Multifractal characterization of urban residential land price in space and time

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A B S T R A C T

The spatial and temporal distribution of land price plays a key role in urban development and redevelopment processes. Identifying the features of land price distribution (LPD) is essential for improving urban planning and modeling land use changes. The purposes of this study were to determine if LPD can be characterized by multifractal models and to develop multifractal methods for characterizing the properties of LPD at various scales. An analysis was performed for a study site in Wuhan City, central China. Land prices were sampled in the years 2001, 2004, and 2007. The LPD patterns were represented by multifractal spectra estimated using the method of moments and characterized by five quantitative multifractal parameters. The results showed that the dimension spectra calculated from the LPD data in various regions and at different times indeed depict multifractality; the curves of the multifractal spectra are continuous, displaying the same characteristics of asymmetric and convex curves at the same times in different regions, where the common transitional trend was from shorter toward the left but much longer toward the right in 2001, comparatively symmetric with a slight right deviation in 2004, shorter toward the right but much longer toward the left in 2007, which implying continuous multifractality observed for LPD, and this trend indicates that the singularity of land prices in the different areas keeps step with urban development. In addition, the horizontal characteristics of the curves also differed in different development stages in the city. These results also demonstrated that we may characterize the spatial and temporal differences of different LPD patterns using multifractal methods, which may thus be utilized as a quantitative measure in understanding how land price affects changes in urban land use.

Introduction

During the past decades, increasing urbanization has resulted in a complex process of changes in land use and land cover from local to global scales. In turn, this process has profoundly affected natural and human systems in cities (Yu & Ng, 2007), with effects such as worsening conditions of crowding, housing shortages, and insufficient or obsolete infrastructure. Thus, these increasing urban ecological and security problems require both effective management and planning for urban regions (Brockheroff, 2000; Herold, Couclelis, & Clarke, 2005). The profound change in the urban environment in contemporary China is characterized by an intense use of the available space. One of the main driving forces for this use is that increasing land price has been impacted tremendously by population growth and the limited availability of arable land (Han, 2010; Morris & Michael, 2008; Tan, Li, Xie, & Lu, 2005). Sustainable urban development and smart growth (American Planning Association, 2002) rely strongly on available information and knowledge about the causes and effects of urban land use change processes (Batty, 2005, 589 pp.; Kaiser, Godschalk, & Chapin, 1995, pp. 493; Longley & Mesev, 2000). In practice, land price plays a significant role in urban expansion in terms of its development orientation and speed (Lv, Long, & Liu, 2007). Therefore, the characterization of land price distribution (LPD) is essential to provide the information needed to improve our understanding and modeling of urban land use change and reveal the drivers and spatial change patterns of urban development (Brueckner & Colwell, 1983).

In another aspect, urban LPD can be regarded as the combined result of regional development processes and land development activities. Land development processes, especially in terms of land operating processes, usually happen repeatedly, and every parcel land price change, or each influential factor change, may possibly
result in changes in the land prices of other nearby parcels or regional land price increases or decreases (Colwell & Sirmans, 1980; Cotteleer & Peerlings, 2011; Ecker & Isaksen, 2005; Witte, Howard, Erekson, 1979). These superimposed effects of spatially related development activities result in a multiple-dimension spatial distribution of land price. Conventional statistical and hedonic pricing methods were widely used in previous studies in the fields of land and house pricing (Beers and Kleijnen., 2003; Banerjee, Gelfand, & Knight, 2004; Cotteleer & Peerlings, 2011; Gillard, 1981; José-Ma and Beatriz., 2006; Kong, Yin, & Nakagoshi, 2007; Li and Brown, 1980; Longley, Higgs, & Martin, 1994; Lou, 2004, 132 pp.; Martínez, Lorenzo, & Rubio, 2000; Mueller et al., 2004; Tang & Yu, 2010; Vural & Fidan, 2009; Waltelt & Schlafper, 2010; Wu, 2005, 150 pp.). For example, statistical characteristics of land price samples are usually used to describe the LPD (David, Johnson, & Jan, 2001); however, due to the uncertainty in land price samples and regional randomness in urban space (Cheng & Agterberg, 1995), it remains a critical challenge for land science researchers to understand the characteristics of LPD based on random samples. Conventional geostatistics methods have also played extremely important roles in this respect, e.g., Kriging methods (José-Ma & Beatriz, 2006; Lou, 2004, 132 pp.; Martínez et al., 2000; Wu, 2005, 150 pp.), but they cannot consider the variability of a spatial measure-scale which changes with the distribution and statistical characteristics of the samples (Cheng & Agterberg, 1996). In addition, conventional geostatistics methods are usually suitable for measuring abundant and ordinary data with normal or lognormal distributions, and it is thus difficult to quantitatively differentiate and describe the spatial singularity properties of the data due to the effects of smoothing functions (Cheng, 2001). Increasing evidence has illustrated that LPD has similar or self-similar and singularly properties (Colwell & Munneke, 1997, 2003; Eugen, 1965; Longley & Dunn, 1988; Liu, Li, Sun, & Ma, 2006; Tang & Yu, 2010; Wang et al., 2005; Wu, 2008); thus, there exists a significant difficulty in implementing any procedure to characterize spatiotemporal LPD.

