Inferring the Components of the Bid-Ask Spread: Theory and Empirical Tests

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ABSTRACT

The relation between the square of the quoted bid-ask spread and two serial covariances—the serial covariance of transaction returns and the serial covariance of quoted returns—is modeled as a function of the probability of a price reversal, π, and the magnitude of a price change, ∂, where ∂ is stated as a fraction of the quoted spread. Different models of the spread are contrasted in terms of the parameters, π and ∂. Using data on the transaction prices and price quotations for NASDAQ/NMS stocks, π and ∂ are estimated and the relative importance of the components of the quoted spread—adverse information costs, order processing costs, and inventory holding costs—is determined.

THE QUOTED BID-ASK spread is the difference between the ask price quoted by a dealer and the bid price quoted by a dealer at a point in time. The realized bid-ask spread is the average difference between the price at which a dealer sells at one point in time and the price at which a dealer buys at an earlier point in time. Theories of the spread are theories of the quoted spread. The current literature implies that the quoted spread must cover three costs faced by a dealer: order processing costs, inventory holding costs, and adverse information costs. Order processing costs receive a greater emphasis in the early literature of Demsetz [5] and Tinic [19], but all researchers on the bid-ask spread recognize the importance of these costs. Inventory holding costs arising from the risk assumed by a dealer are modeled in Stoll [17] and Ho and Stoll [13], and Amihud and Mendelson [1] model the effect of constraints on inventory size. The role of adverse information costs is emphasized by Copeland and Galai [4], Glosten and Milgrom [7], and Easley and O'Hara [6].

Empirical studies have shown that the quoted spread is related to characteristics of securities such as the volume of trading, the stock price, the number of market makers, the risk of the security, and other factors. However, these results are roughly consistent with several theories of the spread and do not give much insight into the evolution of the spread over time or the relative importance of the cost components of the quoted spread.

An implication of both the inventory cost model and the adverse information cost model is that the realized spread earned by a dealer is less than the spread

quoted by the dealer. Under the inventory cost model, this is because the dealer lowers both bid and ask prices after a dealer purchase and raises both bid and ask prices after a sale in order to induce transactions that will equilibrate inventory. Under the adverse information cost model, bid and ask prices are changed in a similar way to reflect the information conveyed by transactions. Transactions convey information under the assumption that some traders are better informed than others. The empirical evidence in Stoll [16, 18] shows that the realized spread is less than the quoted spread in the manner predicted by the inventory holding cost model or the adverse information cost model.

Roll [15] has shown that the covariance of transaction returns is an estimate of the realized spread in an efficient market. (He uses the term "effective spread".) He calculates the realized spread for the New York Stock Exchange stocks and finds, as well, that the realized spread is less than the quoted spread. Roll does not, however, develop the theoretical relation between realized and quoted spreads. Indeed, he assumes a naive order processing cost model of the spread. Since Roll, a number of papers have estimated the realized spread in the stock market and in the option market. Glosten and Harris [8] have modeled the relation between the quoted and realized spreads more explicitly than Roll and have attempted to estimate the components of the spread. However, they do not have data on the quoted spread and thus have difficulty in inferring the components of the spread. In a recent paper, Hasbrouck [11] examines the time-series relation between quotes and transaction prices for NYSE stocks and concludes that there is evidence for inventory effects and information effects. Bhattacharya [2] estimates spreads from stock option transactions data.

In this paper, the time-series behavior of the spread is modeled and the relation between the quoted spread and the realized spread is specified. A relation between the quoted spread and the covariance estimate of the spread that depends upon two parameters—the probability of a price reversal and the amount of a price reversal—is established. Using NASDAQ data on quoted spreads and transaction prices, inferences can be made about these parameters and, as a result, about the relative importance of the components of the spread.

I. Transactions and the Spread

Suppose that, at time zero, ask and bid prices are A_0 and B_0, respectively. The quoted spread at time zero is therefore $S_0 = A_0 - B_0$. The quoted spread depends on the transaction size, which is assumed to be constant for a given stock but which may vary across stocks. Suppose a transaction occurs at B_0. Now examine how the new bid and ask, A_1 and B_1, will be set under alternative views of the spread. Assume that no new information, other than that conveyed by the transaction itself, appears in the marketplace, and assume that the spread, S, is constant. Three alternative views of the trading process are illustrated in Figure 1.

If the spread reflects only order processing cost, A and B always straddle the "true" price (indicated by the solid line), as in Figure 1A. The dealer covers costs by buying at B_0 and selling at A_1 (on average). Sequences of purchases at the bid price are ultimately offset by sequences of sales at the ask price. This is the model depicted in Roll [15]. Further, the realized spread, $A_1 - B_0$ is the same as

Panel A
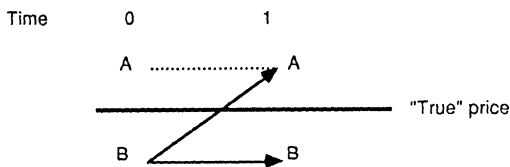
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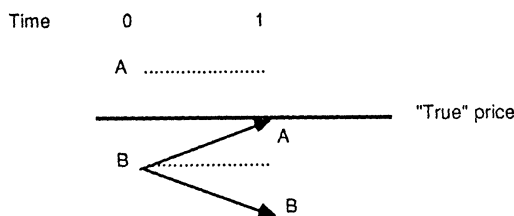
Panel C
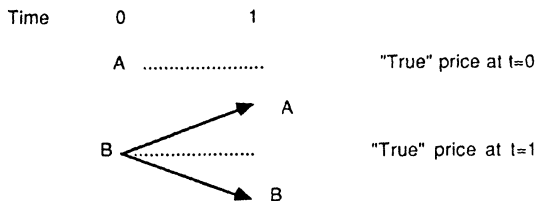
Figure 1. Transactions and the spread. Assuming an initial transaction at the bid, what are the possible transaction prices in the next period? A = ask price; B = bid price. Panel A: Spread determined by order processing costs. Panel B: Spread determined by inventory holding costs. Panel C: Spread determined by adverse information costs.

the quoted spread, $A_0 - B_0$. (Roll recognizes, however, that he estimates an "effective" spread which is smaller than the quoted spread.)

