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Risk in Inventory Models: The Case of the Newsboy Problem— Optimality Conditions

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A recent article in this journal employs the capital-asset pricing model for the analysis of the newsboy problem and shows how the covariance risk affects the optimal inventory policy. The purpose of this paper is to sharpen the optimality conditions given by the article and hence to provide a simple method for finding the solution. Under reasonable assumptions, this paper shows that the optimal ordering policy can be described by a single equation, regardless of the sign of the covariance term.

Key words: finance, inventory, risk

INTRODUCTION

In a recent paper, Anvari[1] employs the capital-asset pricing model for the analysis of the newsboy problem. The author provides a cogent argument on how the risk will affect the optimal inventory policy of firms. The author's approach adds an important dimension to the analysis of inventory management, since it captures an important capital-market notion that rational individuals will hold multiple-asset portfolios and thus will be concerned with the covariance risk. The purpose of this paper is to sharpen the optimality conditions given by Anvari and hence to provide a simple method for finding the solution.

VALUATION AND DERIVATIVES

Given the assumption that demand (i.e. R) is normally distributed, it can easily be shown that the firm's profit function is defined as

$$(1 + r_f)S(x) = C(1 + \alpha) + [b - (1 + \alpha)a]x - (b - p)\sigma^2 f(x) - (b - p)F(x)[x - E(R) + \Omega \operatorname{cov}(R, M)]. \quad (1)$$

Hence, the first and second derivatives are expressed as

$$(1 + r_f)\, dS(x)/dx = b - (1 + \alpha)a - (b - p)F(x) - (b - p)\Omega \operatorname{cov}(R, M)f(x) \quad (2)$$

$$(1 + r_f)\, d^2S(x)/dx^2 = -(b - p)f(x)[1 + \Omega \operatorname{cov}(R, M)\{E(R) - x\}/\sigma^2]. \quad (3)$$

OPTIMALITY CONDITIONS: AN EXTENSION

First-order condition

From the first derivative (2), the first-order necessary condition for optimality can be written as

$$F(x^*) + \Omega \operatorname{cov}(R, M)f(x^*) = \frac{b - (1 + \alpha)a}{b - p}, \quad (4)$$

where x^* is the value of x that satisfies the first-order condition (i.e. the extreme point). The analogous optimality condition of the traditional expected profit-maximization approach is given by

$$F(x^*) = \frac{b - (1 + \alpha)a}{b - p} \quad (5)$$

(see Anvari,[1] p. 629).

Notice the remarkable resemblance between equations (4) and (5). In fact, equation (4) will be reduced to (5) if we assume that the market price per unit of risk (Ω) is zero (i.e. if investors are risk neutral). This is a very intuitive result since expected profit will be identical to the risk-adjusted value if investors are risk neutral.

Second-order condition

Case 1: cov(R, M) > 0. When cov$(R, M) > 0$, the objective function (1) is strictly concave in the region $x \in (0, x_c)$ and is strictly convex in the region $x \in (x_c, \infty)$, where $x_c = E(R) + \sigma^2/\Omega \operatorname{cov}(R, M)$ (see Figs 1 and 2). However, as shown below, the extreme point will occur only in the strictly concave region. To see this, note first from (4) that

$$(b - p)\Omega \operatorname{cov}(R, M) = \frac{b - (1 + \alpha)a - (b - p)F(x^*)}{f(x^*)}. \qquad (6)$$

Substituting (6) into (3), and after simplification, the second derivative at the extreme point can be written as

$$\frac{(1 + r_f)\, d^2S(x^*)}{dx^2} = -(b - p)\left[f(x^*) - \frac{\{x^* - E(R)\}}{\sigma^2}\{H - F(x^*)\}\right], \qquad (7)$$

where $H = \{b - (1 + \alpha)a\}/\{b - p\}$.

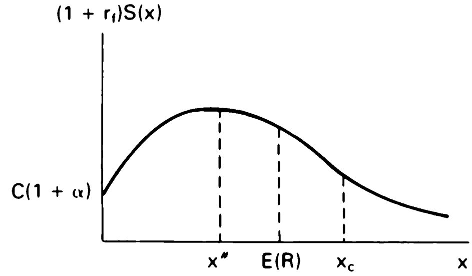

FIG. 1. *When* cov$(R, M) > 0$ *and* $x^* \leqslant E(R)$.