Concepts of fractals and scaling have already widely been used in physical and human geography and seems appropriate for characterizing nonlinear property of LPD (Batty & Longley, 1987a; De Keersmaecker, Frankhauser, & Thomas, 2003; Goodchild & Mark, 1987; MacLennan, Fotheringham, Batty, & Longley, 1991; Mandelbrot, 1983, 468 pp.). While natural spatial objects such as coastlines, plants, and clouds have long been treated as fractals of various dimensions (Falconer, 1990, 316 pp.; Lam and De Cola, 1993; Mandelbrot, 1983, 468 pp.; Orbach, 1986), lots of researches on spatial analysis have concluded that artificially planned and designed spatial objects such as urban forms, internal structure of a city, and transportation networks can be modeled and simulated by means of fractal geometry (Arlingham & Nystuen, 1990; Batty, 1991; Batty & Longley, 1987a, 1987b, 1994; Batty & Xie, 1996, 1999; Benguigui, Czamanski, Marinov, & Portugali, 2000; Benguigui & Daoud, 1991; Chen, 2010; De Keersmaecker et al., 2003; Frankhauser, 2008; Fotheringham, Batty, & Longley, 1989; Shen, 2002; Thomas, Frankhauser, & Keersmaecker, 2007). Fractal analysis offers a different perspective on the urban landscape which takes into account urban spatial complexity (Batty, 2005, 589 pp.; Batty & Longley, 1994, 394 pp.; Batty & Xie, 1996; Cavalliés, Frankhauser, Peeters, & Thomas, 2010; Terzi & Kaya, 2011). The existence of multifractals usually signifies an underlying multiplicative cascade process, whereas additive processes generally produce simple fractals (Grassberger & Procaccia, 1983; Lovejoy & Schertzer, 1980). Multifractal formalisms involve decomposing self-similar measures into intertwined fractal sets, which are characterized by their singularity strength and fractal dimension (Posadas, Gimenez, Quiroz, & Protz, 2003). Multifractal characterization involves not a single dimension but rather a sequence of generalized fractal dimensions (Evertsz & Mandelbrot, 1992). Thus, a combination of all the fractal sets produces a multifractal spectrum that characterizes the variability and heterogeneity of the studied variables (Kravchenko et al., 1999) and, in particular, characterizes the distribution patterns of a field which has self-similar, statistically self-similar and singularity properties (Cheng, 1999). The advantage of the multifractal approach is that multifractal parameters can be independent of the size of the studied objects (Cox & Wang, 1993; Scheuring & Riedl, 1994). Moreover, multifractal analysis has been successfully applied to land use science and agricultural research to quantitatively measure the scaling properties of landforms (Gupta & Waymire, 1989; Mandelbrot, 1983, 468 pp.) and soil structure (Caniego, Espejo, Martín, & José, 2005; Caniego, Ibáñez, & José, 2006; Eghball, Schepers, Negahban, & Schlemmer, 2003; Ibáñez, Pérez-Gómez, & San José Martínez, 2009; Zeleke & Si, 2006) as well as to characterize the spatial and temporal complexities of vegetation patterns (Scheuring & Riedl, 1994) and land use patterns (Verburg & Chen, 2000; Wang et al., 2010; White & Engelen, 1993). However, very few researchers have attempted to use multifractal methods to characterize LPD in space and time. We believe that the characteristics of LPD in space and time can be well described by the fractal spectra of different regions and times. This hope is based mainly on the fact that similar studies have been successfully conducted in other earth science fields with similar characteristics of statistical data distribution, i.e., that most data are found in normal or lognormal distributions but both data tails follow power-law or fractal distributions (Cheng, 1999). These types of measures or fields including LPD are usually caused by cascade processes which generate measures with singularities and self-similarity that can be characterized by multifractal distribution.