If the spread reflects inventory holding costs, dealers tend to change the position of the spread relative to the "true" price in order to induce public transactions that would even out the inventory position of the dealer. Stoll [17] and Ho and Stoll [13] propose a model of the dealer that is based on the risk of holding inventory. In this case, bid and ask prices are lowered after a dealer purchase in order to induce dealer sales and inhibit additional dealer purchases, and bid and ask prices are raised after a dealer sale in order to induce dealer purchases and inhibit dealer sales. New prices are set such that the dealer is indifferent between a transaction at the bid price and a transaction at the ask price. Assuming inventory costs that are linear in inventory and symmetric with respect to purchases and sales implies that price changes are symmetric. In other

words, the bid and ask prices fall by 0.5S after a dealer purchase, and they both increase by 0.5S after a dealer sale.[1] This process, illustrated in Figure 1B, implies that the dealer makes 0.5S if a trade is reversed. Over time, the dealer's inventory position is evened out because the dealer's price adjustments increase the probability of transactions that eliminate inventory the dealer has acquired.

If the spread reflects adverse information costs, as modeled in Copeland and Galai [4] and Glosten and Milgrom [7], bid and ask prices, after a transaction at B_0, shift in the same manner as in the inventory adjustment model but for a different reason. After a sale to the dealer, bid and ask prices are lowered because a transaction at B_0 conveys information that the expected equilibrium price of the security is lower. Such information is conveyed under the assumption that some traders have superior information. At time zero, the equilibrium price conditional on a sale to the dealer is B_0, and the equilibrium price conditional on a purchase from the dealer is A_0. The expected equilibrium price at time zero is therefore $(A_0 + B_0)/2$. A sale to the dealer conveys information that causes the dealer to revise the expected equilibrium price to $(A_1 + B_1)/2$. The adjustment of bid and ask prices when a proportion of all traders have superior information is illustrated in Figure 1C.

It is evident from Figures 1B and 1C that the dealer's realized spread is less than the dealer's quoted spread when the spread depends on inventory holding costs or adverse information costs.

The three views of the spread can be summarized by the value of two parameters, ∂ and π, as shown in Figure 2. The size of a price reversal (conditional on a reversal) is given by $(1 - \partial)S$, where S is the spread and $0 \leq \partial \leq 1$. The probability of a price reversal is given by π. Under the assumption of a constant spread, the price continuation (conditional on a continuation) is ∂S and the probability of a continuation is $(1 - \pi)$. Price reversals are assumed to be symmetric in the sense that a price increase after a transaction at the bid has the same size as a price decrease after a transaction at the ask. Correspondingly, the probability of a price reversal is assumed to be the same whether the last transaction was at the bid or at the ask. Price continuations are assumed to be symmetric in the same way. The three views can now be summarized as follows.

Determinant of Quoted Spread	∂	π
Pure Order Processing (Roll)	0.0	0.5
Pure Adverse Information (Copeland/Galai, Glosten/Milgrom)	0.5	0.5
Pure Inventory Holding Cost (Ho/Stoll)	0.5	$1 > \pi > 0.5$

[1] The price change is 0.5S for the following reason. The spread, $(A - B)$, is twice the inventory cost of a transaction since the dealer is prepared to buy or sell. If the dealer buys, inventory is larger by one transaction amount. With linear inventory costs the next purchase will cost the same. That means that the new bid price will be lower by the inventory cost on one transaction—0.5S. The ask price will also be lower by 0.5S because of symmetry; that is, the dealer saves 0.5S if he or she sells the inventory he or she just acquired. See Ho and Stoll [13, p. 65] for discussion. Ho and Stoll show that price adjustments may not be linear in inventory. However, the nonlinear component is small relative to the linear component.
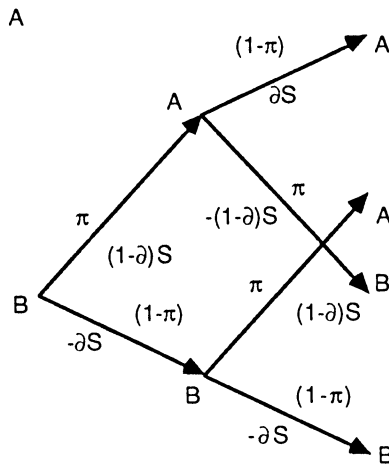
Figure 2. Possible sequences of transaction prices starting at the bid price. A = ask price; B = bid price; π = probability of a price reversal; $1 - \partial$ = amount of a price reversal as a fraction of the spread, S.

Under the pure order processing view of the spread, prices simply move between the bid and the ask, and the price reversal is equal to the spread. In these models, it is usually assumed that the inflow of orders is such that the probability of a purchase equals the probability of a sale.

Under the pure adverse information view of the spread, the spread is determined by the probability that traders with adverse information will trade with the dealer. Because of the price adjustment mechanism described earlier and presented in detail in Glosten and Milgrom and in Copeland and Galai, the price reversal is only half the magnitude of the spread and is equal to the price continuation. On the basis of the information conveyed by the last trade, prices are such such that the probability of a purchase equals the probability of a sale.

Under the pure inventory holding cost view of the spread, the price reversal is half the magnitude of the spread. However, the equilibrium price does not change on the basis of the observed trades, as in the adverse information case. Instead, the adjustment of prices is intended to modify the probability of a transaction in one direction or the other. As a result, the probability, π, of a reversal exceeds 0.5.

In all likelihood, the quoted spread reflects each of these components to some degree. In this paper the relative importance of each of the components of the spread is inferred from the covariance of transaction returns and from the covariance of returns defined over quoted bid or ask prices. This paper is distinguished from Roll's paper and other recent papers inferring the spread for several reasons.

First, a more general model of the relation between the quoted spread and the covariance of returns is developed than is the case in the Roll paper or other

more recent papers. Second, the paper shows that the relation between the quoted spread and the covariance of returns is different according to whether transaction prices or quotations are used in calculating returns. Third, the empirical work in this paper is based on NASDAQ "National Market Securities" (NASDAQ/NMS) for which both bid and ask prices and transaction prices are available. These data make possible the estimation of the parameters ∂ and π and permit inferences about the relative importance of the three components of the spread.

The model developed in this paper and the empirical work are based on the following assumptions:

(A1)   The market is informationally efficient in the sense that the expected price change in a security is independent of current and past information.

(A2)   The spread, S, is constant over the one-month period for which empirical work is carried out. This assumption may be relaxed to allow for random variations in the spread.

(A3)   All transactions are carried out at the highest bid or the lowest ask price available in the market.

