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# ASSET PRICING AND THE BID-ASK SPREAD*

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This paper studies the effect of the bid–ask spread on asset pricing. We analyze a model in which investors with different expected holding periods trade assets with different relative spreads. The resulting testable hypothesis is that market-observed expected return is an increasing and concave function of the spread. We test this hypothesis, and the empirical results are consistent with the predictions of the model.

## 1. Introduction

Liquidity, marketability or trading costs are among the primary attributes of many investment plans and financial instruments. In the securities industry, portfolio managers and investment consultants tailor portfolios to fit their clients' investment horizons and liquidity objectives. But despite its evident importance in practice, the role of liquidity in capital markets is hardly reflected in academic research. This paper attempts to narrow this gap by examining the effects of illiquidity on asset pricing.

Illiquidity can be measured by the cost of immediate execution. An investor willing to transact faces a tradeoff: He may either wait to transact at a favorable price or insist on immediate execution at the current bid or ask price. The quoted ask (offer) price includes a premium for immediate buying, and the bid price similarly reflects a concession required for immediate sale. Thus, a natural measure of illiquidity is the spread between the bid and ask

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prices, which is the sum of the buying premium and the selling concession.[1] Indeed, the relative spread on stocks has been found to be negatively correlated with liquidity characteristics such as the trading volume, the number of shareholders, the number of market makers trading the stock and the stock price continuity.[2]

This paper suggests that expected asset returns are increasing in the (relative) bid–ask spread. We first model the effects of the spread on asset returns. Our model predicts that higher-spread assets yield higher expected returns, and that there is a clientele effect whereby investors with longer holding periods select assets with higher spreads. The resulting testable hypothesis is that asset returns are an increasing and concave function of the spread. The model also predicts that expected returns net of trading costs increase with the holding period, and consequently higher-spread assets yield higher net returns to their holders. Hence, an investor expecting a long holding period can gain by holding high-spread assets.

We test the predicted spread–return relation using data for the period 1961–1980, and find that our hypotheses are consistent with the evidence: Average portfolio risk-adjusted returns increase with their bid–ask spread, and the slope of the return–spread relationship decreases with the spread. Finally, we verify that the spread effect persists when firm size is added as an explanatory variable in the regression equations. We emphasize that the spread effect is by no means an anomaly or an indication of market inefficiency; rather, it represents a rational response by an efficient market to the existence of the spread.

This study highlights the importance of securities market microstructure in determining asset returns, and provides a link between this area and mainstream research on capital markets. Our results suggest that liquidity-increasing financial policies can reduce the firm's opportunity cost of capital, and provide measures for the value of improvements in the trading and exchange process.[3] In the area of portfolio selection, our findings may guide investors in balancing expected trading costs against expected returns. In sum, we demonstrate the importance of market-microstructure factors as determinants of stock returns.

In the following section we present a model of the return–spread relation and form the hypotheses for our empirical tests. In section 3 we test the

[1] Demsetz (1968) first related the spread to the cost of transacting. See also Amihud and Mendelson (1980, 1982), Phillips and Smith (1982), Ho and Stoll (1981, 1983), Copeland and Galai (1983), and West and Tinic (1971). For an analysis of transaction costs in the context of a fixed investment horizon, see Chen, Kim and Kon (1975), Levy (1978), Milne and Smith (1980), and Treynor (1980).

[2] See, e.g., Garbade (1982) and Stoll (1985).

[3] See, e.g., Mendelson (1982, 1985, 1986, 1987), Amihud and Mendelson (1985, 1986) for the interaction between market characteristics, trading organization and liquidity.

predicted relationship, and in section 4 we relate our findings to the firm size anomaly. Our concluding remarks are offered in section 5.

## 2. A model of the return–spread relation

In this section we model the role of the bid–ask spread in determining asset returns. We consider $M$ investor types numbered by $i = 1, 2, \ldots, M$, and $N+1$ capital assets indexed by $j = 0, 1, 2, \ldots, N$. Each asset $j$ generates a perpetual cash flow of $\$d_j$ per unit time ($d_j > 0$) and has a relative spread of $S_j$, reflecting its trading costs. Asset 0 is a zero-spread asset ($S_0 = 0$) having unlimited supply. Assets are perfectly divisible, and one unit of each positive-spread asset $j$ ($j = 1, 2, \ldots, N$) is available.