Therefore, in this study, we are attempting to apply multifractal analysis to characterize urban residential LPD based on the computation of their generalized dimensions and multifractal spectra extracted from different regions and times in a city. Wuhan City of China was selected as the area for this case study. The objectives of the study were set as (1) to determine if LPD can be characterized by multifractal models and (2) to characterize the multifractal properties of LPD at various scales and try to delve into multiscale land price structures existing in cities at different times by means of multifractal spectra and parameters; moreover, we also attempted to explain the variations in the multifractal patterns of LPD associated with urban development processes. The results from this study may provide insights into identification, explanation, and application of underlying physical mechanisms to balance urban land price and land use planning.

Materials and methods

Study area

Wuhan City, the capital of Hubei province, is located in central China. The Yangtze River and the Han River cross the city dividing the city into three separate geographical parts: Wu Chang, Han Kou, and Han Yang. Wu Chang is the educational and provincial administrative center. Han Kou is the commercial, financial and city administrative center in where the central business district is located, and Han Yang is the region dominated by industries. These three regions of Wuhan as a single municipality are linked by a series of bridges and tunnels. There are seven main administrative regions (towns) in the following urban areas: Jiang An, Jiang Han, Qiao Kou, Han Yang, Wu Chang, Qing Shan, and Hong Shan. The city covers a total area of approximately 1040.64 km² (Fig. 1); the building and infrastructures of the city are constructed as three rings; namely, the...
first ring, the second ring and the third ring, from the inner to outer rings, respectively (Fig. 1). The average residential land price in Wuhan city went up by 6.05%, 6.15%, and 7.35% in the years 2005, 2006 and 2007, respectively (Wuhan City government, 2005e2007). In addition, Wuhan City has been chosen by the central Chinese Government as an experimental site for an environmentally friendly and resource-economizing society in China and as one of the most important parts of central China’s development. Therefore, the study area provides a typical example of a city in China with natural landscape and sociometric status for our study.

Dataset used

Three datasets of high-density residential land price samples with details properties were obtained from the Wuhan City municipal, including sale price of land property, sale price of new and used residential apartments, and residential apartment rentals. Because apartment sale price is composed of land price and buildings or other attachments price in China, thus these types of data are used to calculate land price in urban area there. Firstly, the land prices (per square meter) of samples were carefully calculated one by one based on routine methods and relative standards. Every type of samples has their calculation formula which was followed by the land use evaluation standards published by Department of Land Resources in China (2002), which was based on hedonic price model (Maclennan, 1977; Witte et al., 1979). The calculation of the datasets are based on these main processes, the sale price of land property is that individual, institutions or governments pay for using the land for construction or other purposes, the land price (per square meter) of the samples is equal to the total prices of the land use parcel divided into the parcel area. Sale prices of new and used residential apartments are the expenses people pay for owning the apartments and using the shared land of the apartments, so we should subtract the prices of the buildings, other attachments onto the sale prices of apartments in order to calculate the land prices. Residential apartment rental are the fees people pay for using the apartments, so we should adopt the method of rent avulse and the method of profit reduction to get the prices of apartments, then calculate the land prices using the similar method to the samples of the sale prices of new and used residential apartments. Second, the land prices of samples were amended to a standardized land use property price based on factors such as the date, the floor area ratio etc., which has the uniform meaning. Finally, samples and a spatial attributive database were viewed on the map based on GPS data collected by the field work, with the samples addresses located in the centers of parcels. The data of land prices were calculated from samples collected in 2001 (1721 parcels), 2004 (1495 parcels) and 2007 (3460 parcels); Fig. 2 shows the distribution of samples in 2007 as an example. The basic statistics for these data, including mean and standard deviation, are shown in Fig. 3a; a quantile–quantile (Q–Q) plot (Fig. 3b) was constructed to investigate the frequency distribution of land price concentration values. The results indicated that the values of land prices from different regions in various times do not follow a single lognormal distribution, which is similar to other cases demonstrated previously in Huhehaote City of Inner Mongolia by Du, Zhang, Zhang, and Su (2006) and in Beijing by Wang and Zhu (2007). The similar situations are also commonly found