II. Serial Covariance of Price Changes

A. Total Price Change in Efficient Markets

The total price change in a security may be decomposed into three components as follows:

$$\Delta V_t = a + \Delta P_t + e_t, \tag{1}$$

where

ΔV_t = total price change in a security between time $t - 1$ and time t,

a = expected price change in the security in the absence of the bid-ask spread,

ΔP_t = price change due to the spread, and

e_t = price change due to new information; $E(e_t) = 0.0$.

Then

$$\text{cov}(\Delta V_t, \Delta V_{t+1}) = \text{cov}(\Delta P_t, \Delta P_{t+1}) + \text{cov}(\Delta P_t, e_{t+1})$$
$$+ \text{cov}(e_t, \Delta P_{t+1}) + \text{cov}(e_t, e_{t+1}). \tag{2}$$

In an efficient market, changes in prices due to new information are serially uncorrelated and are uncorrelated with lagged or leading values of the price change due to the spread. This implies that

$$\text{cov}(\Delta V_t, \Delta V_{t+1}) = \text{cov}(\Delta P_t, \Delta P_{t+1}). \tag{3}$$

As Roll noted and as equation (3) indicates, the serial covariance in observed price changes in an efficient market is due only to the covariance induced by the spread.

B. Price Changes Due to the Spread

Under the assumption of constant spread, the possible price changes ΔP_t, starting at the bid price, are

$$\Delta P_t = \begin{cases} (A_t - B_{t-1}) = (1 - \partial)S & \text{with prob } \pi, \\ (B_t - B_{t-1}) = -\partial S & \text{with prob } (1 - \pi). \end{cases}$$

Under the assumption of symmetry, the possible price changes starting at the ask price are

$$\Delta P_t = \begin{cases} (B_t - A_{t-1}) = (1 - \partial)S & \text{with prob } \pi, \\ (A_t - A_{t-1}) = -\partial S & \text{with prob } (1 - \pi). \end{cases}$$

Transactions at time $t - 1$ are assumed to occur with equal probability at the bid or ask.

The expected price change conditional on a transaction at the bid is

$$E(\Delta P_t \,|\, B_{t-1}) = \pi(1 - \partial)S + (1 - \pi)(-\partial S) = (\pi - \partial)S.$$

Correspondingly, the expected price change conditional on a transaction at the ask is $-(\pi - \partial)S$. Given equal probability of a transaction at the ask and the bid, the unconditional expected price change is $E(\Delta P_t) = 0$. The realized spread that the dealer expects to earn is defined as the expected price change after a dealer purchase less the expected price change after a dealer sale; that is, the realized spread is

$$2(\pi - \partial)S. \tag{4}$$

Note that the realized spread is the expected revenue on two transactions: a purchase and a sale. It is evident from (4) that only in the "order processing" world, in which $\pi = 0.5$ and $\partial = 0$, is the realized spread the same as the quoted spread. When the quoted spread is determined solely by adverse information, $\pi = 0.5$ and $\partial = 0.5$, so that the realized spread is zero. When the quoted spread is determined by inventory holding costs, $\pi > 0.5$ and $\partial = 0.5$, so that the realized spread is positive but less than the quoted spread.[2]

If it were possible to observe whether a transaction occurs at B or A, the realized spread could be estimated directly, but that information is not available. Instead, the serial pattern of prices is used.

C. Serial Covariance of Price Changes Due to the Spread

Under the assumption of market efficiency, the serial covariance of price changes due to the spread may be inferred from observed price changes. The serial covariance depends on the two-period (three-date) sequence of prices.

The possible two-period sequences of price changes starting with a transaction at the bid are depicted in Figure 2. If a transaction at the bid is followed by

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[2] Only if the probability of reversal is $\pi = 1.0$ does the dealer earn S in the inventory holding cost world. Alternatively, if $\pi = 0.5$, the dealer receives no compensation for holding inventory and the dealer's inventory position has no tendency to return to a desired level.

another transaction at the bid (a continuation), the price change is $-\partial S$, where $0 < \partial < 1$. If a transaction at the bid is followed by a transaction at the ask, the price change is $(1 - \partial)S$, a reversal. The probability of a continuation is $(1 - \pi)$, and the probability of a reversal is π. Under the assumption of constant spread, the difference in price changes is always equal to the spread, S.

Figure 2 depicts not only the possible sequences of transaction prices but also the sequences of bid and ask prices. Since the NASDAQ data used in this study contain transaction data and data on bids and asks, covariances of changes in bids, changes in asks, and changes in transaction prices may be estimated. The separate estimates may then be used to infer the values of ∂ and π.

Consider first the covariance of transaction price changes. On the basis of the transaction process depicted in Figure 2, the covariance of transaction price changes is

$$\text{cov}_T \equiv \text{cov}(\Delta P_t, \Delta P_{t+1}) = S^2[\partial^2(1 - 2\pi) - \pi^2(1 - 2\partial)]. \tag{5}$$

The details of the derivation are in the Appendix. Equation (5) is a more general version of the Roll [15] formula. When $\partial = 0$ and $\pi = 0.5$, $\text{cov}_T = -\frac{1}{4}S^2$, which is the Roll formula.

One can also determine the possible sequences of changes in quoted prices associated with sequences of transaction price changes. The Appendix shows that the serial covariance of quoted price changes, cov_Q, is

$$\text{cov}_Q \equiv \text{cov}(\Delta Q_t, \Delta Q_{t+1}) = S^2\partial^2(1 - 2\pi) \qquad Q = A, B. \tag{6}$$

Under the assumption of a constant spread, cov_Q can be calculated either from bid price changes, as $\text{cov}_B \equiv \text{cov}(\Delta B_t, \Delta B_{t+1})$, or from ask prices changes, as $\text{cov}_A \equiv \text{cov}(\Delta A_t, \Delta A_{t+1})$.

If the observed values of S contained an error because dealers sometimes failed to adjust spreads or because of reporting errors, equations (5) and (6) would be the same except that $S + e$, where e is a random error, would appear wherever S now appears. In the empirical work, an average of daily spread observations is used to eliminate the effect of random errors.

The values of the serial covariances for the alternative models described earlier may now be summarized as follows.