Trading is performed via competitive market makers who quote assets' bid and ask prices and stand ready to trade at these prices. The market makers bridge the time gaps between the arrivals of buyers and sellers to the market, absorb transitory excess demand or supply in their inventory positions, and are compensated by the spread, which is competitively set. Thus, they quote for each asset $j$ an ask price $V_j$ and a bid price $V_j(1 - S_j)$, giving rise to two price vectors: an ask price vector $(V_0, V_1, \ldots, V_N)$ and a bid price vector $(V_0, V_1(1 - S_1), \ldots, V_N(1 - S_N))$.[4]

A type-$i$ investor enters the market with wealth $W_i$ used to purchase capital assets (at the quoted ask prices). He holds these assets for a random, exponentially distributed time $T_i$ with mean $E[T_i] = 1/\mu_i$, liquidates his portfolio by selling it to the market makers at the bid prices, and leaves the market. We number investor types by increasing expected holding periods, $\mu_1^{-1} \le \mu_2^{-1} \le \cdots \le \mu_M^{-1}$, and assets by increasing relative spreads, $0 = S_0 \le S_1 \le \cdots \le S_N < 1$. Finally, we assume that the arrivals of type-$i$ investors to the market follow a Poisson process with rate $\lambda_i$, with the interarrival times and holding periods being stochastically independent.

In statistical equilibrium, the number of type-$i$ investors with portfolio holdings in the market has a Poisson distribution with mean $m_i = \lambda_i/\mu_i$ [cf. Ross (1970, ch. 2)]. The market makers' inventories fluctuate over time to accommodate transitory excess demand or supply disturbances, but their *expected* inventory positions are zero, i.e., market makers are 'seeking out the market price that equilibrates buying and selling pressures' [Bagehot (1971, p. 14); see also Garman (1976)]. This implies that the expected sum of investors' holdings in each positive-spread asset is equal to its available supply of one unit.

Consider now the portfolio decision of a type-$i$ investor facing a given set of bid and ask prices, whose objective is to maximize the expected discounted net

[4] Competition among market makers drives the spread to the level $S_j$ of trading costs. In a different scenario, $V_j$ may be viewed as the sum of the market price and the buying transaction cost, and $V_j(1 - S_j)$ as the price net of the cost of a sell transaction.

cash flows received over his planning horizon. The discount rate $\rho$ is the spread-free, risk-adjusted rate of return on the zero-spread asset. Let $x_{ij}$ be the quantity of asset $j$ acquired by the type-$i$ investor. We call the vector $\{x_{ij}, j = 0, 1, 2, \dots, N\}$ 'portfolio $i$'. The expected present value of holding portfolio $i$ is the sum of the expected discounted value of the continuous cash stream received over its holding period and the expected discounted liquidation revenue. This sum is given by

$$\mathrm{E}_{T_i}\left\{\int_0^{T_i} \mathrm{e}^{-\rho y}\left[\sum_{j=0}^{N} x_{ij} d_j\right] \mathrm{d}y\right\} + \mathrm{E}_{T_i}\left\{\mathrm{e}^{-\rho T_i} \sum_{j=0}^{N} x_{ij} V_j (1 - S_j)\right\}$$

$$= (\mu_i + \rho)^{-1} \sum_{j=0}^{N} x_{ij}\left[d_j + \mu_i V_j (1 - S_j)\right].$$

Thus, for *given* vectors of bid and ask prices, a type-$i$ investor solves the problem

$$\max \sum_{j=0}^{N} x_{ij}\left[d_j + \mu_i V_j (1 - S_j)\right], \tag{1}$$

subject to

$$\sum_{j=0}^{N} x_{ij} V_j \le W_i \quad \text{and} \quad x_{ij} \ge 0 \quad \text{for all} \quad j = 0, 1, 2, \dots, N, \tag{2}$$

where condition (2) expresses the wealth constraint and the exclusion of investors' short positions.[5] Under our specification, the usual market clearing conditions read

$$\sum_{i=1}^{M} m_i x_{ij} = 1, \qquad j = 1, 2, \dots, N \tag{3}$$

(recall that $m_i$ is the expected number of type-$i$ investors in the market).