![Fig. 1. The location of Wuhan in China and Study area. The study area is located in Wuhan, China, an important area for the development of central China.](image1)

![Fig. 2. (a) The distribution of residential land price samples in the study area. (b) The distribution of land prices generated by the MIDW interpolation method.](image2)
in geochemistry dealing with trace element concentration in rocks and soils etc, where it was demonstrated that such data may follow fractal distributions. The majority of the data, the values around mean, often follows normal or lognormal distributions, but values along both tails may follow power-law or fractal distributions (Cheng, 1999; Cheng & Agterberg, 1996). These types of values are typically caused by cascade processes which results in some areas with enriched values and others with depleted values. The land price may depict the similar property with variability caused by many factors involved in cascade processes. Land price is a dynamic variable with value change due to many factors ranging from political, environmental to economical mechanisms. One of the interesting properties of these types of distribution is nonlinear properties of self-similarity and singularity which can be characterized by multifractal models. Under the concept of multifractal distribution, the values around mean may follow normal or lognormal distribution and these data can be analyzed by means of ordinary statistical or multivariate statistical methods. The extreme values along two tails need to be characterized by extreme value distribution such as fractal distribution (Cheng, 1999). This is one of our motivations to apply multifractal modeling in LPD in the current research.

**Multifractal analyses**

There are two ways to represent the multifractal of a measure: a generalized dimension and a singularity spectrum. One of the most popular approaches in urban and spatial analysis for multifractal analysis is the conjunction of the moment method with the box-counting algorithm (Batty & Longley, 1994, 394 pp.; Borda-de-Água, Hubbell, McAllister, 2002; Cheng, 1999, 2001; Halsey, Jensen, Kadanoff, Procaccia, & Shraiman, 1986). Given that the study areas are irregular and some areas without building such as rivers, lakes and etc. the moment-based boxing counting method with edge correction was employed (Agterberg et al., 1996). In order to implement the multifractal model we first interpolate point data of land price into grid (raster format, see Fig. 2b) with a spatial resolution 100 m × 100 m by the multifractal Inverse Distance Weighted method (MIDW) (Cheng, 1999). The land price of grid is defined as the land prices intersecting the box and denote the average price as

\[ u_i(\varepsilon) = \frac{1}{\varepsilon} \left( \sum u_i \right) \]  

where \( \varepsilon \) is the average price per box and \( \varepsilon \) is the grid size. The so-called partition function results from a weighted sum over all boxes, that is,

\[ \chi_q(\varepsilon) = \sum_{i=1}^{n} u_i^q \]  

where \( n(\varepsilon) \) is the number of boxes of linear size \( \varepsilon \) covering the sampling space and \( q \) is the order of moment of the weighted sum (Halsey et al., 1986). For a generalized dimension, \( D_q \) may then be computed for the LPD through the parameter \( q \) by (Hentschel & Procaccia, 1983; Rényi, 1970, 670 pp.):

\[ D_q = \frac{1}{q-1} \times \log \left( \frac{\sum_{i=1}^{n} u_i^q}{\log(\varepsilon)} \right) \]  

For a multifractal measure, the function \( \chi_q(\varepsilon) \propto \varepsilon^{\tau(0)} \), where \( \tau(q) \) is a function of \( q \), gives the mass exponent (Feder, 1988, 283 pp.). For multifractal measures, \( \tau(q) \) is the nonlinear function \( \tau(q) = \alpha(q) - f(\alpha) \), where \( \alpha(q) = dq\tau(q)/dq \) is the global Hölder exponent and \( f(\alpha) \) is the fractal dimension and multifractal spectrum function.

