MTH 142
College Calculus II Course Guide
University at Buffalo

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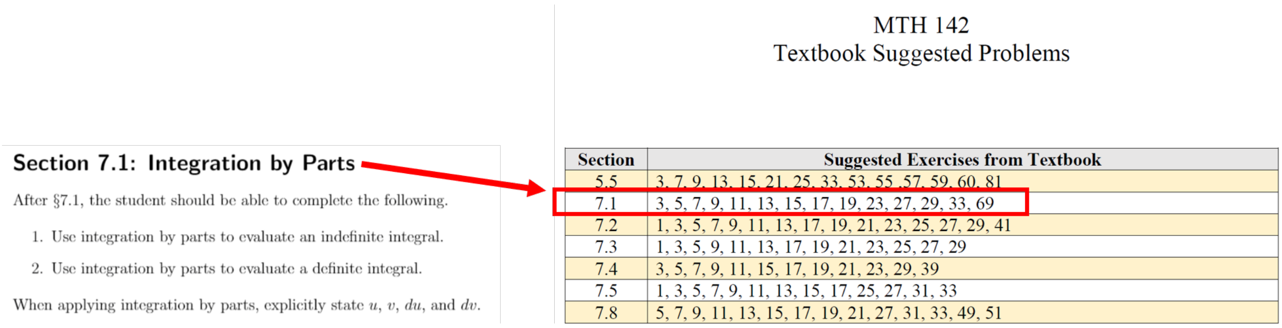
Contents

1 Introduction: How to use the Course Guide

The purpose of this document is to provide to the student the **MTH 142** course learning objectives. Any learning objective listed here may be assessed on the MTH 142 final exam.

This list of learning objectives is broken by section of the textbook (*J. Stewart, Calculus, Early Transcendental MTH 141, 142, 8th custom UB ed.*). The Course Guide should be used together with the MTH 142 Textbook Suggested Problems. Figure 1.1 illustrates how to use the Course Guide and the Textbook Suggested Problems.



Section 7.1: Integration by Parts

After §7.1, the student should be able to complete the following.

1. Use integration by parts to evaluate an indefinite integral.
2. Use integration by parts to evaluate a definite integral.

When applying integration by parts, explicitly state u, v, du, and dv.

MTH 142
Textbook Suggested Problems

Section	Suggested Exercises from Textbook
5.5	3, 7, 9, 13, 15, 21, 25, 33, 53, 55 ,57, 59, 60, 81
7.1	3, 5, 7, 9, 11, 13, 15, 17, 19, 23, 27, 29, 33, 69
7.2	1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 23, 25, 27, 29, 41
7.3	1, 3, 5, 9, 11, 13, 17, 19, 21, 23, 25, 27, 29
7.4	3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 29, 39
7.5	1, 3, 5, 7, 9, 11, 13, 15, 17, 25, 27, 31, 33
7.8	5, 7, 9, 11, 13, 15, 17, 19, 21, 27, 31, 33, 49, 51

Figure 1.1: Learning objectives for §7.1 are mapped to exercises in the textbook.

2 Learning Objectives for Chapter 5

Section 5.5: The Substitution Rule

After §5.5, the student should be able to complete the following.

1. When appropriate, apply the Substitution Rule to evaluate an indefinite integral.
2. When appropriate, apply the Substitution Rule to evaluate a definite integral.

3 Learning Objectives for Chapter 7

Section 7.1: Integration by Parts

After §7.1, the student should be able to complete the following.

1. Use integration by parts to evaluate an indefinite integral.
2. Use integration by parts to evaluate a definite integral.

When applying integration by parts, explicitly state u, v, du, and dv.

Section 7.2: Trigonometric Integrals

After §7.2, the student should be able to complete the following.

1. Evaluate an integral of the form: $\displaystyle\int \sin^m x \cos^n x \; dx$.
2. Evaluate an integral of the form: $\displaystyle\int \tan^m x \sec^n x \; dx$.

Section 7.3: Trigonometric Substitution

After §7.3, the student should be able to complete the following.

1. Use a trigonometric substitution to evaluate an indefinite integral. The answer should involve the same variable as the original given integral (e.g., if the given indefinite integral involves the variable x then the answer should be an explicit function of x).
2. Use a trigonometric substitution to evaluate a definite integral.