When an $M \times (N+1)$ matrix $\boldsymbol{X}^*$ and an $(N+1)$-dimensional vector $\boldsymbol{V}^*$ solve the $M$ optimization problems (1)–(2) such that (3) is satisfied, we call $\boldsymbol{X}^*$ an equilibrium allocation matrix and $\boldsymbol{V}^*$ – an equilibrium ask price vector [the corresponding bid price vector is $(V_0^*, V_1^*(1 - S_1), \dots, V_N^*(1 - S_N))$]. The

[5] In our context, the use of short sales cannot eliminate the spread effect, since short sales by themselves entail additional transaction costs. Note that a constraint on short positions is necessary in models of tax clienteles [cf. Miller (1977), Litzenberger and Ramaswamy (1980)]. Clearly, market makers are allowed to have transitory long or short positions, but are constrained to have zero expected inventory positions [cf. Garman (1976)].

above model may be viewed as a special case of the linear exchange model [cf. Gale (1960)], which is known to have an equilibrium allocation and a unique equilibrium price vector. Our model enables us to derive and interpret the resulting equilibrium in a straightforward and intuitive way as follows.

We define the expected *spread-adjusted return* of asset $j$ to investor-type $i$ as the difference between the gross market return on asset $j$ and its expected liquidation cost per unit time:

$$r_{ij} = d_j/V_j - \mu_i S_j, \tag{4}$$

where $d_j/V_j$ is the gross return on security $j$, and $\mu_i S_j$ is the *spread-adjustment*, or expected liquidation cost (per unit time), equal to the product of the liquidation probability per unit time by the percentage spread. Note that the spread-adjusted return depends on *both* the asset $j$ and the investor-type $i$ (through the expected holding period).

For a given price vector $V$, investor $i$ selects for his portfolio the assets $j$ which provide him the highest spread-adjusted return, given by

$$r_i^* = \max_{j=0,1,2,\ldots,N} r_{ij}, \tag{5}$$

with $r_1^* \le r_2^* \le r_3^* \le \cdots \le r_M^*$, since, by (4), $r_{ij}$ is a non-decreasing function of $i$ for all $j$. These inequalities state that the spread-adjusted return on a portfolio increases with the expected holding period. That is, investors with longer expected holding periods will earn higher returns *net* of transaction costs.[6]

The *gross* return required by investor $i$ on asset $j$ is given by $r_i^* + \mu_i S_j$, which reflects both the required spread-adjusted return $r_i^*$ and the expected liquidation cost $\mu_i S_j$. The equilibrium gross (market-observed) return on asset $j$ is determined by its highest-valued use, which is in the portfolio $i$ with the minimal required return, implying that

$$d_j/V_j^* = \min_{i=1,2,\ldots,M} \left\{ r_i^* + \mu_i S_j \right\}. \tag{6}$$

Eq. (6) can also be written in the form

$$V_j^* = \max_{i=1,2,\ldots,M} \left\{ d_j/\left(r_i^* + \mu_i S_j\right) \right\}, \tag{7}$$

[6] This is consistent with the suggestions that while the illiquidity of investments such as real estate [Fogler (1984)] coins [Kane (1984)] and stamps [Taylor (1983)] excludes them from short-term investment portfolios, they are expected to provide superior performance when held over a long investment horizon (the same may apply to stock-exchange seats) [Schwert (1977)]. See also Day, Stoll and Whaley (1985) on the clientele of small firms, and Elton and Gruber (1978) on tax clienteles.