To calculate the multifractal parameters, the choice of range of \( q \) will strongly influence the multifractal properties to be estimated including the width of singularity spectrum and asymmetry of \( f(\alpha) \) which are important properties characterizing the multifractal distribution. After comparing the characteristics of the curves of \( \alpha(q) \), \( \alpha(q) \) and \( f(\alpha) \) obtained using different ranges of \( q \), the range from \(-3.5 \) to \( 3.5 \) was selected for \( q \) value on the basis of 5% of standard error associated with the estimation of \( \tau(q) \). Then \( q \)-value from \(-3.5 \) to \( 3.5 \) with an interval of 0.1 were used to construct the partition functions \( \chi_q(\varepsilon) \). Considering the size of the study area and the sample density of each region, the minimum box (square) was sized at 100 m × 100 m. The valid box sizes and \( q \)-values were determined by setting the threshold of standard errors associated with the linear regression between log(\( \chi_q(\varepsilon) \)) and log(\( \varepsilon \)) less than 0.05 and a correlation coefficient \( |r| > 0.96 \). The slopes of the valid straight lines fitted using a least-squares between log(\( \chi_q(\varepsilon) \)) and log(\( \varepsilon \)) were estimated and denoted as \( \tau(q) \). The generalized dimension is known as the capacity dimension or box-counting dimension for \( D_0 \), the information dimension for \( D_\xi \), and the correlation dimension for \( D_2 \). The singularity exponent \( \alpha(q) \) and the multifractal spectrum \( f(\alpha) \) are then obtained. Multifractality, represented by \( \tau'(1) = \tau(2) - 2\tau(1) + \tau(0) \), is calculated and proven to be associated with the spatial analysis parameters (Cheng & Agterberg, 1996). In case of \( \tau(1) = 0 \) then \( \tau'(1) = D_2 - D_0 = -\Delta D \). This index measures the degree of multifractality, for example, if the LPD follows multifractal distribution then \( \tau'(1) < 0 \) implying the spectrum function \( f(\alpha) \) in a convex curve otherwise if LPD follows a single
1996, 1999), the widths of the singularity spectrum $D$ and $R$ for fractal or ordinary nonfractal distribution then the value of $\tau''(1) = 0$ and the curve $f(\alpha)$ reduces to a single spike. Therefore, we can also use the width of curve $f(\alpha)$ to measure the degree of multifractality. Previous authors have used various indexes to measure the degree of multifractality such as curvature of function $\tau(q)$ around 1 (Cheng, 1996, 1999). The widths of the singularity spectrum $f(\alpha)$, i.e., $\Delta\alpha = \alpha_{q=3.5} - \alpha_{q=3.5}$, are also calculated. To characterize the shapes of the multifractal spectrum, the asymmetry index, $R = (\Delta \alpha_{\text{left}} - \Delta \alpha_{\text{right}}) / (\Delta \alpha_{\text{left}} + \Delta \alpha_{\text{right}})$, is also obtained. Furthermore, the values of $\Delta \alpha_{\text{left}}$, $\Delta \alpha_{\text{right}}$, and $R$ for land price are summarized.

**Results**

**Multifractality of LPD**

The dimension spectra calculated from the LPD data in various regions and at different times have shown that LPD indeed depict multifractality. For example, the dimension spectra of land prices in the study area in the years 2001, 2004, and 2007 shown in Fig. 4 and Table 1. Fig. 4 illustrate the spectra functions are all curve rather than single spike, implying the LPD following multifractal distribution which are also proved by the multifractality indexes for the LPD in these three years, $\tau''(1) = 0.034$, 0.014, and 0.016, respectively. The fluctuation range of the $D_{q-q}$ curves differs each year. The fluctuation range of $D_{q}$ in 2001 and 2007 is bigger than in 2004. The information dimension ($D_1$) and correlation dimension ($D_2$) in 2004 are larger than those in 2001 and 2007, indicating that the LPD in 2004 is smoother than in the other two years and that the LPDs in 2001 and 2007 are both significantly different from that in 2004.

**Spatial and temporal variability of multifractality spectra of LPD**

**Multifractality spectra of LPD in ring areas**

The results in Fig. 5a–c show the multifractal spectra calculated for LPD from the first ring area to the third ring area in Wuhan City in the years 2001, 2004 and 2007. The multifractal spectra of each ring area show different shapes, but their curves are continuous, implying continuous multifractality observed for LPD. The $f(\alpha)$ functions are generally asymmetric convex curves, which indicates that the LPD was processed by a different local superposition (Cheng & Agterberg, 1996). Moreover, the results obtained from LPD in 2001 show the spectrum curve with asymmetry shorter toward the left but much longer toward the right. This difference indicates that the variation of singularities of land price in the low-value area is bigger than that in high-value area. However, the curve in 2004 is comparatively symmetric with a slight right deviation, indicating that there is no significant difference between singularities in the high or low values. In 2007, where the curve is shorter toward the right but much longer toward the left, indicating that the land price singularity in the higher-value area was larger than in the lower-value area, which is contrary to the circumstances in 2001. Similar results were found in the first and second rings, as shown in Fig. 5a and b, respectively.

