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GROWTH OPPORTUNITIES AND INVESTMENT DECISIONS: A NEW PERSPECTIVE ON THE COST OF CAPITAL

Kee H. Chung and Kyu H. Kim*

INTRODUCTION

Recently studies have suggested that growth opportunities possessed by a firm can be regarded as real options and applied contingent-claims analysis to evaluate them in conjunction with the firm's operating environment (see Stulz, 1982; Baldwin, Mason and Ruback, 1983; Kester, 1984 and 1986; Brennan and Schwartz, 1985; Mason and Merton, 1985; Majd and Pindyck, 1987; Pindyck, 1988; and Chung and Charoenwong, 1991; among others). Empirical results suggest that a significant portion of the market value of firms is accounted for by growth opportunities. For instance, Kester (1984 and 1986) finds that the value of growth opportunities frequently accounts for more than 50 percent of the market value of firms. Furthermore, Kester finds that the fraction is about 70 to 80 percent in industries with high demand volatility. More recently, Pindyck (1988) argues that the fraction of market value attributable to future growth options may be one-half or more for firms with reasonable demand volatility.

Although these studies have provided a significant insight into the nature of growth opportunities and their relative importance as a component of corporate value, none of these studies has examined the implications of an options interpretation of growth opportunities for the firm's cost of capital and investment decisions. The purpose of this paper is to closely examine the implications of this new insight (i.e., the options interpretation of growth opportunities) for the hurdle rate for the firm's capital budgeting analysis.

Conventional financial theory has suggested that one should make an upward adjustment to the stock beta in calculating the hurdle rate for capital budgeting when the project under consideration is riskier than existing assets. Theory also advises managers to use capitalization rates calculated from stock betas as long as the riskiness of projects is the same as that of existing assets. The results of our analysis suggest, however, that these traditional

* The authors are respectively, Professor of Finance at Fogelman College of Business and Economics, the University of Memphis; and Professor of International Business and Finance at the College of Social Science, Chung-Ang University, Seoul, Korea. They are grateful to the anonymous referee for useful comments. (Paper received June 1995, accepted September 1996)

Address for correspondence: Kee H. Chung, Department of Finance, Insurance and Real Estate, The Fogelman College of Business and Economics, The University of Memphis, Memphis, Tennessee 38152, USA.

prescriptions for firms' investment decisions are problematic once the options feature of the firm's growth opportunities is recognized.

Specifically, this study finds that it is not always correct to make an upward adjustment to the stock beta in calculating the hurdle rate for capital budgeting even when the project under consideration is riskier than existing assets. It is also shown that a downward adjustment to the stock beta is necessary when the project has the same risk as existing assets. Furthermore, it is shown that the equilibrium capitalization rate calculated from the firm's stock beta will be an underestimate (overestimate) of the correct hurdle rate when the risk of future assets is greater (smaller) than both the risk of assets in place and that of future capital expenditures. These results are all direct consequences of the insight that the firm has an option to undertake or pass up future investment opportunities as their net present values are revealed.

The rest of the paper is organized as follows. The next section presents a contingent-claim valuation of the firm's growth opportunities and examines the relationships among the stock beta, asset beta, and growth opportunities. The third section examines the implications of the options interpretation of growth opportunities for the hurdle rate for capital budgeting. Numerical illustrations are presented in the fourth section. The final section presents a brief summary and concluding remarks.

GROWTH OPTIONS AND SYSTEMATIC RISK

Let V be the current equilibrium market value of the firm's *equity*. V can then be expressed as the sum of two parts:

$$V = VA + PVGO, \tag{1}$$

where VA is the portion of the market value of equity which is accounted for by assets already in place, that is, the present value of expected net cash flows that existing assets will generate, and $PVGO$ is the value of the firm's growth opportunities.[1] Notice that $PVGO$ is the summation of net present values (NPV) of all future projects, i.e.,

$$PVGO = \sum G_i, \tag{2}$$

where G_i is the NPV of growth opportunity i and Σ denotes the summation over i.