Section 7.4: Integration of Rational Functions by Partial Fractions

After §7.4, the student should be able to complete the following.

1. Using Cases I - III of the section, find a partial fraction decomposition of a rational function.
2. Evaluate the integral of a rational function using a partial fraction decomposition.

Section 7.5: Strategy for Integration

After §7.5, the student should be able to complete the following.

1. Use strategies for integration to evaluate a given indefinite integral. The student determines the method.

2. Use strategies for integration to evaluate a given definite integral. The student determines the method.

Section 7.8: Improper Integrals

After §7.8, the student should be able to complete the following.

1. Determine whether an improper integral is converge or divergent. If convergent, evaluate the integral.

2. Determine for which values of p the improper integral $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ is convergent and for which values of p the integral is divergent.

3. Use the Comparison Theorem to determine whether an improper integral is convergent or divergent.

4 Learning Objectives for Chapter 8

Section 8.1: Arc Length

After §8.1, the student should be able to complete the following.

1. Use an arc length formula to find the exact length of a curve over an interval.

5 Learning Objectives for Chapter 10

Section 10.1: Curves Defined by Parametric Equations

After §10.1, the student should be able to complete the following.

1. Given parametric equations $x = f(t)$, $y = g(t)$ for a curve, sketch the curve using the parametric equations to plot points. Indicate, using an arrow, the direction in which the curve is traced as t increases.
2. Given parametric equations $x = f(t)$, $y = g(t)$ for a curve, eliminate the parameter to find a Cartesian equation of the curve.

Section 10.2: Calculus with Parametric Curves

After §10.2, the student should be able to complete the following.

1. Given parametric equations $x = f(t)$, $y = g(t)$ for a curve, find dy/dx if $dx/dt \neq 0$.
2. Given parametric equations $x = f(t)$, $y = g(t)$ for a curve, find d^2y/dx^2 if $dx/dt \neq 0$.
3. Given parametric equations $x = f(t)$, $y = g(t)$ of a curve for $\alpha \leq t \leq \beta$, find the exact length of the curve.

Section 10.3: Polar Coordinates

After §10.3, the student should be able to complete the following.

1. Plot points given polar coordinates (r, θ)
2. Convert a point $P(r, \theta)$ in polar coordinates to Cartesian coordinates. Use $x = r\cos\theta$, $y = r\sin\theta$.
3. Convert a point (x, y) in Cartesian coordinates to polar coordinates. Use $x^2 + y^2 = r^2$ and $\tan\theta = y/x$.
4. Sketch the region in the plane consisting of points whose polar coordinates satisfy inequality conditions.
5. Sketch a polar curve $r = f(\theta)$.

Section 10.4: Areas and Lengths in Polar Coordinates

After §10.4, the student should be able to complete the following.

1. Use the definite integral to find the area of a region enclosed by a polar curve.
2. Use the definite integral to solve the following area problems involving two polar curves.
   a) Find the area of the region that lies inside the first curve and outside the second curve.
   b) Find the area of the region that lies inside both polar curves.
3. Use the definite integral to find the length of a polar curve.

6 Learning Objectives for Chapter 11

Section 11.1: Sequences

After §11.1, the student should be able to complete the following.

1. Determine whether a sequence $\{a_n\}$ is convergent or divergent.
2. List first terms of a sequence given the general term a_n. For example, list the first 5 terms of the sequence with general term $a_n = \frac{1}{n^2}$.

Section 11.2: Series

After §11.2, the student should be able to complete the following.

1. Given an infinite series, find partial sums of the series. For example, given the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ find the first 4 partial sums s_1, s_2, s_3, and s_4.
2. Given a geometric series:
   a) Find the common ratio r.
   b) Use the common ratio r to determine whether the geometric series is convergent or divergent. If the geometric series is convergent, find its sum.
3. Use the Test for Divergence to show a series Σa_n is divergent by showing either $\lim_{n\to\infty} a_n$ DNE or $\lim_{n\to\infty} a_n \neq 0$, if possible.

Section 11.3: The Integral Test and Estimates of Sums

After §11.3, the student should be able to complete the following.