The multifractal spectrum curves of different ring areas in the same year show similar tendencies but different patterns across different years. To more clearly compare the differences of multifractal spectra in time, Fig. 6a–c show the multifractal spectrum curves of the LPD from 2001 to 2007 in the three ring areas, which show the similar tendencies within different ring areas within each year but not a pattern because they have some singularities. For example, in 2001, the curve is generally symmetric in the first ring area, but asymmetry in the second and third ring areas, indicating that the land price singularity in the downtown area was different than in the other urban areas. Moreover, the

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**Table 1**


<table>
<thead>
<tr>
<th>Time</th>
<th>$D_{q=1.5}$</th>
<th>$D_{q=3.5}$</th>
<th>$D_0$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$\Delta(D_0 - D_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>2.117</td>
<td>1.969</td>
<td>2</td>
<td>1.991</td>
<td>1.984</td>
<td>0.016</td>
</tr>
<tr>
<td>2004</td>
<td>2.036</td>
<td>1.964</td>
<td>2</td>
<td>1.994</td>
<td>1.986</td>
<td>0.014</td>
</tr>
<tr>
<td>2007</td>
<td>2.021</td>
<td>1.866</td>
<td>2</td>
<td>1.990</td>
<td>1.970</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Where $D_{q=3.5}$ is the maximum of $D$ within the range of $q$; $D_0$ is the minimum of $D$ within the range of $q$; $D_0$ is box-counting dimension; $D_1$ is information dimension; $D_2$ is correlation dimension.

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Fig. 4. The $D_{q-q}$ curves in study area, showing nonlinear relationships between $q$ and $D_{q}$ in 2001, 2004 and 2007.

Fig. 5. Changes in the multifractal spectrum curves of $f(\alpha)-\alpha$ in three ring regions; (a) first ring area, (b) second ring area, and (c) third ring area.
horizontal span ($\Delta a$) of the curves in Table 2 also differs at different times, which means that the variation of land price varied in the same area at different times. For example, in 2001, the magnitude of $\Delta a$ and $\Delta a_R$ ranged from large to small was in the second, third and first ring areas, respectively. In 2004, was in the order of first, third and second ring areas. In 2007, was in the order third, first, and second ring areas. This implies that the variation of singularities of land price in the different ring area keeps step with urban development.

**Multifractality spectra of LPD in towns**

As an alternative to the above analysis using natural geographic boundaries, we here use administrative boundaries to characterize the multifractality of regional differences in detail at a different scale. We thus calculated the multifractal spectra of the land price based mainly on seven towns in Wuhan City, as shown in Figs. 7 and 8. The results also show that (1) there were continuous multifractality from 2001 to 2007, and the $f(a)$ functions are generally asymmetric convex curves each year; and (2) there were different degrees of horizontal span ($\Delta a$) in Table 2. For example, in 2001, the $\Delta a$ for Han Yang town is the smallest, indicating that the least significant singularity of LPD was in Han Yang; however, for the towns of Jiang An, Jiang Han, Qiao Kou, and Qing Shan, the $\Delta a$ are comparatively larger, some of the difference may stem from the fact Jiang An, Jiang Han, and Qiao Kou are the more developed towns in Wuhan City, and Moreover, in 2007, the $\Delta a$ values are higher in Wu Chang, Qing Shan and Hong Shan towns, which may rely deeply on the subcenters of the cities of Zhong Nan, Xu Dong and Guang Gu, respectively.

Thus, we determined that the general patterns of multifractal spectra are shorter toward the left but much longer toward the right in 2001, comparatively symmetric with a slight right deviation in 2004, and shorter toward the right but much longer toward the left in 2007, results similar to those from the analysis of the three ring areas. This similarity demonstrates that the development trends for LPD in the different towns changed over time from low value with significant singularity to a balance between high value and low value and then to high value with significant singularity. These trends match the increasing speed of land price and urban development orientations in those years in Wuhan City. In addition, there remain somewhat different patterns in special regions, for example, in Han Yang, the multifractal spectra are symmetric in 2001 and 2007, but the spectrum is slightly right deviation in 2004.

**Table 2**

Multifractal analysis parameters for land price in different areas and time.