Determinant of Quoted Spread	cov_T	cov_Q
Order Processing		
($\partial = 0$, $\pi = 0.5$)	$-0.25S^2$	0.0
Adverse Information		
($\partial = 0.5$, $\pi = 0.5$)	0.0	0.0
Inventory Cost		
($\partial = 0.5$, $\pi > 0.5$)	$-0.25S^2 < \text{cov}_T < 0.0$	$-0.25S^2 < \text{cov}_Q < 0.0$

The inventory holding cost model has a different prediction from the other two models—namely, that the serial covariance of bid or ask price changes is negative. The other two models predict an absence of serial covariance in bid or ask price changes. The NASDAQ data make it possible to estimate cov_T and cov_Q and to observe the value of S. Also, these data make it possible to estimate the values of ∂ and π and to assess the importance of alternative views of the spread.

In order to permit comparisons across stocks, spreads and covariances are stated in proportional terms rather than in dollar magnitudes. The use of returns simply means that in equations (5) and (6) the returns corresponding to ΔP_t, ΔB_t, and ΔA_t are used. The proportional spread is the dollar spread divided by the midpoint of the spread.

III. Data and Empirical Procedures

The data set contains transaction prices and price quotations for National Market System (NMS) securities on the NASDAQ system in the months of October, November, and December 1984. Transaction prices available on the data set are the closing price each day and the last two transaction prices prior to the closing price. Price quotations are the daily closing bid and ask prices. Quotations are the "inside" quote on the NASDAQ system. The inside quote is not necessarily the quote of any single dealer since it may represent the bid of one dealer and the ask of another. Competition among dealers, the desire of investors to trade with the dealer at the "inside", knowledge by all dealers of quotes of other dealers, and knowledge of transaction prices cause the inside quote and transaction prices to behave as if there were one dealer. For example, a sale at the inside bid will cause the dealer at the inside to lower the bid. This change is observed by all dealers, who may interpret this as adverse information and recognize that it will be more difficult to sell any shares they hold in inventory. As a result, other dealers may lower their bid prices. The dynamic interaction of competing dealers causes market bid-ask quotes and transaction prices to behave as if there were one dealer (who prices competitively).[3]

Each of the months (October, November, and December) is taken as a separate sample, and covariances are estimated using time-series data for each stock for each month. The analysis is restricted to NMS stocks for which at least fifteen observations were available on the return series underlying each of the covariances estimated for the stock. The number of trading days, the number of NMS stocks, and the number of NMS stocks in the sample are shown below for each month.

	October 1984	November 1984	December 1984
Trading Days	23	21	20
No. of NMS Stocks	872	934	959
No. of Stocks in Sample	765	779	821

Stocks were excluded from the sample primarily because sufficient intraday transaction prices were not available.

The data set makes it possible to estimate two serial covariances of transaction returns. One serial covariance, cov$_C$, is based on daily closing prices. A second serial covariance, cov$_I$, is based on the intraday prices at the end of the day. Two returns were calculated from the last three transaction prices each day, and the covariance between these two returns was estimated for each stock. In addition to the serial covariances of transaction returns, the serial covariances of returns

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[3] See Ho and Stoll [14] for a model of the dynamic interaction of competing dealers.

based on closing bid prices, cov_B, and on closing ask prices, cov_A, are also estimated.[4]

The quoted proportional spread for each stock is the average of the daily quoted proportional spreads. The daily proportional spread is calculated from the closing bid and ask prices.

IV. Empirical Results

Equations (5) and (6) may be written in a regression framework as

$$cov_T = a_0 + a_1 S^2 + u, \tag{7}$$

$$cov_Q = b_0 + b_1 S^2 + v, \tag{8}$$

where

u, v are random errors,

$$a_1 = \partial^2 (1 - 2\pi) - \pi^2 (1 - 2\partial), \tag{9}$$

$$b_1 = \partial^2 (1 - 2\pi). \tag{10}$$

Under the assumption of market efficiency, one expects $a_0 = b_0 = 0.0$.

Regression (7) uses covariances estimated from transaction prices—either cov_C, calculated from daily closing prices, or cov_I, calculated from intraday prices. Regression (8) uses covariances estimated from quoted prices—either cov_B, calculated from closing bid prices, or cov_A, calculated from closing ask prices. The independent variable, S^2, is the squared value of the average proportional spread.

Summary statistics for these variables are present in Table I along with summary statistics on four other characteristics of stocks: average daily dollar volume ($\$VOL$), average number of market makers (MM), average stock price (P), and share turnover ($TURN$). $TURN$ is the average daily share volume divided by the number of shares outstanding.

The regression results for October, November, and December 1984 are presented in Table II. The estimated value of a_1 is given by the first two regressions

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[4] The covariances are calculated only from returns adjacent in time. The restriction that at least fifteen returns are available thus does not necessarily imply that covariances are based on fifteen observations.

The use of daily price changes and daily quote revisions rather than trade-to-trade price changes and quote revisions may affect the empirical results. Roll [15] has shown, however, that in an order processing world, the differencing interval does not affect the covariance (if the market is efficient). It is also evident from the time-series process implied by the adverse information world that the serial covariance of prices and quotes is zero regardless of the differencing interval. Only in the holding cost world will the differencing interval have an effect on the serial covariance. In that world the probability of a price reversal increases with the differencing interval because the dealer sets prices to induce transactions that sooner or later bring his or her inventory back to the desired level. The probability of a price reversal is thus greater the longer the differencing interval. That implies that the covariance would also be more negative the longer the differencing interval. The data imply, however, that differencing over days or over transactions does not affect the results. Comparisons of cov_I and cov_C in Table I indicate that except for December the covariance is more negative for the trade-to-trade covariance (cov_I) than the daily covariance (cov_C).

