PARALLEL IMAGING: SOME SIGNAL PROCESSING ISSUES AND SOLUTIONS

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ABSTRACT
Parallel imaging using multiple receiver coils has emerged as an effective tool to reduce imaging time in various MRI applications. Mathematically, the imaging equation can be expressed as a weighted Fourier transform, and the image reconstruction formula can be derived from Papoulis’ generalized sampling theorem. Although perfect reconstructions can be obtained under ideal conditions, several signal processing problems exist in practical settings. This paper discusses some of these problems. Specifically, it analyzes the effect of data truncation, addresses the problem of estimating the coil sensitivity functions, and proposes a regularization scheme to cope with the ill-conditioned inverse problem associated with achieving high acceleration factors.

1. INTRODUCTION
The idea of using multiple receiver coils to improve imaging speed in MRI dates back to the late 80’s and early 90’s [1–4]. However, practical methods for parallel imaging using multiple receiver coils emerged only recently with the development of the SMASH (Simultaneous Acquisition of Spatial Harmonics) technique [5], and the SENSE (SENsitivity Encoding) technique [6]. Since then, many variations of SMASH and SENSE have been proposed, including PILS [7], SPACE-RIP [8], K-SENSE [9], spiral SENSE [10], and regularized SENSE [11, 12], which incorporate different coil sensitivity assumptions and k-space sampling patterns.

The k-space data acquired using an array of L receiver coils with sensitivity \( S_\ell(\vec{r}) \) for \( 1 \leq \ell \leq L \) can be expressed in general as

\[
d_\ell(\vec{k}_m) = \int \rho(\vec{r}) S_\ell(\vec{r}) e^{-i2\pi \vec{k}_m \cdot \vec{r}} d\vec{r} \tag{1}
\]

where \( \rho(\vec{r}) \) is the desired image function and \( d_\ell(\vec{k}_m) \) is the data measured at k-space locations \( \vec{k}_m \) by the \( \ell \)th coil. The term parallel imaging comes from the fact that \( d_\ell(\vec{k}_m) \) are acquired simultaneously for \( 1 \leq \ell \leq L \), and sensitivity encoding refers to the spatial encoding effect of \( S_\ell(\vec{r}) \) in Eq. (1). For simplicity, we consider only Cartesian sampling in this paper, and because of the separability of the Fourier transform we can further reduce the problem to a 1D problem (e.g., along the phase encoding direction only). As a result, we can rewrite the imaging equation as

\[
d_\ell(n \Delta \hat{k}) = \int_{-W/2}^{W/2} \rho(x) S_\ell(x) e^{-i\omega m \Delta \hat{k}_x} dx \tag{2}
\]

where \( \rho(x) \) is assumed to be supported on \( |x| < W/2 \), and \( \Delta \hat{k} \) is often chosen to be \( R \Delta \hat{k} = R/W \), with \( R \) being the data acquisition acceleration factor. Clearly, the Nyquist sampling criterion is satisfied when \( R = 1 \); otherwise, the k-space signal \( d_\ell(n \Delta \hat{k}) \) is undersampled by a factor of \( R \). When there is no data truncation, the Fourier image of the \( \ell \)th channel is given by

\[
\hat{\rho}_\ell(x) = \sum_{m=0}^{R-1} \rho(x - m \hat{W}) S_\ell(x - m \hat{W}), \tag{3}
\]

where \( \ell = 1, 2, \cdots, L, \hat{W} = W/R \) and \( W/2 - \hat{W} < x < W/2 \). Assuming that \( R \leq L \), we can solve for \( \rho(x) \) pixel by pixel from the above equations. More specifically, rewriting Eq. (3) in matrix form

\[
S \hat{\rho} = \tilde{\rho}
\]

where

\[
S = \begin{bmatrix}
S_1(x) & S_1(x - \hat{W}) & \cdots & S_1(x - (R-1)\hat{W}) \\
S_2(x) & S_2(x - \hat{W}) & \cdots & S_2(x - (R-1)\hat{W}) \\
\vdots & \vdots & \ddots & \vdots \\
S_L(x) & S_L(x - \hat{W}) & \cdots & S_L(x - (R-1)\hat{W})
\end{bmatrix}
\]

\[
\tilde{\rho} = \begin{bmatrix}
\hat{\rho}_1(x) \\
\hat{\rho}_2(x) \\
\vdots \\
\hat{\rho}_L(x)
\end{bmatrix}, \quad \text{and} \quad \hat{\rho} = \begin{bmatrix}
\rho(x) \\
\rho(x - \hat{W}) \\
\vdots \\
\rho(x - (R-1)\hat{W})
\end{bmatrix}
\]
Equation (4) is known as the SENSE reconstruction formula [6], which is a direct result of Papoulis’ generalized sampling theorem [13], although such a connection has not been made in the parallel MRI literature. Clearly, perfect reconstruction of $\rho(x)$ requires: (a) precise knowledge of $S(\ell)$ to form $S$, (b) $S$ is non-singular for $W/2 - \tilde{W} < x < W/2$, and (c) $\rho(x)$ is noiseless and not corrupted by the data truncation artifact. When these conditions are not met in practice, reconstruction errors result. These issues are discussed next.