1. Use the Integral Test to test the convergence or divergence of an infinite series.
2. State for which values of the parameter p the p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

   is convergent and for which values of p the series is divergent.

Section 11.4: The Comparison Tests

After §11.4, the student should be able to complete the following.

1. Use The Comparison Test to test the convergence or divergence of a series $\sum a_n$ with positive terms. Explicitly state the series $\sum b_n$ to which $\sum a_n$ is compared.
2. Use The Limit Comparison Test to test the convergence or divergence of a series $\sum a_n$ with positive terms. Explicitly state the series $\sum b_n$ to which $\sum a_n$ is compared.

Section 11.5: Alternating Series

After §11.5, the student should be able to complete the following.

1. Determine if a given series $\sum a_n$ is an alternating series.
2. Use the Alternating Series Test to show an alternating series is convergent (when the series satisfies to hypotheses of the test).
3. For a convergent alternating series $\sum(-1)^n b_n$, use the Alternating Series Estimation Theorem to estimate the error

$$|R_n| = |s - s_n|$$

   where $s = \sum(-1)^n b_n$ and s_n is the nth partial sum of the series. (So $s_n \approx s$.)

Section 11.6: Absolute Convergence and the Ratio and Root Tests

After §11.6, the student should be able to complete the following.

1. Determine if a series $\sum a_n$ is absolutely convergent or conditionally convergent.
2. Use the Ratio Test to determine if a series $\sum a_n$ is convergent or divergent.
3. Use the Root Test to determine if a series $\sum a_n$ is convergent or divergent.

Section 11.7: Strategy for Testing Series

After §11.7, the student should be able to complete the following.

1. Choose an appropriate method (test) to to test a series for convergence or divergence. When testing a series for convergence or divergence, the student should expect to complete the following.
   (a) State which test is being used.
   (b) Explicitly show that the assumptions of a test are satisfied (if any assumptions). For example, if applying the Integral Test the student should address conditions that the function $f(x)$ must be continuous, positive, and decreasing on $[1, \infty)$.

Section 11.8: Power Series

After §11.8, the student should be able to complete the following.

1. Given a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$:
   a) Find the radius of convergence R using either the Ratio Test or the Root Test.
   b) Find the interval of convergence using either the Ratio Test or Root Test. If $0 < R < \infty$, the student must test convergence of the power series at each end point of $(a-R, a+R)$.

Section 11.9: Representations of Functions as Power Series

In §11.9, the student learns how to manipulate the power series shown below in Equation 6.1 to construct power series for functions related to $f(x) = \frac{1}{1-x}$ through multiplication by a x^k (k a positive integer), composition, differentiation, and/or antidifferentiation.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n \qquad |x| < 1 \tag{6.1}$$

After §11.9, the student should be able to complete the following.

1. Find a power series expression for a function f that can be generated from Equation 6.1 through multiplication by x^k and/or composition of functions (e.g., replace x by $g(x)$ in Equation 6.1).

2. Use term-by-term differentiation to express a function as a power series.

3. Use term-by-term integration to express a function as a power series.

4. If the function f can be expressed as a power series obtained from the power series in Equation 6.1, approximate $\int_a^b f(x)\,dx$ using the power series.

Section 11.10: Taylor and Maclaurin Series

After §11.10, the student should be able to complete the following.

1. Find a Taylor series of a function f centered at a. Find the associated radius of convergence. [Do not show that $R_n(x) \to 0$ as $n \to \infty$.]

2. Find Maclaurin series for the following functions. Find the associated radius of convergence for each function. [Do not show that $R_n(x) \to 0$ as $n \to \infty$.]

$$f_1(x) = e^x, \quad f_2(x) = \sin x, \quad f_3(x) = \cos x, \quad f_4(x) = \tan^{-1} x$$

3. Use a Maclaurin series found in Table 1 of §11.10 to find a Maclaurin series for a function.

Section 11.11: Applications of Taylor Polynomials

After §11.11, the student should be able to complete the following.

1. Assuming f is equal to the sum of its Taylor series centered at a, use the nth-degree Taylor polynomial, T_n, of f at a to approximate f near a.