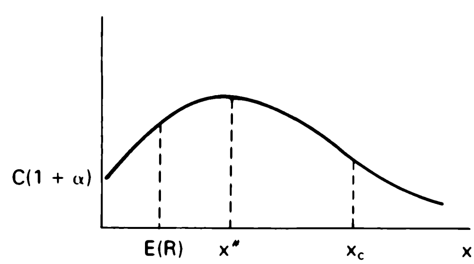

FIG. 2. *When* cov$(R, M) > 0$ *and* $x^* > E(R)$.

For the case when $x^* \leqslant E(R)$ (see Fig. 1), it should be noted that $d^2S(x^*)/dx^2 < 0$ if $H > F(x^*)$. However, from equation (4) we know that $H > F(x^*)$ since, by assumption, cov$(R, M) > 0$. Thus the sufficiency condition is satisfied when $x^* \leqslant E(R)$. On the other hand, for the case when $x^* > E(R)$ (see Fig. 2), note the following property of the normal distribution (see Feller,[2] p. 175):

$$f(x) > \frac{\{x - E(R)\}}{\sigma^2}\{1 - F(x)\}. \qquad (8)$$

Because $H < 1$ by assumption (since $p < a$), it is also clear that $d^2S(x^*)/dx^2 < 0$. Thus, for the case of positive covariance, the sufficiency condition is always satisfied. Furthermore, it should be noted that $S(x^{\#}) > S(x_c)$ and $S(x^{\#}) > S(R_{\max})$. Therefore, Anvari's equation (19) is simplified to

$$S(x^*) = S(x^{\#}). \tag{9}$$

Case 2: cov(R, M) < 0. When $\text{cov}(R, M) < 0$, the objective function (1) is strictly convex in the region $x \in (0, x_c)$ and is strictly concave in the region $x \in (x_c, \infty)$ (see Figs 3 and 4). However, as shown below, the extreme point will occur only in the strictly concave region. First note that if $x^{\#} > E(R)$ (see Fig. 3), then in (7), $x^{\#} - E(R) > 0$ and $H - F(x^{\#}) < 0$ from (4). Hence the second term in the large bracket is negative, and therefore the sufficiency condition is satisfied.

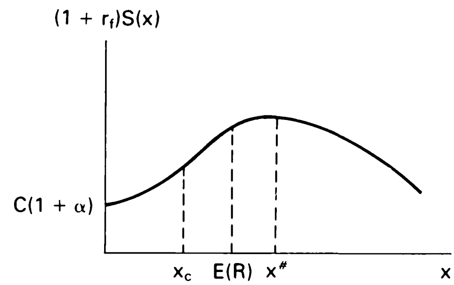

FIG. 3. *When* $\text{cov}(R, M) < 0$ *and* $x^{\#} > E(R)$.

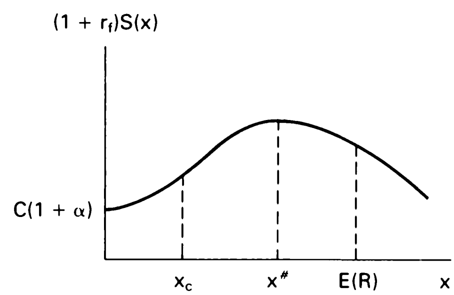

FIG. 4. *When* $\text{cov}(R, M) < 0$ *and* $x^{\#} \leqslant E(R)$.

For $x^{\#} \leqslant E(R)$ (see Fig. 4), it is important to note that $x^{\#}$ cannot be less than x_c because $(1 + r_f)\, dS(0)/dx = b - (1 + \alpha)a > 0$ since, by assumption, $F(0) = f(0) = 0$. Therefore $x^{\#}$ will be occurring in the strictly concave region of the objective function. As a result, the sufficiency condition will always be satisfied when $\text{cov}(R, M)$ is negative. Furthermore, it should be noted that $S(x^{\#}) > S(x_c)$ and $S(x^{\#}) > S(0)$. Therefore, Anvari's equation (20) is simplified to

$$S(x^*) = S(x^{\#}). \tag{10}$$

Case 3: cov(R, M) = 0. When $\text{cov}(R, M) = 0$, the objective function (1) is strictly concave everywhere. Thus the optimal policy is described by

$$S(x^*) = S(x^{\#}). \tag{11}$$

Thus in all cases the optimal order policy will be described by the single equation (4), regardless of the sign of the covariance term.

SUMMARY AND CONCLUDING REMARKS

This paper sharpens the optimality conditions given by Anvari, and hence provides a simple method for finding the solution. Under reasonable assumptions, this study shows that the optimal ordering policy can be described by a single equation, regardless of the sign of the covariance term.

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