Table I

Mean and Standard Deviation of NASDAQ/NMS Stock Characteristics in the
Months of October, November, and December 1984[a]

Characteristic	October 1984 (765 Stocks)		November 1984 (779 Stocks)		December 1984 (821 Stocks)	
	Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
$cov_B(\times 10,000)$	0.3586	2.83	0.1784	3.32	0.0750	3.74
$cov_A(\times 10,000)$	0.4230	2.45	0.1484	2.83	0.0648	3.00
$cov_I(\times 10,000)$	−2.0365	4.24	−2.2132	3.16	−2.5132	5.48
$cov_C(\times 10,000)$	−1.4907	4.58	−1.8596	5.66	−2.5206	8.54
$S(\times 100)$	2.83	1.94	2.88	4.16	3.10	2.29
$\$VOL\ (\times 1/10,000)$	57.48	4.47	53.08	3.94	51.11	3.97
MM	13.45	5.76	13.77	5.90	13.79	6.00
$Price$	14.67	10.77	14.41	11.17	13.69	10.96
$TURN\ (\times 1000)$	4.82	8.41	4.91	9.65	4.91	7.17

[a] The sample includes all NASDAQ/NMS stock for which at least fifteen days of data are available
in the month.
cov_B = serial covariance of daily closing inside bid prices.
cov_A = serial covariance of daily closing inside ask prices.
cov_I = serial covariance of intraday transaction prices, based on the last three transaction prices
in the day.
cov_C = serial covariance of daily closing transaction prices.
S = average bid-ask spread as a fraction of the average of the bid and ask prices.
$\$VOL$ = average daily dollar volume of trading in the month.
MM = average number of market makers during the month.
$Price$ = average stock price in the month.
$TURN$ = average daily volume divided by shares outstanding.

in each panel. All estimates of a_1 are negative and statistically significant. All
theories of the spread imply that $cov_T \leqq 0$, 0, and that is certainly the case. The
values of a_1 estimated using covariances based on intraday transaction prices are
−0.12297, −0.17055, and −0.15676 in the three months, an average of −0.15. The
average value of a_1 using covariances based on closing transaction prices is also
−0.15. A large proportion of the cross-sectional variation in covariances is
explained by S^2. In December, for example, fifty-two percent of the cross-
sectional variation in cov_I is explained by S^2. These results give the first direct
support for Roll's [15] proposition that the covariance of transaction returns
may be used to infer spreads.[5] However, the coefficient, a_1, is less negative than
the value of −0.25 assumed by Roll.

The estimated value of b_1 is given by the slope coefficient in the last two
regressions in each panel of Table II. The value is statistically significant in
three of six cases and is negative in those three cases. There is thus some support
for the inventory holding cost model in that b_1 appears to be negative. However,
the support is very weak, and the proportion of the variation in cov_Q explained

[5] Roll showed that his covariance estimate of the spread was related to stock volume, but he did
not have data on quoted spreads to make a direct comparison.

Table II

Regressions of Serial Covariance Measures, cov_i, Against the Square of the Proportional Spread, S^2, for a Sample of NASDAQ/NMS Stocks in the Months of October, November, and December 1984:

$$cov_i = k_0 + k_1 S^2, \quad i = I, C, B, A^a$$

Dependent Variable	$k_0{}^b$ (t)	$k_1{}^b$ (t)	Adjusted R^2
October 1984 (765 Stocks)			
cov_I	−0.00005896 (−4.14)	−0.12297 (−20.79)	0.3607
cov_C	−0.00002189 (−1.33)	−0.10809 (−15.76)	0.2446
cov_B	0.00004698 (4.03)	−0.00945 (−1.95)	0.0037
cov_A	0.00004171 (4.05)	−0.00050 (0.12)	0.0
November 1984 (779 Stocks)			
cov_I	−0.00009368 (−0.92)	−0.17055 (−40.75)	0.6808
cov_C	0.00000360 (−0.91)	−0.15254 (−19.07)	0.3180
cov_B	0.00002070 (1.53)	−0.00230 (0.42)	0.0
cov_A	0.00002054 (1.78)	−0.00458 (−0.97)	0.0
December 1984 (821 Stocks)			
cov_I	−0.00001858 (1.20)	−0.15676 (−29.87)	0.5207
cov_C	0.00004846 (1.75)	−0.20236 (−21.52)	0.3604
cov_B	0.00002819 (1.89)	−0.01407 (−2.771)	0.0081
cov_A	0.00002067 (1.71)	−0.00963 (2.345)	0.0055

[a] cov_I is based on intraday transaction prices, cov_C on closing transaction prices, cov_B on closing bid prices, and cov_A on closing ask prices.

[b] The parameters k_0, k_1 are estimates of a_0, a_1 in

$$cov_T = a_0 + a_1 S^2 + \mu, \quad T = I, C, \tag{7}$$

and they are estimates of b_0, b_1 in

$$cov_Q = b_0 + b_1 S^2 + \mu, \quad Q = A, B. \tag{8}$$

by S^2 is small. The average value of b_1 based on all six estimates is $b_1 = -0.007$. The small value of b_1 is consistent with a probability, π, close to 0.5 in (10). A small value of π for the average transaction size is not surprising. One would expect to find more negative values of b_1 in an analysis of large transactions.

For the most part, the intercepts of the regressions are not significantly different from zero, which is what market efficiency requires. In an information-ally efficient market the only source of serial covariance ought to be the spread. However, the cov_I and cov_C regressions tend to have negative intercepts, while
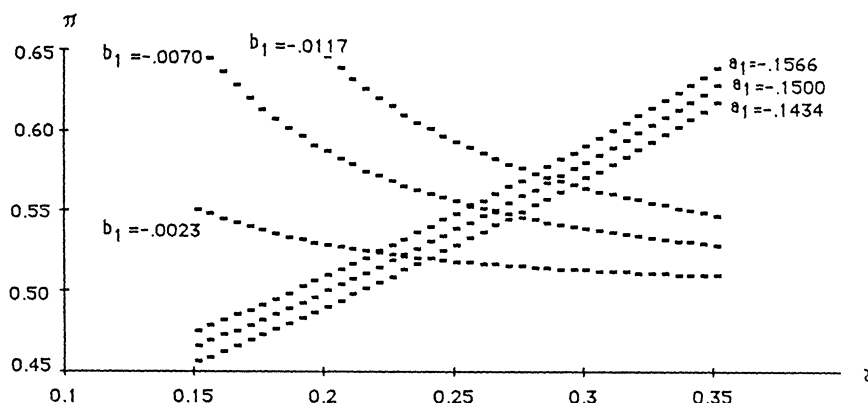
Figure 3. Plots of equations (9) and (10) using estimated values of a_1 and b_1 plus and minus one standard error. The estimates are based on regressions for NASDAQ/NMS stocks in October, November, and December 1984. $a_1 = \partial^2(1 - 2\pi) - \pi^2(1 - 2\partial)$ (9); $b_1 = \partial^2(1 - 2\pi)$ (10); π = probability of a price reversal; ∂ = price continuation as a fraction of the spread.

the cov_A and cov_B regressions tend to have positive intercepts. The positive intercept may imply a lag in the adjustment of price quotations, a form of market inefficiency.[6] The negative intercepts are more difficult to explain, but they may reflect actual transaction sizes that exceed the transaction size implicit in the average quoted spread, or they may reflect market inefficiencies.