Following previous studies, we employ contingent-claims analysis to evaluate the firm's growth opportunities.[2] Consider that growth opportunity i requires a capital outlay of $I(t)$ at time t which will, in turn, create an asset of value $X(t)$. Here, $X(t)$ can be interpreted as the present value (as of time t) of the stream of uncertain future net cash flows generated from the asset purchased at time t. The present value will, of course, fluctuate stochastically over time, reflecting new information about future cash flows. Following

McDonald and Siegel (1986), we assume that $X(t)$ follows a geometric Brownian motion process of the form:

$$dX(t)/X(t) = (\mu + \delta)dt + \sigma dw, \tag{3}$$

where μ is the instantaneous equilibrium rate of return on a security or dynamic portfolio of assets whose price is perfectly correlated with $X(t)$, $\mu + \delta$ is the instantaneous expected growth rate of $X(t)$, σ is the instantaneous standard deviation of the growth rate of $X(t)$, and dw is a Wiener process.[3] Intuition underlying equation (3) is that the proportional change in asset value (i.e., $dX(t)/X(t)$) during a small interval of time is composed of a non-random drift term (i.e., $(\mu + \delta)dt$) and a random shock driven by the Wiener process (i.e., δdw).[4]

Similarly, due to technological uncertainty, we assume that the required capital expenditure, $I(t)$, follows the process given by:

$$dI(t)/I(t) = (\Omega + \tau)dt + \phi dv, \tag{4}$$

where Ω is the instantaneous equilibrium rate of return on a security or dynamic portfolio of assets whose price is perfectly correlated with $I(t)$, $\Omega + \tau$ is the instantaneous expected growth rate of $I(t)$, ϕ is the instantaneous standard deviation of the growth rate of $I(t)$, and dv is a Wiener process.[5]

For both $X(t)$ and $I(t)$, the geometric Brownian motion assumption is crucial for the derivation of the formulae below. This assumption is reasonable for the project value $X(t)$, but may be less so for the investment cost $I(t)$. The project value in many applications is the market value of an asset; if the project were undertaken and a company owned only this asset, $X(t)$ is the price for which the company's asset will sell. Thus the assumption of geometric Brownian motion for $X(t)$ is as reasonable as assuming that a stock price obeys geometric Brownian motion (a standard assumption in the finance literature). The investment cost $I(t)$ is typically the price of a physical asset and not a present value. McDonald and Siegal (1986) suggest, however, that $I(t)$ can also be interpreted as the present value under certain conditions.

Then the growth opportunity may be viewed as a European call option with a stochastic exercise price $I(t)$ and a terminal cash flow max$[0, X(t) - I(t)]$.[6] Using the solution technique in McDonald and Siegel (1985), it can be shown that the net present value (at time zero) of growth opportunity i, G_i, is expressed as:

$$G_i = X(0)e^{\delta t}\mathcal{N}(d_{1t}) - I(0)e^{\tau t}\mathcal{N}(d_{2t}), \tag{5}$$

where $d_1 t = [\ln\{X(0)/I(0)\} + (\delta - \tau)t]/\Theta\sqrt{t} + (1/2)\Theta\sqrt{t}$,

$d_{2t} = [\ln\{X(0)/I(0)\} + (\delta - \tau)t]/\Theta\sqrt{t} - (1/2)\Theta\sqrt{t}$,

$\mathcal{N}(\;)$ = the cumulative distribution function of a standard normal random variable,

$\Theta^2 = \sigma^2 - 2\sigma\phi r_{12} + \phi^2$, and

r_{12} = the correlation between the Wiener processes dw and dv.

The present value of the firm's growth opportunities, $PVGO$, is defined as the summation of G_i over i, i.e.,

$$PVGO = \sum \{X(0)e^{\delta t}\mathcal{N}(d_{1t}) - I(0)e^{\tau t}\mathcal{N}(d_{2t})\}. \tag{6}$$

The systematic risk of the firm's common stock (β_M) is the weighted average of the systematic risk of assets in place (β_A) and the systematic risk of growth opportunities (β_G):

$$\beta_M = (VA/V)\beta_A + (PVGO/V)\beta_G; \tag{7}$$

with $\beta_A = \mathrm{Cov}(ROE_A, ROE_M)/\mathrm{Var}(ROE_M)$ and $\beta_G = \Sigma\ (G_i/PVGO)\beta_i$, where ROE_A is the instantaneous rate of return on equity generated from assets already in place, ROE_M is the market equivalent of ROE_A, and β_i is the systematic risk of growth opportunity i.[7]