implying that the equilibrium value of asset $j$, $V_j^*$, is equal to the present value of its perpetual cash flow, discounted at the gross return $(r_i^* + \mu_i S_j)$. Alternatively, $V_j^*$ can be written as the difference between (i) the present value of the perpetual cash stream $d_j$ and (ii) the present value of the expected trading costs for all the present and future holders of asset $j$, where both are discounted at the spread-adjusted return of the holding investor. To see this, assume that the available quantity of asset $j$ is held by type-$i$ investors; then (7) can be written as

$$V_j^* = d_j/r_i^* - \mu_i V_j^* S_j/r_i^*,$$

where the first term is, obviously, (i). As for the second, the expected quantity of asset $j$ sold per unit time by type-$i$ investors is $\mu_i$, and each sale incurs a transaction cost of $V_j^* S_j$; thus, $\mu_i V_j^* S_j/r_i^*$ is the expected present value (discounted at $r_i^*$) of the transaction-cost cash flow.

The implications of the above equilibrium on the relation between returns, spreads and holding periods are summarized by the following propositions.

*Proposition 1 (clientele effect). Assets with higher spreads are allocated in equilibrium to portfolios with (the same or) longer expected holding periods.*

*Proof.* Consider two assets, $j$ and $k$, such that in equilibrium asset $j$ is in portfolio $i$ and asset $k$ is in portfolio $i+1$ (recall that $\mu_i \geq \mu_{i-1}$). Applying (5), we obtain $r_{ij} \geq r_{ik}$ and $r_{i+1,k} \geq r_{i+1,j}$; thus, substituting from (4), $d_j/V_j^* - \mu_i S_j \geq d_k/V_k^* - \mu_i S_k$ and $d_k/V_k^* - \mu_{i+1} S_k \geq d_j/V_j^* - \mu_{i-1} S_j$, implying that $(\mu_i - \mu_{i+1})(S_k - S_j) \geq 0$. It follows that if $\mu_i > \mu_{i-1}$, we must have $S_k \geq S_j$. The case of non-consecutive portfolios immediately follows. Q.E.D.

*Proposition 2 (spread–return relationship). In equilibrium, the observed market (gross) return is an increasing and concave piecewise-linear function of the (relative) spread.*

*Proof.* Let $f_i(S) = r_i^* + \mu_i S$. By (6), the market return on an asset with relative spread $S$ is given by $f(S) \equiv \min_{i=1,2,\ldots,M} f_i(S)$. Now, the proposition follows from the fact that monotonicity and concavity are preserved by the minimum operator, and that the minimum of a finite collection of linear functions is piecewise-linear. Q.E.D.

Proposition 2 is the main testable implication of our model. Intuitively, the positive association between return and spread reflects the compensation required by investors for their trading costs, and its concavity results from the clientele effect (Proposition 1). To see this, recall that transaction costs are amortized over the investor's holding period. The longer this period, the

Table 1

An example of the equilibrium relation between asset bid–ask spreads, returns and values (see section 2). There are 10 assets ($j$), each generating $1 per period, with relative bid–ask spreads $S_j$ (= dollar spread divided by asset value) ranging from 0 to 0.045 (column 2), and 4 investor types ($i$) with expected holding periods, $\mu_i^{-1}$, of 1/12, 1/2, 1 and 5 periods.[a] The return on the zero-spread asset is $\rho$; all returns are measured in excess of $\rho$. A type-$i$ investor chooses the assets $j$ which maximize his spread-adjusted return, $r_{ij}$, given by the difference between the gross market return on asset $j$ and its expected liquidation cost per unit time. The equilibrium solution gives the excess spread-adjusted returns, $r_{ij} - \rho$, in columns 3–6, where the boxes highlight the assets with the highest excess spread-adjusted return for each investor-type. The equilibrium portfolio for each investor-type is composed of the boxed assets. Column 7 shows the assets' equilibrium excess gross returns observed in the market, which include the expected liquidation cost to their holders. Column 8 shows the resulting asset values, obtained by discounting the perpetuity by the respective equilibrium market return, as a fraction of the value of the zero-spread asset.

