KerNL: Parallel imaging reconstruction using Kernel-based NonLinear method

Jingyuan Lyu¹, Yihang Zhou¹, Ukash Nakarmi¹, Chao Shi¹, and Leslie Ying^{1,2}

¹Department of Electrical Engineering, State University of New York at Buffalo, Buffalo, NY, United States, ²Department of Biomedical Engineering, State University of New York at Buffalo, Buffalo, NY, United States

Target audience: Scientists and clinicians interested in new and improved parallel imaging techniques

Purpose: Parallel imaging (PI) has been used routinely for many clinical MR applications. The conventional calibration-based parallel imaging method (e.g., GRAPPA [1] and SPIRiT [2]) assumes a linear relationship between the acquired undersampled k-space data and the unacquired missing k-space data, where the linear coefficients are estimated using some auto-calibration signal (ACS). Such a linear model is valid in the ideal case but not in practice because both the ACS data and the undersampled data contain measurement noise. As a result, the model error leads to poor reconstruction at high accelerations. NL-GRAPPA [3] has used a truncated 2nd-order polynomial model to describe the nonlinear relationship between the missing and acquired k-space data and shown improved reconstruction quality. In this

work, a much more general nonlinear framework is proposed for auto-calibrated parallel imaging. In this framework, kernel tricks are employed to represent the general nonlinear relationship between acquired and unacquired k-space data without increasing the computational complexity. Identification of the nonlinear relationship is still performed by solving linear equations. We name the proposed method Kernel-based NonLinear (KerNL) reconstruction method. Experimental results demonstrate that the proposed method can achieve reconstruction quality superior to GRAPPA and NL-GRAPPA at high net reduction factors.

Theory and Methods: In k-space-based parallel imaging methods [1,2], we need to find the relationship between each unacquired k-space data point y and its neighboring acquired data points \mathbf{x} from all coils for reconstruction. The relationship is usually estimated using some additionally acquired auto-calibration data (ACS). Conventional methods assume a linear relationship between y and x: $y=x^{T}b$, where b is estimated through calibration using the ACS. In this work, we propose a general nonlinear relationship $f(\bullet)$ between **x** and y such that $y = f(\mathbf{x})$. With all the \mathbf{x}_i and y_i pairs obtained from the auto-calibration data, finding the nonlinear relationship can be formulated as finding a function $f(\bullet)$ such that $\sum_i (y_i - f(\mathbf{x}_i))^2 + \lambda ||f||_{\mathrm{H}}$ (1) is minimized, where $\|\bullet\|_{H}$ defines a norm in the Hilbert space. According to the Representer's theorem [4], the minimizer $f(\bullet)$ of (1) always takes the form of $f(\mathbf{x}) =$ $\sum_{i=1}^{m} \alpha_i k(\mathbf{x}_i, \mathbf{x})$ (2), where $k(\cdot, \cdot)$ is a positive definite kernel function. The significance of the theorem is that although we are searching for functions in an infinitedimensional Hilbert space, it states that the solution lies in the span of m particular kernels – those centered on the calibration data points x_i . Given the ACS, the coefficients $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_m]^T$ in (2) can be found analytically by $\boldsymbol{\alpha} = (\mathbf{K} + \lambda \mathbf{I})^{-1} y$ (3), where the kernel matrix **K** is calculated by $\mathbf{K}_{i,j} = k(\mathbf{x}_i, \mathbf{x}_j)$ from the calibration data (as illustrated in Fig. 1), I is the identity matrix, and λ is the regularization parameter. After we obtain the coefficient α , the missing k-space data is reconstructed by $y = \sum_{i=1}^{m} \alpha_i k(\mathbf{x}_i, \mathbf{x})$, which is a nonlinear function of **x**. Although many different kernel functions (e.g., Gaussian) are applicable, here we use a polynomial kernel $k(\mathbf{x}_i, \mathbf{x}_i) = (\mathbf{x}_i^H \mathbf{x}_i + a)^2$ (4), where *a* is a constant, and $(\bullet)^H$ denotes the Hermitian transpose. Here we choose a to be equal to the maximum of $\mathbf{x}_i^H \mathbf{x}_j$ for all i and j. To speed up calculation, random projection [5] is used in solving Eq. (3).

Results: To evaluate the performance of the proposed method, a set of brain data was acquired from a multiple sclerosis patient on a GE 3T scanner (GE Healthcare, Waukesha, WI) with an 8-channel head coil. The dataset was an axial brain image acquired using a 2D spin echo sequence (TE/TR=11/700 ms, matrix size=256×256, FOV=220×220 mm²). The data were fully acquired and then retrospectively undersampled to simulate the partial acquisition. The proposed KerNL method, GRAPPA and NL-GRAPPA were used to reconstruct the image. All code were written in MATLAB and run on a PC with 3.4GHz CPU and 16GB memory. Fig. 2 compares the





Fig. 2. Comparison of proposed method with GRAPPA and NL-GRAPPA

reconstructed images from GRAPPA, NL-GRAPPA, and KerNL. Experimental results demonstrate that, compared with GRAPPA and NL-GRAPPA, KerNL achieves lower NMSEs and better image quality for all net reduction factors. The proposed KerNL method is also more efficient computationally.

Discussion: NL-GRAPPA has shown to significantly improve the SNR over GRAPPA. However, it might fail when too few ACS are acquired to solve for the increased number of unknown weights. As a new nonlinear method, the proposed KerNL addresses this issue by introducing kernel functions. As seen in Fig. 2, when NL-GRAPPA fails at very few ACS (16 lines), the proposed method is still superior to GRAPPA.

Conclusion: We have proposed a nonlinear, non-iterative approach to parallel imaging reconstruction, named KerNL. The method is able to improve both image quality and computation efficiency at high reduction factors, compared with GRAPPA and NL-GRAPPA.

References: [1] Griswold MA, et al., MRM, 47(6):1202-1210, 2012. [2] Lustig M, et al., MRM, 64(2):457-471, 2010. [3] Chang Y, et al., MRM, 68(3): 730-740, 2012. [4] Kimeldorf GS, et al., J of Math. Anal. and Appl., 33: 82-95, 1971. [5] Lyu J, et al., MRM, DOI: 10.1002/mrm.25373, 2014.