Next, notice that $\beta_i = \mathrm{Cov}(R_i, R_M)/\mathrm{Var}(R_M)$, where R_i and R_M are instantaneous returns on growth opportunity i (i.e., dG_i/G_i) and on the market portfolio, respectively. From McDonald and Siegel (1985), the instantaneous return on growth opportunity i is defined as:

$$R_i = (G_{iX}/G_i)X_iR_{Xi} + (G_{iI}/G_i)I_iR_{Ii}, \tag{8}$$

where G_{iX} and G_{iI} are partial derivatives of G_i with respect to X_i and I_i, $R_{Xi} = dX_i/X_i$, and $R_{Ii} = dI_i/I_i$. Substituting $G_{iX} = e^{\delta t}\mathcal{N}(d_{1t})$ and $G_{iI} = -e^{\tau t}\mathcal{N}(d_{2t})$ into equation (8), we obtain:

$$R_i = \{e^{\delta t}\mathcal{N}(d_{It})/G_i\}X_i(0)R_{Xi} - \{e^{\tau t}\mathcal{N}(d_{2t})/G_i\}I_i(0)R_{Ii}. \tag{9}$$

Substituting equation (9) into the definition of β_i and subsequently substituting β_i into the definition of β_G, we obtain:

$$\beta_G = \sum e^{\delta t}\mathcal{N}(d_{It})\{X_i(0)/PVGO\}\beta_{Xi} - \sum e^{\tau t}\mathcal{N}(d_{2t})\{Ii(0)/PVGO\}\beta_{Ii,} \tag{10}$$

where $\beta_{Xi} = \mathrm{Cov}(R_{Xi}, R_M)/\mathrm{Var}(R_M)$ and $\beta_{Ii} = \mathrm{Cov}(R_{Ii}, R_M)/\mathrm{Var}(R_M)$.

Suppose that the relationship between the risk of future investment opportunities and the risk of existing assets can be described as $\beta_{Xi} = \Phi_1\beta_A$ and $\beta_{Ii} = \Phi_2\beta_A$ for all i. Then equation (10) becomes:

$$\beta_G = \Phi_1\beta_A \sum e^{\delta t}\mathcal{N}(d_{1t})\{X_i(0)/PVGO\} - \Phi_2\beta_A \sum e^{\tau t}\mathcal{N}(d_{2t})\{I_i(0)/PVGO\}. \tag{11}$$

Finally, substituting equation (11) into equation (7), and after simplification, we obtain:

$$\beta_M = \beta_A[1 - \{PVGO - (\Phi_1 PVRG - \Phi_2 PVIG)\}/V], \tag{12}$$

where $PVGO = \Sigma\{e^{\tau t}N(d_{1t})X_i(0) - e^{\tau t}N(d_{2t})I_i(0)\}$, $PVRG = \Sigma e^{\delta t}N(d_{1t})X_i(0)$, and $PVIG = \Sigma e^{\tau t}N(d_{2t})I_i(0)$.

Equation (12) describes the relationship between the risk of equity and characteristics of growth opportunities when both future asset values and capital expenditures are stochastic. The existence of growth opportunities may imply either a higher or lower equity risk for firms with the same assets in place, depending on characteristics of their growth opportunities as represented by Φ_1 and Φ_2.

IMPLICATIONS FOR CAPITAL BUDGETING ANALYSIS

When Capital Expenditures are Nonstochastic

In many practical settings, it may be reasonable to assume that uncertainty associated with capital expenditures is substantially less than that associated with the value of created assets. Hence we begin our analysis by assuming that capital expenditures are not stochastic [i.e., $\Phi_2 = 0$ in equation (12)]. Then, equation (12) becomes:

$$\beta_M = \beta_A[1 - (PVGO1 - \Phi_1 PVRG)/V], \tag{13}$$

where $PVGO1 = \Sigma\,\{e^{\delta t}N(d_{1t})X_i(0) - e^{-rt}N(d_{2t})I(t)\}$, $PVRG = \Sigma\, e^{\delta t}N(d_{1t})X_i(0)$, $d_{1t} = [\ln\{X(0)/I(t)\} + (r+\delta)t]/\sigma\sqrt{t} + (1/2)\sigma\sqrt{t}$, $d_{2t} = [\ln\{X(0)/I(t)\} + (r+\delta)t]/\sigma\sqrt{t} - (1/2)\sigma\sqrt{t}$, and r = the instantaneous risk-free rate.