| | | Investor type, $i$ | | | | Market return in excess of $\rho$, the return on the zero-spread asset | Value of asset $j$ relative to that of the zero-spread asset, $V_j/V_0$ |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | | |
| | Relative bid–ask spread, $S_j$ | Length of holding period, $\mu_i^{-1}$ | | | | | |
| Asset, $j$ | | 1/12 | 1/2 | 1 | 5 | | |
| | | Excess spread-adjusted return, $r_{ij} - \rho$ | | | | | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| 0 | 0 | $\boxed{0}$ | 0 | 0 | 0 | 0 | 1 |
| 1 | 0.005 | $\boxed{0}$ | 0.05 | 0.055 | 0.059 | 0.06 | 0.943 |
| 2 | 0.01 | $\boxed{0}$ | $\boxed{0.10}$ | 0.11 | 0.118 | 0.12 | 0.893 |
| 3 | 0.015 | −0.05 | $\boxed{0.10}$ | 0.115 | 0.127 | 0.13 | 0.885 |
| 4 | 0.02 | −0.10 | $\boxed{0.10}$ | $\boxed{0.12}$ | 0.136 | 0.14 | 0.877 |
| 5 | 0.025 | −0.155 | 0.095 | $\boxed{0.12}$ | 0.140 | 0.145 | 0.873 |
| 6 | 0.03 | −0.21 | 0.09 | $\boxed{0.12}$ | 0.144 | 0.15 | 0.870 |
| 7 | 0.035 | −0.265 | 0.085 | $\boxed{0.12}$ | $\boxed{0.148}$ | 0.155 | 0.866 |
| 8 | 0.04 | −0.324 | 0.076 | 0.116 | $\boxed{0.148}$ | 0.156 | 0.865 |
| 9 | 0.045 | −0.383 | 0.067 | 0.112 | $\boxed{0.148}$ | 0.157 | 0.864 |

[a] Investors have the same wealth, and the expected number of investors of each type is 1.

smaller the compensation required for a given increase in the spread. Since in equilibrium higher-spread securities are acquired by investors with longer horizons, the added return required for a given increase in spread gets smaller. In terms of our model, longer-holding-period portfolios contain higher-spread assets and have a lower slope $\mu_i$ for the return–spread relation.

A simple numerical example can illustrate the spread–return relation. Assume $N = 9$ positive-spread assets and $M = 4$ investor types whose expected holding periods are $1/\mu_1 = 1/12$, $1/\mu_2 = 1/2$, $1/\mu_3 = 1$, and $1/\mu_4 = 5$. For simplicity we set $\lambda_i = \mu_i$, implying that the expected number of investors of each type $i$ is $m_i = 1$. Assets yield $d_j = \$1$ per period, and all investors have equal wealth. The relative spread of asset $j$ is $S_j = 0.005j$, $j = 0, 1, 2, \ldots, 9$; thus, asset percentage spreads range from zero to 4.5%.

Using this data, we solve (1)–(3) and obtain the results in table 1 and figs. 1 and 2. Note that the additional excess return per unit of spread goes down