V. Composition of the Quoted Spread

The empirical results imply values of $a_1 = -0.152$ and $b_1 = -0.007$. (These are the average values for the three months.) Using these values, one can solve (9) and (10) for ∂ and π. These values are

$$\pi = 0.550,$$

$$\partial = 0.265.$$

The sensitivity of these values to errors in estimation of a_1 and b_1 is given in Figure 3 and Table III. In Table III, values of π and ∂ are calculated for values of a_1 and b_1 deviating by a standard error from the mean estimate. (The average of the standard errors in the three panels of Table II is used.) Figure 3 plots equations (9) and (10) for the estimated values of a_1 and b_1 plus and minus a standard error.

A. Realized versus Quoted Spread

The average realized spread implied by the calculated value of π and ∂ is

$$2(0.550 - 0.265)S = 0.57S.$$

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[6] Delays in price adjustments mean that some or all of the last three terms in (2) are nonzero. Papers that model price adjustment delays include Goldman and Beja [9], Cohen, Maier, Schwartz, and Whitcomb [3, Chapter 6], and Hasbrouck and Ho [12]. No attempt is made in this paper to model price adjustment delays.

To give more insight into the costs of trading, Table IV classifies the December sample of stocks into dollar volume deciles and calculates quoted and realized spreads in cents per share based on the average percentage spread, the average stock price in each decile, and the estimated relation between the realized and quoted spread. (It is shown later in this paper that the coefficients a_1 and b_1 do not depend on volume of trading or certain other stock characteristics. Therefore, use of the same function, $0.57S$, for the realized spread in all volume categories is appropriate.) What is evident from the table is that the quoted spread, stated in cents per share, is nearly constant across all volume categories at thirty-three cents. As a result, the realized spread is also nearly constant at nineteen cents. The decline in the percentage spread with higher volume is offset by an increase in the stock price.

Although the estimated relation between the realized spread and the quoted spread is for NASDAQ/NMS securities, it is roughly the same as that calculated for NYSE securities. Stoll [18] calculates a realized spread based on specialist

Table III

Values of π, the Probability of a Price Reversal, and ∂, the Amount of a Price Continuation as a Fraction of the Spread, Derived from Estimates of $a_1 = \partial^2(1 - 2\pi) - \pi^2(1 - 2\partial)$ and $b_1 = \partial^2(1 - 2\pi)$ Plus and Minus One Standard Error[a]

a_1	b_1	π	∂
$\hat{a}_1 = -.15$	$\hat{b}_1 = -0.007$	0.550	0.265
$\hat{a}_1 + \hat{\sigma}$	$\hat{b}_1 + \hat{\sigma}$	0.520	0.240
$\hat{a}_1 - \hat{\sigma}$	$\hat{b}_1 - \hat{\sigma}$	0.575	0.280
$\hat{a}_1 + \hat{\sigma}$	$\hat{b}_1 - \hat{\sigma}$	0.567	0.295
$\hat{a}_1 - \hat{\sigma}$	$\hat{b}_1 + \hat{\sigma}$	0.524	0.220

[a] The estimates of a_1 and b_1 are based on regressions for NASDAQ/NMS stocks in October, November, and December 1984. $\hat{a}_1 = -0.15$, with a standard error of $\hat{\sigma} = 0.0066$. $\hat{b}_1 = -0.007$, with a standard error of $\hat{\sigma} = 0.0047$.

Table IV

Quoted and Realized Spreads Stated in Cents per Share by Dollar Volume Decile (December 1984, 820 NASDAQ/NMS Stocks)

Average $ Volume Decile	Average Percentage Spread	Stock Price	Quoted Spread in Cents	Realized Spread in Cents
Smallest	6.87	4.82	33	19
2	4.80	6.92	33	19
3	4.22	7.75	33	19
4	3.19	10.38	33	19
5	2.75	12.38	34	19
6	2.59	12.65	33	19
7	2.09	16.98	35	20
8	1.93	17.53	34	19
9	1.39	23.77	33	19
Largest	1.16	23.76	28	16

trading income for the years 1960, 1975–1980. He finds that percentage realized spread of about 0.327% to be half the percentage quoted spread of 0.65% on a value-weighted portfolio of NYSE stocks. That relationship is quite similar to the NASDAQ/NMS relationship of $0.57S$. The value-weighted average NYSE stock price in 1986 was \$37, which implies quoted and realized spreads of 0.0065 (3700) = 24 cents and 0.0037 (3700) = 12 cents.

B. Components of the Quoted Spread

Consider now the decomposition of the quoted spread according to order costs, holding costs, and adverse information costs. We know that the realized spread, $2(\pi - \partial)S$, which is the expected profit per trade (as a percentage of the stock price), covers only order costs and holding costs. The expected profit per trade is zero when the quoted spread is determined by adverse information costs. The adverse information cost component of the quoted spread is thus the difference between the quoted spread and the realized spread: $S - \partial(\pi - \partial)S$, or $0.43S$ under the estimate of $\pi = 0.550$ and $\partial = 0.265$.

Derivation of the order cost and holding cost components requires a division of the realized spread. Under the holding cost view of the spread, $\partial = 0.5$. (Compensation of the dealer for the risk of holding stock results from $\pi > 0.5$.) Using that value of ∂ and the observed value of $\pi = 0.550$, it is possible to calculate the realized spread in the absence of order costs: $2(\pi - 0.5)S = 2(0.550 - 0.5)S = 0.10S$.[7] For example, at a quoted spread of 2.5 percent, the compensation for bearing inventory risk is $(0.10)(2.5) = 0.25$ percent of the stock price. This number is consistent with the theoretical risk premium component of the spread calculated by Ho and Stoll [13, Table 1]. The holding cost component of the spread compensates the dealer for bearing the risk resulting from a loss of diversification. It does not reflect the normal risk premium associated with holding a security. That risk premium is earned by holding the security.

The order cost component of the spread is the remainder of the realized spread after netting out the holding cost component: $0.57S - 0.10S = 0.47S$. The three components of the quoted spread may now be summarized as follows.