Proposition 1:
The stock beta will be greater than the beta of new assets when $\Phi_1 < \{1 - (PVGO1/V)\}/\{1 - (PVRG/V)\}$.

Proof: Substituting $\beta_A = (1/\Phi_1)\beta_{Xi}$ into equation (13), the relation between the stock beta and the beta of new assets (i.e., β_{Xi}) can be written as:

$$\beta_M = \beta_{Xi}[(1/\Phi_1) - \{(1/\Phi_1)PVGO1 - PVRG\}/V]. \tag{14}$$

Hence, the stock beta will be greater than the beta of new assets when:

$$[(1/\Phi_1) - \{(1/\Phi_1)PVGO1 - PVRG\}/V] > 1, \text{ or alternatively,}$$

$$\Phi_1 < \{1 - (PVGO1/V)\}/\{1 - (PVRG/V)\}.\ \text{Q.E.D.} \tag{16}$$

Since the value of $\{1 - (PVGO1/V)\}/\{1 - (PVRG/V)\}$ is greater than one, the above result has an important implication for firms' capital budgeting analysis. Conventional wisdom suggests that one should make an *upward* adjustment to the stock beta in calculating the hurdle rate for capital budgeting when the project under consideration is riskier than existing assets (i.e., $\Phi_1 > 1$). It is important to recognize, however, that even when $\Phi_1 > 1$, if $\Phi_1 < \{1 - (PVGO1/V)\}/\{1 - (PVRG/V)\}$, the conventional wisdom fails. This is because the stock beta, for this case, is greater than the beta of new projects (i.e., β_{Xi}) and thus gives an overestimate of the correct hurdle rate, and as a result, contrary to the conventional wisdom, one should make a *downward* adjustment to the stock beta in calculating the hurdle rate. Economic intuition underlying this result is that the beta of the new project before the acceptance/rejection decision is greater than that of the same project when it is accepted since the former is an option written on the latter. Since the stock beta is the weighted average of the beta of assets in place and that of growth opportunities, it is possible that β_M turns out to be greater than β_{Xi} even when $\beta_{Xi} > \beta_A$ if β_G is sufficiently larger than β_{Xi}.

It is important to note that the correct beta to use in capital budgeting analysis is that of the asset *once it has been created* since it is the cash flow stream from the asset in place that is being discounted. For growth firms, the market capitalization rate calculated from the firm's stock beta may overstate the hurdle rate in evaluating new projects since the market-based beta contains the extra volatility of growth opportunities. If the decision being made concerns the creation of a new asset, this extra volatility should not enter the project valuation since it will not be present in the cash flow stream from the asset, *once created*.

Presumably, if it is possible to calculate the beta of the project under consideration it would not be necessary to use the stock beta to calculate the correct hurdle rate. Managers are, however, unlikely to know the beta of the project. Instead, managers are more likely to know whether the project, once undertaken, will be riskier or less risky than existing assets and to use this knowledge in assessing the correct hurdle rate by making a necessary adjustment to the stock beta. Viewed in this context, the primary implication of preceding results is that managers should not always make an upward adjustment to the stock beta in calculating the hurdle rate (which they would do if they ignore the options feature of growth opportunities) when they feel that the project under consideration is riskier than existing assets. This is because the risk of the project, once undertaken, can still be less than the stock beta even when the former is greater than the risk of existing assets.