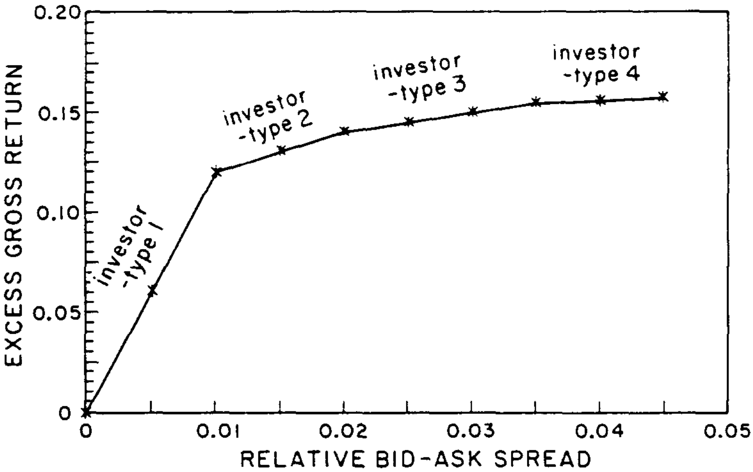


Fig. 1. An illustration of the relation between observed market return in excess of the return on the zero-spread asset (the excess gross return) and the relative bid–ask spread (see the numerical example of section 2 and table 1, column 7). There are 10 assets, each generating \$1 per period, with relative bid–ask spreads (= dollar spread divided by asset value) ranging from 0 to 0.045, and 4 investor types with expected holding periods ranging from 1/12 to 5 periods. Investors have equal wealth, and the expected number of investors of each type is 1.
The relation between asset returns and bid–ask spreads is piecewise-linear, increasing and concave, with each linear section corresponding to the portfolio of a different investor type.

from $\mu_1 = 12$ in portfolio 1 to $\mu_2 = 2$ for portfolio 2, then to $\mu_3 = 1$ in portfolio 3, and finally to $\mu_4 = 0.2$ in portfolio 4. The behavior of the excess market return as a function of the spread is shown in fig. 1, which demonstrates both the positive compensation for higher spread and the clientele effect which moderates the excess returns, especially for the high-spread assets. This figure summarizes the main testable implications of our model: The observed market return should be an increasing and concave function of the relative spread. The piecewise-linear functional form suggested by our model provides a specific and detailed set of hypotheses tested in the next section. The effect of the spread on asset values (or prices) is demonstrated in fig 2: the equilibrium values are decreasing and convex in the spread.

While the above model provides a lucid demonstration of the spread–return (or spread–price) relation, our main results do not hinge on its specific form, and hold as well under different specifications. Consider $(N+1)$ assets, each generating the same stochastic (gross) cash flow given by the process $\{X(t), t \geq 0\}$. Assume that each transaction in asset $j$ entails a cost of $\$c_j$, with $0 = c_0 < c_1 < c_2 < \cdots < c_N$ (asset 0 having zero spread). There are $M$ investor types numbered by $i = 1, 2, \ldots, M$, and the transaction epochs of type-$i$ investors follow a renewal process with given parameters (depending on $i$).[7]

[7]An investor could be viewed as owning a number of portfolios with different liquidation horizons, without changing the results.

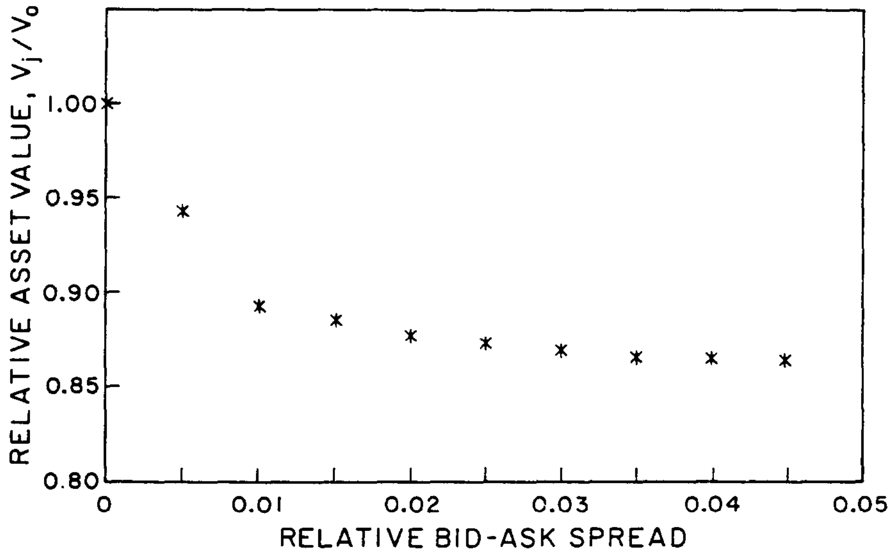


Fig. 2. The relation between asset values and bid–ask spreads for the numerical example of section 2 (see table 1, column 8, and fig. 1). The figure depicts the value of each asset $j$ relative to the value of the zero-spread asset, $V_j/V_0$, as a function of the bid–ask spread relative to the asset's value. Asset values are a decreasing function of the spread.