2. TRUNCATION EFFECTS

To analyze the truncation effects in SENSE reconstruction, we use as a reference the Fourier reconstruction of $\rho(x)$ from a single, uniform channel with $N$ points taken at the Nyquist rate. Such a reconstruction can be expressed as

$$\hat{\rho}(x) = \rho(x) \ast h(x)$$

where $h(x) = \Delta k \sum_{m=0}^{N-1} \rho(x - m\Delta W)S(\ell - m\Delta W)$, and $\ast$ denotes the periodic convolution operator. When $M$ data points are taken for each channel at $\Delta k$ with $M = N/R$ and $\Delta k = R\Delta k$, it can be shown, based on Eq. (3), that

$$\hat{\rho}_T(x) = \sum_{m=0}^{M-1} \rho(x - m\Delta W)S_T(x - m\Delta W) \ast \hat{h}(x)$$

where $\hat{h}(x) = \Delta k \sum_{m=0}^{M-1} \rho(x - m\Delta W)S(\ell - m\Delta W)$.

Equation (7) suggests that the SENSE reconstruction with truncation effects can be expressed as

$$\hat{\rho}_T(x) \approx \rho(x) \ast \hat{h}(x) \approx \rho(x) \ast h(x) = \hat{\rho}(x)$$

Therefore, when $S(\ell)$ is a smooth function relative to $h(x)$ and $S$ is non-singular, the truncation effect in SENSE is approximately equal to that in a single, uniform channel (Figs. 1a and c). Some additional points are worth noting: (a) As $N$ increases, the effective width of $h(x)$ decreases and $S(\ell)$ appears smoother with respect to $h(x)$. Therefore, $\hat{\rho}_T(x)$ is closer to $\hat{\rho}(x)$; (b) Any discontinuities on the image borders (resulting from periodic extension by the Fourier series) can be enhanced after being weighted by $S_T(x)$. Consequently, the Gibbs ringing from these discontinuities will be aliased back, creating “new” artifacts in $\rho(x)$, which may not be present in $\hat{\rho}(x)$ (Figs. 1c and d); (c) If $S(\ell)$ has zero-order discontinuities, additional Gibbs ringing will be introduced by $S_T(x)$. But this is not a concern in practice because $S_T(x)$ is always a smooth function; and (d) When $S$ is not known precisely, aliased Gibbs ringing may also be present in $\hat{\rho}_T(x)$.

3. IMPROVED SENSE RECONSTRUCTION

In addition to data truncation errors, both uncertainties about $S(\ell)$ and data noise introduce errors in $S$ and $\hat{\rho}$. These errors can lead to significant reconstruction artifacts when $S$ is ill-conditioned ($\|S\|\|S^+\| \gg 1$).

3.1. Improved estimation of $S(\ell)$

The $S(\ell)$ are often obtained from a “reference” scan. The reference image obtained by each coil is divided by that obtained with a whole-body coil (if available) or by the “sum-of-squares” of all the references to yield the estimated sensitivity map $\hat{S}(\ell)$. To reduce noise, $\hat{S}(\ell)$ is often fitted with a polynomial model [6]. Recently, a variational method has also been proposed for coil sensitivity estimation [14]. Here we propose to use wavelet denoising, which is computationally fast, and near optimal in a statistical minimax sense [15, 16]. Specifically, we first filter $\hat{S}(\ell)$ with a median filter to remove any noise spikes (resulting from division), and then transform it to the wavelet domain where wavelet shrinkage [16] is applied for noise removal. More specifically, let $\mathcal{M}(\cdot)$ denote a median filter, $\mathcal{W}(\cdot)$ and $\mathcal{W}^{-1}(\cdot)$ denote, respectively, the forward and backward wavelet transforms, and $\mathcal{T}_\delta(\cdot)$ denote the shrinking operator with thresholding.
old $\theta$, i.e., $T_\ell(x) = \text{sign}(x) \max(0, |x| - \theta)$. Selection of the threshold $\theta$, has been well investigated in the literature. We have found that the universal threshold $[17]$ is rather effective for our application, although more advanced techniques $[18, 19]$ can also be used. The denoised $S_\ell(x)$ is expressed as

$$
\hat{S}_\ell = W^{-1}(T_\ell(W(M(\hat{S}_\ell))))
$$

(9)