Adverse Information Cost	$0.43S$
Holding Cost	$0.10S$
Order Cost	$0.47S$
Total	S

VI. Stock Characteristics

It is well known that the quoted bid-ask spread, S, and the realized spread differ significantly across securities according to characteristics such as volume of trading, stock price, return variance, number of market makers, turnover, and other characteristics. It is possible that the relation between the quoted and

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[7] Using the observed value of π is appropriate under the assumption that the other two theories of the spread assume that $\pi = 0.5$. That is, the only reason for $\pi > 0.5$ is the presence of inventory adjustment pricing by the dealer.

realized spread, as reflected in the coefficients a_1 and b_1, also depends on stock characteristics. This question is examined by allowing the coefficients to depend linearly on a stock characteristic, X:[8]

$$a_1 = \alpha_1 + \alpha_2 X, \tag{11}$$

$$b_1 = \beta_1 + \beta_2 X, \tag{12}$$

which implies that

$$\text{cov}_T = a_0 + \alpha_1 S^2 + \alpha_2 S^2 X, \tag{13}$$

$$\text{cov}_Q = b_0 + \beta_1 S^2 + \beta_2 S^2 X. \tag{14}$$

The regressions (13) and (14) are estimated for

$X = P$, the average stock price during the month,
$X = TURN$, the average daily volume divided by shares outstanding,
$X = \$VOL$, the average daily dollar volume of trading, and
$X = MM$, the average number of market makers in the stock.

No strong theoretical argument can be made for most of these variables. Only *TURN* has some theoretical justification in that it ought to reflect the degree of informational trading and should therefore reflect adverse information costs. In the absence of informational trading, stocks would be traded in proportion to the amount outstanding. Thus, large values of *TURN* might imply greater adverse information and thus a larger adverse information cost component. It is possible that different stocks have different dollar adverse information costs and the same values of a_1, b_1. That will be the case if other costs also tend to be higher so that all costs are a constant proportion of the quoted spread.

The regression results for (14), the regression using covariances calculated from quoted prices, are, for the most part, not statistically significant. Out of a total of six regressions, P is significant in one, *TURN* in two, $\$VOL$ in two, and *MM* in three. The signs on $\$VOL$ are not consistent. The greatest increase in R^2 relative to the corresponding regression in Table II was 0.0380. Because of their lack of significance, these regression results are not presented here and are available from the author.

The regression results for (13) are more interesting because the previous regressions involving cov_I and cov_C, the covariances based on transaction returns, were very significant. The results for equation (13) are in the Table V. It is evident from the table that there is no dependence of the parameter a_1 on the stock characteristics, *TURN* and $\$VOL$.

The stock price, P, is significant in each of the regressions involving cov_I and has a positive sign. The stock price is not significant in the regressions involving cov_C. A positive sign on P implies that the greater the stock price, the less

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[8] An alternative approach to assessing the role of stock characteristics is to run the regressions (7) and (8) for subsamples of stocks classified into deciles on each of the variables P, *TURN*, $\$VOL$, and *MM*. That procedure results in 480 regressions (four different dependent variables × four stock characteristics × ten deciles × three months). An analysis of the coefficients across deciles did not disclose any systematic tendencies not also reflected in the somewhat more structured procedure reported in this paper.

Table V

Regression of Serial Covariance Measures, cov_i, against the Square of the Proportional Spread, S^2, and a Multiplicative Variable XS^2, Where X Represents a Stock Characteristic[a]

$$cov_i = a_0 + \alpha_1 S^2 + \alpha_2 XS^2$$

Dependent Variable	X	October 1984 (764 Stocks)				November 1984 (778 Stocks)				December 1984 (820 Stocks)			
		a_0	α_1	α_2	Adjusted R^2	a_0	α_1	α_2	Adjusted R^2	a_0	α_1	α_2	Adjusted R^2
cov_I[b]	P[c]	-0.00008458 (-4.790)	-0.1455 (-13.245)	0.00598 (2.432)	0.3647	-0.00003742 (-3.081)	-0.1908 (-30.000)	0.0063 (4.200)	0.6874	-0.00004444 (-2.401)	-0.1724 (-21.192)	0.0055 (2.575)	0.5237
	$TURN$[d]	-0.00005597 (-3.868)	-0.1199 (-18.616)	-1.5496 (-1.222)	0.3610	-0.00000962 (-0.937)	-0.1725 (-35.674)	0.5084 (0.798)	0.6805	-0.00002396 (-1.563)	-0.1780 (-25.049)	5.7628 (4.381)	0.5311
	$\$VOL$[e]	-0.00006098 (-3.656)	-0.1231 (-20.595)	0.0000 —	0.3598	-0.00001396 (-1.185)	-0.1715 (-39.322)	3.340E-08 (0.788)	0.6805	-0.00001279 (-0.719)	-0.1557 (-28.783)	-3.656E-08 (-0.658)	0.5203
	MM[f]	-0.00000459 (-0.315)	-0.02891 (-2.574)	-0.0123 (-9.654)	0.4296	0.00001449 (1.402)	-0.1059 (-11.555)	-0.0072 (-7.858)	0.7039	0.00000525 (0.337)	-0.0887 (-7.454)	-0.0074 (-6.337)	0.5425
cov_C[g]	P	-0.00000385 (-0.187)	-0.0921 (-7.216)	-0.0042 (-1.483)	0.2457	0.00004346 (1.863)	-0.1237 (-10.128)	-0.0089 (-3.110)	0.3254	-0.00002665 (-0.809)	-0.2480 (-17.125)	0.0159 (4.116)	0.3726
	$TURN$	-0.0002088 (-1.244)	-0.1070 (-14.328)	-0.5226 (-0.355)	0.2437	-0.00000218 (0.112)	-0.1648 (-17.915)	3.2434 (2.674)	0.3232	0.00004924 (1.773)	-0.1998 (-15.515)	-0.7009 (-0.294)	0.3597
	$\$VOL$	-0.00002614 (-1.351)	-0.1085 (-15.65)	0.0000 —	0.2437	-0.00002837 (-1.267)	-0.1592 (-19.200)	2.339E-07 (2.903)	0.3243	0.00004187 (1.313)	-0.2034 (-20.957)	4.220E-08 (0.424)	0.3598
	MM	0.00001041 (0.589)	-0.0523 (-3.844)	-0.0073 (-4.734)	0.2652	0.00000213 (0.104)	-0.1565 (-8.603)	0.0004 (0.245)	0.3170	0.00008097 (2.876)	-0.0100 (-5.108)	-0.0100 (-4.751)	0.3768