<table>
<thead>
<tr>
<th>Region</th>
<th>Time (year)</th>
<th>$a_{q=3.5}$</th>
<th>$a_{q=-3.5}$</th>
<th>$\Delta a$</th>
<th>$\Delta a_R$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Ring</td>
<td>2001</td>
<td>1.8160</td>
<td>2.2400</td>
<td>0.4240</td>
<td>0.2222</td>
<td>0.2019</td>
</tr>
<tr>
<td></td>
<td>2004</td>
<td>1.9312</td>
<td>2.7951</td>
<td>0.3411</td>
<td>0.2703</td>
<td>0.0709</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>1.7200</td>
<td>2.1058</td>
<td>0.3857</td>
<td>0.0919</td>
<td>0.2938</td>
</tr>
<tr>
<td>Second Ring</td>
<td>2001</td>
<td>1.9414</td>
<td>2.4260</td>
<td>0.4846</td>
<td>0.4163</td>
<td>0.0684</td>
</tr>
<tr>
<td></td>
<td>2004</td>
<td>1.9266</td>
<td>2.4963</td>
<td>0.5696</td>
<td>0.4814</td>
<td>0.0883</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>1.8160</td>
<td>2.2400</td>
<td>0.4240</td>
<td>0.2222</td>
<td>0.2019</td>
</tr>
<tr>
<td>Third Ring</td>
<td>2001</td>
<td>1.9414</td>
<td>2.4260</td>
<td>0.4846</td>
<td>0.4163</td>
<td>0.0684</td>
</tr>
<tr>
<td></td>
<td>2004</td>
<td>1.9175</td>
<td>2.1357</td>
<td>0.2182</td>
<td>0.1301</td>
<td>0.0883</td>
</tr>
<tr>
<td></td>
<td>2007</td>
<td>1.6278</td>
<td>2.0639</td>
<td>0.4361</td>
<td>0.3786</td>
<td>0.0684</td>
</tr>
<tr>
<td>Han Yang</td>
<td>2001</td>
<td>1.9706</td>
<td>2.0651</td>
<td>0.0945</td>
<td>0.0334</td>
<td>0.0611</td>
</tr>
<tr>
<td></td>
<td>2004</td>
<td>1.9550</td>
<td>2.2051</td>
<td>0.2502</td>
<td>0.0506</td>
<td>0.1996</td>
</tr>
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Where $a_{q=3.5}$ represents minimum value of $a$ in the range of $q$; $a_{q=-3.5}$ represents maximum value of $a$ in the range of $q$; $\Delta a = a_{q=3.5} - a_{q=-3.5}$, $\Delta a_R = a_{0} - a_{q=3.5}$, which is the width of right part of the multifractal spectrum curve; $a_0$ represents the value of $a$ when $q = 3$, $\Delta a_R = a_0 - a_{q=3.5}$, which is the width of left part of the multifractal spectrum curve. $\Delta R = a_0 - a_{q=3.5}$, which is the width of right part of the multifractal spectrum curve; $R = (\Delta a_L + \Delta a_R)/(\Delta a_L + \Delta a_R)$.

**Fig. 6.** Differences in the multifractal spectrum curves of $f(a)\sim a$ in the three ring regions: (a) 2001, (b) 2004, and (c) 2007.

**Fig. 7.** Differences in the multifractal spectrum curves of $f(a)\sim a$ of seven towns in 2001, 2004 and 2007.
This is due to its special regional background and geographical location, i.e., because Han Yang is traditionally industrial area; most areas are divided by the Yangtze River and the Han River, so as to develop slowly and in a balanced way. As described above, we found that the multifractal spectra cannot only effectively characterize the LPD patterns but also indirectly show the urban development differences among regions.

**Discussion**

Multifractal spectra are graphs showing a pattern behaves if amplified in certain ways (Mendoza et al., 2010). In this work, we showed that multifractal spectra of LPDs from different regions and times, which indicate urban residential land prices, are multifractally distributed and that the structure and patterns of the LPDs changed with land use processes over time.

The generalized dimensions $D_q$ were obtained using the method of moments for the multiscale analysis of the LPD. The generalized dimensional spectra illustrate that the LPD has multifractal characteristics. It was possible to quantitatively determine their scaling properties and microstructure differences in relation to their LPD spectra. These results provide a picture of how the land price structure of a region can vary in cities and towns. The LPD analysis using $D_q$ parameters suggested that the LPDs of each region and time are multifractally distributed and that the structure and patterns of the LPDs changed with land use processes over time.