### 3.2. Tikhonov Regularization

Tikhonov regularization is perhaps the most common regularization scheme. To solve Eq. (4) with Tikhonov regularization, we form a weighted sum of the data misfit term $\| S \hat{\rho} - \hat{d} \|^2$ and a regularization term $\| W (\hat{\rho} - \hat{\rho}_r) \|^2$ using a weighting factor $\lambda^2$, and find the solution $\hat{\rho}^*$ that minimizes this sum, i.e.,

$$
\hat{\rho}^* = \arg \min \left\{ \| S \hat{\rho} - \hat{d} \|^2 + \lambda^2 \| H (\hat{\rho} - \hat{\rho}_r) \|^2 \right\}
$$

(10)

where $\lambda$ is often referred to as the regularization parameter and $\hat{\rho}_r$ is a regularization image. A closed-form solution for $\hat{\rho}^*$ exists for $L_2$-norm and is given by

$$
\hat{\rho}^* = \hat{\rho}_r + (S^H S + \lambda^2 W^H W)^{-1} S^H (\hat{d} - S \hat{\rho}_r)
$$

(11)

A straightforward way to generate $\hat{\rho}_r$ is to use the conventional SENSE algorithm to obtain an initial reconstruction $\hat{\rho}^*$, which is then filtered by a median filter to suppress any residual aliasing artifacts $\hat{\rho}$. A more elaborate way is to use a generalized series model $[20]$ to derive $\hat{\rho}_r$. Specifically, the method collects several (say, 8) additional encodings at the Nyquist rate in the center of $k$-space, from which $\hat{\rho}_r$ is obtained using the GS model $[20]$ whose basis functions are formed from the reference data (collected for estimating $S_\ell$). This method is particularly suitable for dynamic imaging applications, where the GS model can often generate high-quality images with a small number of encodings $[20]$. Selecting good $\lambda$ values is important for the regularization scheme. We first set $\lambda_{\text{min}} \leq \lambda \leq \lambda_{\text{max}}$. The lower bound $\lambda_{\text{min}}$ is determined by the minimal requirement to avoid $S$ being overly ill-conditioned (e.g., $\lambda_{\text{min}} = 10^{-4} \sigma_M$, with $\sigma_M$ being the maximum eigenvalue of $S$). The upper bound $\lambda_{\text{max}}$ is set manually (e.g., $\lambda_{\text{max}} = 2$) or using the L-curve method. Within these bounds, $\lambda$ is set point-by-point for $W/2 - \hat{W} < x < W/2$. To do so, we first generate a synthetic data set $\tilde{d}_\ell(n \Delta k)$ based on $\hat{\rho}_r$ and $S_\ell$. An error signal: $e_\ell(n \Delta k) = d_\ell(n \Delta k) - \tilde{d}_\ell(n \Delta k)$, is then calculated for each coil, from which an aliased error image $e_\ell(x)$ is obtained after Fourier transformation. An important observation/argument is that $e_\ell(x)$ has large values at the locations where $\hat{\rho}$ contains significant aliasing errors. We can therefore set $\lambda$ based on $e_\ell(x)$. To improve the signal-to-noise ratio for $e_\ell(x)$, we average the $L$ error maps: $e(x) = \frac{1}{L} \sum_{\ell=1}^{L} e_\ell(x)$. We then rescale $|e(x)|$ to $(\lambda_{\text{min}}, \lambda_{\text{max}})$, which provides the desired value for $\lambda$ at each location.

Figure 2 shows a set of reconstructions from real experimental data acquired with 3 receiver coils and $R = 2$. As can be seen from Fig. 2a, the basic SENSE reconstruction has noticeable residual aliasing artifacts and a loss of signal-to-noise ratio. Both problems were significantly alleviated by the proposed method (Fig. 2b), where wavelet denoising helps improve the signal-to-noise and Tikhonov regularization suppresses the aliasing artifacts. Another example is shown in Fig. 3, where the data were collected using three coils in a dynamic contrast-enhanced MRI experiment. In addition to the usual SENSE data, 8 encodings were collected at the Nyquist rate in central $k$-space for each data frame, from which $\hat{\rho}_r$ was derived using the GS model. As can be seen, the SENSE reconstruction using the proposed algorithm (Fig. 3b) is significantly better than that from the standard SENSE algorithm (Fig. 3a).

![Fig. 2. SENSE reconstructions from a real data set acquired with 3 coils and $R = 2$. (a) basic SENSE image reconstructed with $W = I$ and $\hat{\rho}_r = 0$, and (b) improved SENSE image reconstructed using the proposed method.](image)

### 4. CONCLUSION

This paper discusses some signal processing issues in parallel MRI using multiple receiver coils in the presence of non-ideal conditions. It provides a systematic analysis of the truncation effects, and proposes an improved algorithm for image reconstruction. The paper should provide some useful insight into interpreting as well as improving SENSE images, especially when larger acceleration factors are used.

### 5. ACKNOWLEDGMENTS

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Fig. 3. Dynamic images of a chest tumor at two time points after injection of a contrast agent: (a) SENSE reconstruction ($R = 3$, $L = 3$), and (b) improved SENSE reconstruction by the proposed method.

6. REFERENCES