Equation (13) also suggests that the stock beta is greater than the beta of assets in place even when the firm remains in the same business risk class (i.e., all future assets have the same risk as existing assets). If the firm remains in the same business risk class (i.e., $\beta_{Xi} = \beta_A$ for all i), then $\Phi_1 = 1$, and thus equation (13) is simplified to $\beta_M = \beta_A[1 + (PVIG1/V)]$ or $\beta_M = \beta_{Xi}[1 + (PVIG1/V)]$, where $PVIG1 = \Sigma\, e^{-rt} N(d_{2t}) I(t)$.[8] Since $PVIG1/V > 0$, it follows that $\beta_M > \beta_A$. Hence, even when the firm's future projects have the same risk as existing assets, the risk of stock is greater than that of existing assets. Economic intuition underlying this result is simple. Since call options (i.e., growth opportunities) are riskier than the assets on which they are written,[9] and since the stock beta is the weighted average of the beta of assets in place and the beta of growth opportunities, the stock beta will be greater than the beta of existing assets even when all (i.e., existing and future) assets have the same risk characteristics.[10] Managerial implication of this result is clear: one should make a downward adjustment to the market capitalization rate calculated from the stock beta to obtain the correct hurdle rate even when the project under consideration has the same risk as existing assets.

Notice that if we ignore the options feature of growth opportunities and thus equate the beta of growth opportunities (β_G) to that of future assets (β_{Xi}), the stock beta will be *same* as the beta of assets in place. This is because the stock beta is simply the weighted average of the beta of assets in place and that of future assets, where the latter two are equal. The problem with this approach, however, is that it fails to recognize that the beta of a growth opportunity differs from that of the new asset acquired when the growth opportunity is exercised.

Viewing the firm's growth opportunities as real call options and thereby recognizing its ramification that the stock beta of the firm's common stock is greater than that of assets in place even when the firm's future projects have the same risk as existing assets has important implications for the firm's investment decisions. Managers are frequently advised in finance textbooks (see, e.g., Brealey and Myers, 1988, Ch. 9; and Copeland and Weston, 1988, p. 204) to use capitalization rates calculated from stock betas as long as the riskiness of projects under consideration is the same as that of existing assets. The results of this paper however, show that the advice is problematic.

When Capital Expenditures are Stochastic

This section considers the situation when capital expenditures as well as the value of future assets are stochastic.

Proposition 2:

(i) The stock beta is greater than the beta of assets in place when the risk of future assets is greater than both the risk of assets in place and that of future capital expenditures, i.e., $\beta_{Xi} > \max[\beta_A, \beta_{Ii}]$ for all i.

(ii) The stock beta is smaller than the beta of assets in place when the risk of future assets is smaller than both the risk of assets in place and that of future capital expenditures, i.e., $\beta_{Xi} < \min[\beta_A, \beta_{Ii}]$ for all i.

Proof.

(i) Suppose $\beta_{Xi} > \beta_A > \beta_{Ii}$ for all i, or equivalently, $\Phi_1 > 1 > \Phi_2$. Then, since $\Phi_1 > \Phi_2$, it follows that $\Phi_1 PVRG - \Phi_2 PVIG > PVRG - PVIG = PVGO$. Hence the numerator in the large bracket of equation (12) is negative, and thus $\beta_M > \beta_A$. Suppose now $\beta_{Xi} > \beta_{Ii} > \beta_A$ for all i, or equivalently, $\Phi_1 > \Phi_2 > 1$. Then, since $\Phi_1 > \Phi_2$, it follows that $\Phi_1 PVRG - \Phi_2 PVIG > PVRG - PVIG = PVGO$. Hence the numerator in the large bracket of (12) is negative, and thus $\beta_M > \beta_A$.

(ii) Suppose $\beta_{Xi} < \beta_A < \beta_{Ii}$ for all i, or equivalently, $\Phi_1 < 1 < \Phi_2$. Then, since $\Phi_1 < \Phi_2$, it follows that $\Phi_1 PVRG - \Phi_2 PVIG < PVRG - PVIG = PVGO$. Hence the numerator in the large bracket of equation (12) is positive, and thus $\beta_M < \beta_A$. Suppose now $\beta_{Xi} < \beta_{Ii} < \beta_A$ for all i, or, equivalently, $\Phi_1 < \Phi_2 < 1$. Then, since $\Phi_1 < \Phi_2$, it follows that $\Phi_1 PVRG - \Phi_2 PVIG < PVRG - PVIG = PVGO$. Hence the numerator in the large bracket of equation (12) is positive, and thus $\beta_M < \beta_A$. Q.E.D.

