$$

$$= \frac{(0)(\sqrt{2}/2) + (-(1)(2))(\sqrt{2}/2)}{-(0)(\sqrt{2}/2) + (-(1)(2))(\sqrt{2}/2)}$$

$$= \frac{\frac{-2\sqrt{2}}{2}}{\frac{-2\sqrt{2}}{2}}$$

$$= \boxed{1}$$

[10] **3.9.**

# Math 142V final exam practice test 4 solutions

*You must show all of your work and reasoning to receive full credit.*

[10] **4.1.**

$\int x^2 \sin 2x\,dx$

**Work:**

$\to x^2 \int \sin 2x\,dx - \int \left(\frac{dx^2}{dx}\left(\int \sin 2x\,dx\right)\right)dx$

$\to -x^2\left(\frac{\cos 2x}{2}\right) + \int (2x)\left(\frac{\cos 2x}{2}\right)dx$

$\to -\frac{x^2\cos 2x}{2} + \int x(\cos 2x\,dx)$

$= \frac{x^2\cos 2x}{2} + \int x\cos 2x\,dx$

$\hookrightarrow x \cdot \int \cos 2x\,dx - \int \left(\frac{dx}{dx}\int \cos 2x\,dx\right)dx$

$\to x\left(\frac{\sin 2x}{2}\right) - \int \frac{\sin 2x}{2}\,dx$

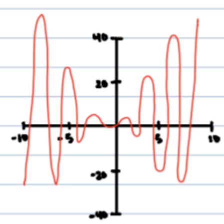
$\to \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + C$

combine

$\int x^2 \sin 2x\,dx = \frac{-x^2\cos 2x}{2} + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + \cancel{C}^{\,0}$

$\to \frac{(1-2x^2)\cos 2x}{4} + \frac{x\sin 2x}{2}$ = Anti-derivative

**Graph:**




[10] **4.2.** Use integration by parts with $u = \ln r$. Answer: convergent, with value $-\frac{1}{4}$.

[15] **4.3.** Use trigonometric substitution with $x = \tan\theta$. Answer: $\frac{2+\pi}{8}$.

[10] **4.4.** Use partial fraction decomposition. Answer:

$$\frac{1}{2}\ln\left(x^2+1\right) + \tan^{-1}x - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \qquad (5)$$

[10] **4.5.**

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!} = \frac{2^n \cancel{n!}}{(n+2)(n+1)\cancel{n!}} = \sum_{n=1}^{\infty} \frac{2^n}{(n+2)(n+1)}$$

Ratio Test

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\cancel{2^{n}}2}{(n+2+1)(n\cancel{+1}+1)} \cdot \frac{(n\cancel{+2})(n+1)}{\cancel{2^n}}\right|$$

$$= \lim_{n\to\infty} \frac{2(n+1)}{n+3} = \lim_{n\to\infty} \frac{2n+1}{n+3} = \frac{\frac{2n}{n} + 1/n}{\frac{n}{n} + 3/n} = 2 > 1$$

Since $2 > 1$, based on the ratio test the series diverges

[15] **4.6.** Use the ratio test. Answer: radius $4$, interval $[-4, 4]$

[10] **4.7.**

Solution: $f'(x) = -\frac{1}{1-x} = \sum_{n=0}^{\infty} -x^n \quad \text{if } |x| < 1$

$$\ln(1-x) = \int \sum_{n=0}^{n} x^n \, dx = \sum_{n=0}^{\infty} -\frac{x^{n+1}}{n+1} + C$$

$$C = f(0) = \ln(1) = 0$$

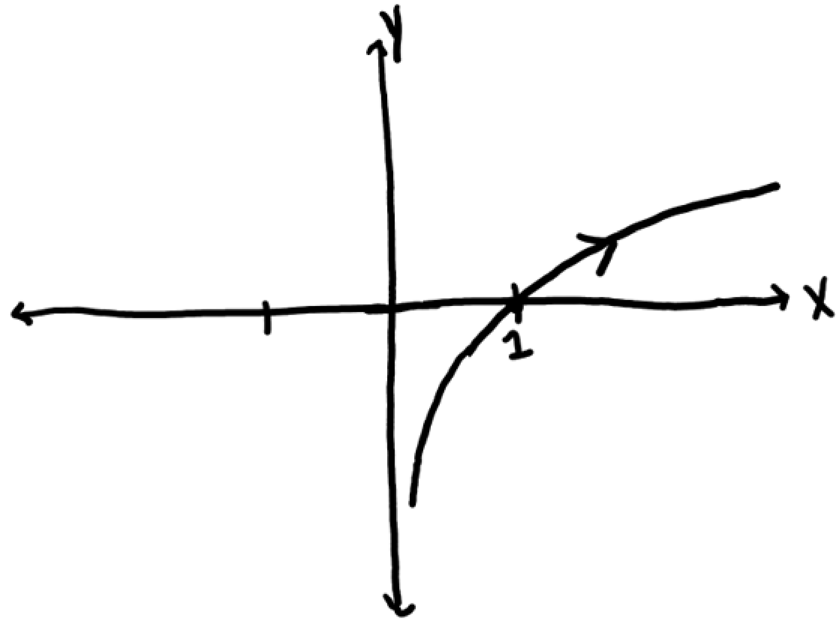
$$\ln(1-x) = \sum_{n=0}^{\infty} -\frac{x^{n+1}}{n+1} \qquad ROC = 1$$

[10] **4.8.**

a) $t = \sqrt{x}$

y= $\ln(\sqrt{x}) = \frac{1}{2}\ln(x)$

b)




[10] **4.9.** Use the integral formula for the arc length of a polar curve. Answer: $\frac{8}{3}\left((\pi^2+1)^{\frac{3}{2}}-1\right)$