# Math 142V final exam practice test 1

*You must show all of your work and reasoning to receive full credit.*

[10] **1.1** (7.2.45) Evaluate the integral.

$$\int_0^{\frac{\pi}{6}} \sqrt{1+\cos(2x)}\,\mathrm{d}x \tag{1}$$

[10] **1.2.** Determine whether the improper integral is convergent or divergent.

$$\int_1^{\infty} \frac{\cos^2 x + 1}{x}\,\mathrm{d}x \tag{2}$$

[15] **1.3.** Evaluate the integral.

$$\int \frac{\sqrt{1+x^2}}{x}\,\mathrm{d}x \tag{3}$$

[10] **1.4.** (7.4.23) Evaluate the integral.

$$\int \frac{10}{(x-1)(x^2+9)}\,\mathrm{d}x \tag{4}$$

[10] **1.5.** Determine whether the sequence converges or diverges. If it converges, find its limit.

$$a_n = \frac{(2n-1)!}{(2n+1)!}6n^2 \tag{5}$$

[15] **1.6.** Find the radius and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{\sqrt[3]{n}} x^n \tag{6}$$

[10] **1.7.** (11.9.3) Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{1}{1+x} \tag{7}$$

[10] **1.8.** Determine the slope of the tangent line to the polar curve at the point where $\theta = \frac{\pi}{2}$.

$$r^2 = 17\cos^2\theta - 8\cos\theta + \sin^2\theta \tag{8}$$

[10] **1.9.** Find the area of the region that lies inside the first polar curve and outside the second polar curve.

$$r = 1 - \sin\theta, \quad r = 1 \tag{9}$$

# Math 142V final exam practice test 2

*You must show all of your work and reasoning to receive full credit.*

[10] **2.1.** Solve the integral with the use of integration by parts.

$$\int_0^{\frac{\pi}{2}} 3x\cos(2x)\ \mathrm{d}x \tag{10}$$

[10] **2.2.** Determine whether the improper integral is convergent or divergent.

$$\int_1^{\infty} e^{-x}\,\mathrm{d}x \tag{11}$$

[15] **2.3.** Evaluate the integral.

$$\int \tan^6 x \sec^4 x\,\mathrm{d}x \tag{12}$$

[10] **2.4.** (7.4.21) Evaluate the integral.

$$\int \frac{\mathrm{d}t}{(t^2-1)^2} \tag{13}$$

[10] **2.5.** Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{\cos(8n)}{3+4^n} \tag{14}$$

[15] **2.6.** (11.8.15) Find the radius and interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1} \tag{15}$$

[10] **2.7.** (11.9.7) Find a power series representation of the function and determine the interval of convergence.

$$f(x) = \frac{x^2}{x^4+16} \tag{16}$$

[10] **2.8.** Find the polar coordinates of the point whose Cartesian coordinates are $\left(-2, -2\sqrt{3}\right)$.

[10] **2.9.** (10.4.9) Sketch the curve and find the area that it encloses.

$$r = 2\sin\theta \tag{17}$$

# Math 142V final exam practice test 3

*You must show all of your work and reasoning to receive full credit.*

[10] **3.1.** (7.1.10) Evaluate the integral.

$$\int \ln \sqrt{x}\, \mathrm{d}x \tag{18}$$

[10] **3.2.** Determine whether the improper integral is convergent or divergent. If it is convergent, find its value.

$$\int_1^2 \frac{2x}{\sqrt[3]{x^2-4}}\, \mathrm{d}x \tag{19}$$

[15] **3.3.** (7.3.4) Evaluate the indefinite integral.

$$\int \frac{x^2}{\sqrt{9-x^2}}\, \mathrm{d}x \tag{20}$$

[10] **3.4.** (7.4.9) Evaluate the integral.

$$\int \frac{5x+1}{(2x+1)(x-1)}\, \mathrm{d}x \tag{21}$$

[10] **3.5.** (11.3.15) Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2} \tag{22}$$

[15] **3.6.** Determine the radius and interval of convergence of the power series.

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \tag{23}$$

[10] **3.7.** Find a power series representation of the function, and determine its

radius of convergence.

$$f(x) = \frac{x^3}{3 - x^2} \tag{24}$$

[10] **3.8.** (10.3.59) Find the slope of the tangent line to the given polar curve at the point specified by the value of $\theta$.

$$r = \cos(2\theta), \quad \theta = \tfrac{\pi}{4} \tag{25}$$

[10] **3.9.** (10.4.45) Find the exact length of the polar curve.

$$r = 2\cos\theta, \quad 0 \leq \theta \leq \pi \tag{26}$$

# Math 142V final exam practice test 4

*You must show all of your work and reasoning to receive full credit.*

[10] **4.1.** Evaluate the integral.

$$\int x^2 \sin(2x)\ \mathrm{d}x \tag{27}$$

[10] **4.2.** (7.8.37) Determine whether the integral is convergent or divergent. Evaluate it if it is convergent.

$$\int_0^1 r \ln r\ \mathrm{d}r \tag{28}$$

[15] **4.3.** (7.3.14) Evaluate the integral.

$$\int_0^1 \frac{\mathrm{d}x}{\left(x^2+1\right)^2} \tag{29}$$

[10] **4.4.** (7.4.27) Evaluate the integral.

$$\int \frac{x^3+4x+3}{x^4+5x^2+4}\ \mathrm{d}x \tag{30}$$

[10] **4.5.** Determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{2^n n!}{(n+2)!} \tag{31}$$

[15] **4.6.** (11.8.9) Find the radius and interval of convergence of the power series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n} \tag{32}$$

[10] **4.7.** Find a power series representation of the function and determine its radius of convergence.

$$f(x) = \ln(1-x) \tag{33}$$

[10] **4.8.** (10.1.15) Eliminate the parameter to find a Cartesian equation of the

curve. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

$$\begin{aligned} x(t) &= t^2 \\ y(t) &= \ln t \end{aligned} \tag{34}$$

[10] **4.9.** (10.4.47) Find the exact length of the polar curve.

$$r = \theta^2, \quad 0 \leq \theta \leq 2\pi \tag{35}$$