Hence, when the risk of future assets dominates both the risk of assets in place and that of future capital expenditures, the stock beta will be greater than the beta of assets in place. Since $\beta_M > \beta_A$ implies $\beta_G > \beta_M$ (this is because β_M is the weighted average of β_A and β_G), it follows that the equilibrium capitalization rate calculated from the firm's stock beta will be an *underestimate* of the correct hurdle rate for capital budgeting when the risk of future assets is greater than both the risk of assets in place and that of future capital expenditures. Conversely, when the risk of future assets is less than both the risk of assets in place and that of future capital expenditures, the stock beta will be smaller than the beta of assets in place. Hence, for this case, the equilibrium capitalization rate from the firm's stock beta will be an *overestimate* of the correct hurdle rate. In short, when the risk of future assets and that of future capital expenditures are different from that of existing assets, it will be again incorrect to use the equilibrium capitalization rate computed from the stock beta as the hurdle rate for capital budgeting.

Proposition 3:
The stock beta is the same as the beta of assets in place when the risk of future assets and that of future capital expenditures are the same as that of existing assets, i.e., $\beta_{Xi} = \beta_{Ii} = \beta_A$ for all i.

Proof.
If $\beta_{Xi} = \beta_{Ii} = \beta_A$ for all i (i.e., $\Phi_1 = \Phi_2 = 1$), then $PVGO - (\Phi_1 PVRG - \Phi_2 PVIG) = 0$ since $PVRG - PVIG = PVGO$, and thus $\beta_M = \beta_A$. Q.E.D.

In the previous section, the stock beta is shown to be larger than the beta of assets in place even when the firm remains in the same business risk class. The above result suggests, however, that when both future asset values and future capital expenditures are uncertain, the stock beta will be less than that of the case when there exists only asset value uncertainty. The underlying economic intuition of this result is as follows: if the value of future assets covaries positively with the market return, then investment expenditures that also covary positively with the market return will moderate the uncertainty associated with future assets, and thus reduce the overall risk of growth opportunities. In particular, if the uncertainty associated with capital expenditures and that associated with future assets are the same as the risk of assets in place, the risk of growth opportunities will be identical to that of existing assets. Hence, for this case, the market capitalization rate calculated from the firm's stock beta will give the correct hurdle rate for capital budgeting.[11]

NUMERICAL ILLUSTRATION

This section illustrates the results presented in the previous section using numerical examples. For the clarity of illustration, we assume that capital expenditures are known to decision makers at time zero. Only the value of future assets is assumed to be stochastic.

Example 1: Suppose that a firm with current market value of $25,000,000 evaluates a project which has the same risk as the firm and an internal rate of return (IRR) of 12.5%. Suppose also that the systematic risk of the firm's stock is 1.2, the risk-free rate is 4%, and the expected return on the market portfolio is 12%. It is also estimated that the present value of the firm's future capital expenditures (i.e., $PVIG1$) is approximately $10,000,000. Given these data, the textbook approach suggests that the hurdle rate for the proposed project can be calculated from the firm's stock beta since the proposed project has the same risk as the firm's existing assets. Hence, according to the Capital Asset Pricing Model (CAPM), the minimum required rate of return for the project is $4 + (12 - 4)1.2 = 13.6\%$, and the project should be rejected since the required rate of return of the project is greater than the IRR.

Once the options feature of growth opportunities is recognized, however, the above procedure is erroneous since the stock beta overestimates the risk of the proposed project. The correct beta for the proposed project is $\beta_{Xi} = \beta_M/[1 + (PVIG1/V)] = 1.2/[1 + (10{,}000{,}000/25{,}000{,}000)] = 0.86$, and thus the minimum required rate of return for the proposed project is $4 + (12 - 4)(0.86) = 10.88\%$. Since the IRR is greater than the hurdle rate, the correct decision is to accept the project.