[a] The sample is NASDAQ/NMS stocks in the months of October, November, and December 1984.
[b] cov_I is based on intraday transaction prices.
[c] P = average stock price in month.
[d] $TURN$ = average daily volume divided by shares outstanding.
[e] $\$VOL$ = average daily dollar volume of trading in the month.
[f] MM = average number of market makers in the stock during the month.
[g] cov_C is based on closing transaction prices.

negative the covariance. That result would be consistent with greater adverse information costs in high-priced stocks, but such an explanation is implausible.[9]

A substantial effect seems to be associated with the number of market makers in a stock. The variable MM is significant and consistently negative in each of of the cov_I regressions, and it is significant and negative in two of the cov_C regressions. However, the independent variables S^2 and $MM \cdot S^2$ are highly correlated. (Correlations are 0.87, 0.90, and 0.90 in October, November, and December, respectively.) The lack of stability in the coefficient of S^2 and the reduced significance of the coefficient reflect that collinearity.[10] The negative sign on $MM \cdot S^2$ appears therefore to reflect the same effect as S^2.

On the basis of these diagnostics, it does not appear that stock characteristics influence the coefficient, a_1, very much, if at all. Cross-sectional differences are reflected in S but not in the coefficient, a_1, that relates S^2 to the serial covariance.

VII. Summary and Conclusions

Serial covariances of returns calculated from NASDAQ/NMS transactions data are negative and strongly negatively associated with the square of quoted spreads, as predicted by Roll [15].

Serial covariances of returns calculated from price quotations tend to be negatively associated with the square of quoted spreads, although this association is much weaker than in the case of covariances calculated from transactions data. This result is evidence of an inventory adjustment component of spreads in which quoted prices are adjusted to induce transactions that tend to equilibrate a dealer's inventory.

The model developed in this paper and the empirical estimates imply a transaction process in which prices have a probability, $\pi = 0.550$, of reversing and exhibit a reversal magnitude of $0.734S$, where S is the quoted proportional spread of the stock.

The average realized spread—the expected return to a round trip transaction in a stock—is approximately $0.57S$ is NASDAQ/NMS stocks. This relation between the realized and quoted spread is consistent with results for NYSE stocks.

On the basis of the empirical estimates in the paper, the quoted spread, S, may be decomposed into the following three components.

Adverse Information Costs	$0.43S$
Inventory Holding Costs	$0.10S$
Order Processing Costs	$0.47S$

While the quoted spread varies considerably across stocks, the components of the spread appear to be an invariant proportion of the quoted spread.

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[9] Alternatively, the result may reflect a tendency for discreteness in stock price changes to induce negative serial dependence that is more for low-priced stocks. (See Harris [10].) Inclusion of the price variable makes the coefficient of S^2 somewhat more negative than in Table II in the cov_I regression.

[10] The variable $P \cdot S^2$ is also collinear with S^2. That correlation exceeds 0.75 in each month. The variables $\$VOL \cdot S^2$ and $TURN \cdot S^2$ are not correlated with S^2 to the same degree.

Appendix: Derivation of Serial Covariances

A. Serial Covariance of Transaction Price Changes

Based on Figure 2, the joint distribution of successive transaction price changes may be depicted as follows.

		ΔP_t			
		Initial Trade at Bid		Initial Trade at Ask	
		$-\partial S$ (BB)	$(1 - \partial)S$ (BA)	∂S (AA)	$-(1 - \partial)S$ (AB)
ΔP_{t+1}	$-\partial S$ (BB)	$(1 - \pi)^2$	0	0	$\pi(1 - \pi)$
	$(1 - \partial)S$ (BA)	$\pi(1 - \pi)$	0	0	π^2
	∂S (AA)	0	$\pi(1 - \pi)$	$(1 - \pi)^2$	0
	$-(1 - \partial)S$ (AB)	0	π^2	$\pi(1 - \pi)$	0

In parentheses next to each possible price change, ∂S or $(1 - \partial)S$, is an indication of whether the transaction price change is a continuation (AA or BB) or a reversal (AB or BA). Assuming that the last trade occurs with equal probability at the bid or ask, the serial covariance of transaction price changes is

$$\text{cov}_T \equiv \text{cov}(\Delta P_t, \Delta P_{t+1}) = (1 - \pi)^2 \partial^2 S^2 - \partial S(1 - \partial)S\pi(1 - \pi)$$

$$+ \partial S(1 - \partial)S\pi(1 - \pi) - (1 - \partial)^2 S^2 \pi^2,$$

$$\text{cov}_T = S^2[\partial^2(1 - 2\pi) - \pi^2(1 - 2\partial)]. \tag{5}$$

B. Serial Covariance of Quoted Price Changes

The joint distribution of successive changes in bid prices is simpler than the joint distribution of successive transaction price changes because the initial change in bid price, $\Delta B_t = B_t - B_{t-1}$, can take on only two values: $-\partial S$ if the transaction at $t - 1$ occurred at the bid price and $+\partial S$ if the transaction at $t - 1$ occurred at the ask price. The next change in bid prices $\Delta B_{t+1} = B_{t+1} - B_t$, can again take on the same two values, depending on whether the transaction at t takes place at B_t or A_t. The following table gives the joint distribution of successive changes in the bid price.

		ΔB_t	
		Initial Trade at Bid	Initial Trade at Ask
		$-\partial S$	∂S
$\Delta B_t + 1$	Next Trade at Bid	$-\partial S$	$1 - \pi$	π
	Next Trade at Ask	∂S	π	$1 - \pi$

Assuming that the initial trade occurs with equal probability at the bid and ask, the serial covariance of changes in the bid price is

$$\text{cov}_B \equiv \text{cov}(\Delta B_t, \Delta B_{t+1}) = (1 - \pi)\partial^2 S^2 - \pi\partial^2 S^2,$$

$$\text{cov}_B = \partial^2 S^2(1 - 2\pi).$$

Under the assumption of symmetry and constant spread,

$$\text{cov}_A \equiv \text{cov}(\Delta A_t, \Delta A_{t+1}) = \text{cov}_B.$$

As a result,

$$\text{cov}_Q \equiv \text{cov}(\Delta Q_t, \Delta Q_{t+1}) = \partial^2 S^2 (1 - 2\pi), \qquad Q = A, B. \tag{6}$$

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