The different multifractal spectra are not simple to implement or explain. In this work, the success in obtaining differentiated multifractal spectra and correlations among different regions and times can be mainly attributed to three factors. First, we collected land price samples from a representative metropolitan city (Wuhan) which has a typical landscape and policy background as described above and calculated the price of residential land samples based on uniform definitions (Department of Land Resources, 2002), which is a precondition for comparing multifractality in different regions and times. Secondly, LPDs were interpolated from many samples with a high spatial resolution (100 m × 100 m pixels) using the multifractal Inverse Distance Weighted method, which was developed and successfully applied in the geochemical field. In particular, we took advantage of the MIDW, which has been demonstrated to generate

![Fig. 8. Changes in the multifractal spectrum curves of $f_\alpha(\alpha)$ in seven towns.](image-url)
a suitable spatial distribution for values having similarity and self-similarity (Cheng, 1999; Hu, 2009, 134 pp.; Xie et al., 2010). Third, since the study area included various irregular boundaries, it may create bias in the multifractal spectra when the method of moments was used (Agterberg, Cheng, Brown, & Good, 1996), therefore, we did a number of experiments to compare the multifractal spectra with different size of cells, and then choose a size of 100 × 100 m which yields relatively stable results. Edge effect correction were also considered using the approach by Agterberg et al. (1996).

However, it is important to mention that the structure of land price exhibits great complexity (Colwell & Munneke, 1997). Land price cannot be regarded as a homogenous distribution; the differences in landscape, urban economic, special landmarks and natural environment strongly contribute to the anisotropy and heterogeneity of an LPD (Parrinello & Vaughan, 2002). The LPD has also a critical influence on the trends of changing land use (Lv et al., 2007), and its quantification is important for modeling land use change in terms of three dimensions. We found that multifractal spectra from different regions and times differ in $D_q$, right deviation and left deviation, and therefore, the LPD displayed multifractal patterns at different times but had the same tendencies at a given time in different regions, showing the characteristics of asymmetric and convex curves. The general trend was from right deviation in 2001 to left deviation in 2007, which indicates that the singularity of land price in the low-value marginal area increased gradually, whereas that of the high-value downtown area decreased. In addition, the horizontal characteristic of the curves differed at different development stages in the city.

Finally, it is important to note that urban development and land use researchers currently require novel experimental tools and analytic methods based on physical or mathematical models to describe the LPD in a quantitative way (Mendoza et al., 2010). Although a number of articles have been published on the fractal method, including those in which the moment method was applied to land use distribution issues (Gupta & Waymire, 1989; Mandelbrot, 1983, 468 pp.; Thielen, San José, Montes, & Lairnet, 2008; Verburg & Chen, 2000; Wang, Zhong, Liu, & Li, 2008), the multifractal analysis proposed in this study is a step forward, as it presents a better method to describe the LPD. All of the results of this study strongly demonstrate that multifractal analysis has significant benefits for the quantitative analysis of the complex structure of land price. This multiscale procedure and complex spatial structural analysis method should provide valuable opportunities for further research in other distributions not only in physical science but also socio-economic fields, especially for analyzing factors that are distributed complexly and singularly (Cheng, Xu, & Grunsky, 1999). This method may help to improve our understanding of how these factors affect land use processes, for example, when determining whether land price or rent changes caused a large and famous factory to move from downtown to the suburban area.

Conclusions

Our study supplies positive proof that there exists a multifractal scaling law in the LPD. Multifractal parameters were found to demonstrate the major aspects of variability in the regional scale of LPD and provided a unique quantitative characterization of the spatial distribution of the data. The irregularity and complexity of land price structure, together with its scale-invariant properties, suggest that a multifractal model is suitable for urban LPD. In this work, the generalized dimensions of the LPD were computed by the box-counting moment method using data obtained from the interpolation results by the MIDW method for samples in the years 2001, 2004 and 2007. The multifractal spectrum can be advantageously used to describe the scaling properties of the heterogeneity of the LPD. The variation of $D_q$ with respect to $q$ and the shape of the generalized dimension spectrum revealed that the LPD structure has properties similar to multifractal self-similarity measures, and the multifractal spectrum $f(\alpha)$ successfully characterized the LPD and the heterogeneity of land price structure based on the boundaries of towns and ring areas in the three years. These results show that multifractal analysis is an appropriate tool for characterizing the heterogeneity of land price structure; the multifractal spectra are capable of characterizing spatial and temporal differences among different land price distribution patterns. As the distribution patterns of multifractal spectrum well fit the processes of land use change, the use of multifractal models is expected to be a viable approach for modeling land use processes and improving urban planning in the future.

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