Example 2: Now suppose that the same firm evaluates a project which is riskier than existing assets. Specifically, suppose that managers perceive that the new project is 1.5 times riskier than existing assets (i.e., $\Phi_1 = 1.5$). It is estimated that approximately ten percent of the market value of the firm is accounted for by the present value of growth opportunities (i.e., $PVGO1/V = 0.1$ and thus $PVGO1 = \$2{,}500{,}000$ since $V = \$25{,}000{,}000$). Finally, suppose that the systematic risk of the firm's stock is 1.3, the risk-free rate is 4%, the expected return on the market portfolio is 12%, and the project's IRR is 14%.

Conventional wisdom would suggest that the appropriate hurdle rate for this project should at least be greater than the market capitalization rate for the firm's stock (i.e., 14.4%) calculated using the stock beta (i.e., 1.3) since the project is riskier than existing assets. Since the IRR of the project is 14% which is less than the minimum market capitalization rate, the conventional approach suggests that the project should be rejected.

However, since $\Phi_1 < 1.8 = \{1 - (2{,}500{,}000/25{,}000{,}000)\}/\{1 - (12{,}500{,}000/25{,}000{,}000)\}$, the contingent-claims approach suggests that the stock beta gives an overestimate of the correct capitalization rate and thus one should make a *downward* adjustment to the stock beta to obtain the correct capitalization rate for evaluating future investment projects. In fact, the correct beta to use can be calculated from the equation $\beta_{Xi} = \beta_M/[(1/\Phi_1) - \{(1/\Phi_1)PVGO1 - PVRG\}/V] = 1.3/[(1/1.5) - \{(1/1.5)2{,}500{,}000 - 12{,}500{,}000\}/25{,}000{,}000] = 1.18$. Hence the correct minimum required rate of return for the proposed project becomes 4 + (12 - 4)1.18 = 13.44%. Since the IRR is greater than the hurdle rate, the correct decision is to accept the project.

These examples illustrate the point that the traditional capital budgeting procedure, which ignores the options feature of investment projects, could erroneously reject a project when the project is, in fact, a profitable one. This is because the procedure uses an inflated hurdle rate calculated from the stock beta in evaluating a new project.

SUMMARY AND CONCLUDING REMARKS

This study finds that conventional wisdom on risk-adjusted hurdle rates for firms' investment decisions will not be a reliable guide once we recognize the options feature of firm's growth opportunities. Managers are advised in finance textbooks that the market capitalization rate calculated from the firm's stock beta may be used for capital budgeting analysis as long as the firm remains in the same business-risk class. They are also advised that an upward adjustment must be made to the market capitalization rate when the project under consideration is riskier than existing assets. In this study, it is shown that these guidelines suggested by the traditional capital budgeting analysis are erroneous once we view the firm's growth opportunities as real call options.

The results of our analysis suggest the following managerial implications for firms' investment decisions: (i) managers should make a downward adjustment to the market capitalization rate calculated from the stock beta to obtain the correct hurdle rate for capital budgeting when the asset under consideration has the same risk as existing assets; (ii) even when managers feel that the project under consideration is riskier than existing assets, they should not always make an upward adjustment to the stock beta in calculating the hurdle rate; and (iii) when future capital expenditures as well as the value of future assets are stochastic, managers should make an upward (downward) adjustment to the market capitalization rate calculated from the stock beta if the risk of future assets is greater (smaller) than both the risk of existing assets and that of future capital expenditures.

In the real world environment managers are unlikely to have all the information necessary for the calculation of the exact hurdle rate. It would be fair to say, however, that the results of this study provide at least some indication regarding the direction of adjustments managers should make on market capitalization rates given their judgment on the relative risk of proposed project as compared to that of existing assets and future capital expenditures.

NOTES

1 If the firm has outstanding debt, the risk of stock will also reflect financial risk. Since the primary purpose of this paper is to examine the effect of the firm's growth opportunities on the systematic risk of its stock and its implications for the cost of equity capital, however, we will abstract from the effect of financial leverage on the firm's cost of capital by assuming that the firm is all-equity financed.

2 The approach taken here is similar to that in Chung and Charoenwong (1991). Chung and Charoenwong, however, do not address the implications of the options interpretation of growth opportunities for the firm's investment decisions. Furthermore, our approach is more general than that of Chung and Charoenwong (1991) in that: (i) this study assumes that the risk of future assets can be different from that of existing assets whereas they assume that all future assets have the same risk as existing assets, and (ii) this study assumes that both the firm's future asset values and capital expenditures are stochastic whereas they assume that all future capital expenditures are known at time zero. As a result, our model yields richer implications than theirs.

3 The Wiener process is a particular type of Markov stochastic process. It has been used in physics to describe the motion of a particle that is subject to a large number of small molecular shocks and is sometimes referred to as Brownian motion. The growth rate of $X(t)$ is typically less than the rate of return (i.e., μ) on a financial asset with comparable risk since the growth rate of $X(t)$ equals the rate of return on a comparable asset less the cash flow that is earned on the project and paid out. Hence δ is a negative constant (see McDonald and Siegel, 1986, p. 710). See also Chung (1990) for similar applications.

4 The behavior of the variable w, which follows a Wiener process can be understood by considering the changes in its value in small intervals of time. Consider a small interval of time of length dt and define dw as the change in w during dt. There are two basic properties of dw: (1) dw is related to dt by the equation $dw = \omega\sqrt{dt}$, where ω is a random sample from a standardized normal distribution; (2) The values of dw for any two different short intervals of time dt are independent.

5 We assume that stochastic changes in $X(t)$ and $I(t)$ are spanned by existing assets, that is, there are assets or dynamic portfolios of assets whose prices are perfectly correlated with $X(t)$ and $I(t)$. This assumption implies that the firm can value its growth options independently of other assets and that there are securities in the market that can be combined to give a portfolio at time zero that will have the same value as the underlying real asset. With the spanning assumption, we can evaluate the value of growth opportunities using the contingent-claim valuation which avoids assumptions regarding risk preferences or discount rates. For notational simplicity, we do not add the subscript i to t, σ, μ, δ, Ω, τ, and ϕ. However, t, σ, μ, δ, Ω, τ, and ϕ should be interpreted as those of growth opportunity i.

6 A variation of this cash flow pattern would be obtained if we assume that the initial investment will result in more than one mutually exclusive projects. Then the terminal cash flow will be expressed as $\max[0, \max\{X_1(t), X_2(t), X_3(t), \ldots\} - I(t)]$, where $X_j(t)$ is the value of asset j. Stulz (1982) and Johnson (1987) provide the solution to this problem when $I(t)$ is a constant. In this paper, however, we keep the model as parsimonious as possible to convey the intended message without invoking unnecessary complications.

7 We employ the CAPM beta as an appropriate measure of risk throughout this paper. We believe, however, that our analysis can be applied to other measures of risk as long as they are additive.

8 Remember here that $I(t)$ is a constant. Note that $e^{-rt}I(t)$ is the present value of investment in

growth opportunity i and $N(d_{2t})$ is the probability of undertaking the project. Hence *PVIG*1 can be interpreted as the *expected* present value of all future investments in growth opportunities. It should be noted that, in general, $N(d_{2t})$ represents the probability of undertaking the project only if investors are risk-neutral (see Smith, 1976, p. 23, fn. 22).

9 In order to see this point, note that $\beta_i = \{e^{\delta t}N(d_{1t})/G_i\}X_i(0)\beta_{Xi}$. Since $\{e^{\delta t}N(d_{1t})/G_i\}X_i(0) > 1$, it follows that $\beta_i \;\; \beta_{Xi}$.

10 Chung and Charoenwong (1991) report that the stock beta is indeed positively related to growth opportunities in a cross-section of firms.

11 Myers (1977, p. 171) argues that the valuation model of Miller and Modigliani (1961) is misspecified since they fail to recognize that the discount rate applicable to the future cash flows generated from existing assets should be different from that applicable to the cash flows generated from future growth opportunities. However, the above result indicates that the risk of growth opportunities and that of assets in place will be identical if the risk associated with the future asset value and that associated with future investment expenditures are the same as that of existing assets. Since Miller and Modigliani implicitly assume that the risk associated with future investment expenditures and that associated with the future cash flows are identical, their model is, in fact, correctly specified, contrary to the Myers' argument.

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