Denote the highest price a type-$i$ investor will pay for asset $j$ by $V_{ij}$. When the price of each asset $j$ is determined by its highest-valued use, we have $V_j = \max_{i=1,2,\dots,M} V_{ij}$ with $V_{ij} = V_{i0} - c_j\theta_i$, where $\theta_i$ is the value (for investor-type $i$) of \$1 at each transaction epoch. Letting $f_i(c) = V_{i0} - c\theta_i$, and following the arguments of Proposition 2, we obtain that the price [given by $\max_{i=1,2,\dots,M} f_i(c)$] is decreasing and convex in $c$. Further, it can be shown that the price is a decreasing and convex function of the relative transaction cost, thus demonstrating the robustness of our results. Qualitatively, similar results will hold as long as a longer investment horizon mitigates the burden of transaction costs by enabling their amortization over a longer holding period.

The next section presents empirical tests of our main testable hypotheses (Proposition 2).

## 3. Empirical tests

This section presents an empirical examination of the relation between expected returns and bid–ask spreads of NYSE stocks, focusing on the particular functional relationship predicted by our model. Specifically, our hypothesis is that expected return is an increasing and concave function of the spread.

### 3.1. The data and the derivation of the variables

Our data consist of monthly securities returns provided by the Center for Research in Security Prices and relative bid–ask spreads collected for NYSE stocks from *Fitch's Stock Quotations on the NYSE*. The relative spread is the dollar spread divided by the average of the bid and ask prices at year end. The actual spread variable used, $S$, is the average of the beginning and end-of-year relative spreads for each of the years 1960–1979 [the data is the same as in Stoll and Whaley (1983)].

The relationship between stock returns, relative risk[8] ($\beta$) and spread[9] is tested over the period 1961–1980. Following the methodology developed by Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Black and Scholes (1974), we first formed portfolios by grouping stocks according to their spread and relative risk, and then tested our hypotheses by examining the cross-sectional relation between average excess return, spread and relative risk over time. We divided the data into twenty overlapping periods of eleven years each, consisting of a five-year $\beta$ estimation period $E_n$, a five-year portfolio formation period $F_n$, and a one-year cross-section test period $T_n$ ($n = 1, 2, \ldots, 20$).[10] The three subperiods of each eleven-year period are now considered in detail:

(i) *The beta estimation period* $E_n$ was used to estimate the $\beta$ coefficients from the market model regressions

$$R^e_{jt} = \alpha_j + \beta_j R^e_{mt} + \varepsilon_{jt}, \qquad t = 1, \ldots, 60,$$

where $R^e_{jt}$ and $R^e_{mt}$ are the month-$t$ excess returns (over the 90-day T-bill rates) on stock $j$ and on the market,[11] respectively, and $\beta_j$ is the estimate of the relative risk[12] of stock $j$.

(ii) *The portfolio formation period* $F_n$ was used to form the test portfolios and estimate their $\beta$ and spread parameters. All stocks traded through the

[8] By the CAPM, the $\beta$ risk is the major determinant of asset returns. Our analysis in section 2 dealt with certainty-equivalent rates of return.

[9] The cost of transacting also includes brokerage commissions. In Stoll and Whaley (1983), the correlation between portfolio spreads and brokerage fees was 0.996, hence we omitted the latter.

[10] To illustrate, $E_1 = 1951$–$1955$, $F_1 = 1956$–$1960$, $T_1 = 1961$; $E_2 = 1952$–$1956$, $F_2 = 1957$–$1961$, $T_2 = 1962$; … $E_{20} = 1970$–$1974$, $F_{20} = 1975$–$1979$, $T_{20} = 1980$.

[11] Throughout this study, $R_m$ and the test portfolios are equally weighted. See Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Stoll and Whaley (1983, p. 71).

[12] Jensen (1968) has shown that the measure of relative risk, $\beta_j$, may be used for a holding period of any length (p. 189).

entire eleven-year period $n$ and for which the spread was available for the last year of $F_n$ were ranked by that spread and divided into seven equal groups. Within each of the seven spread groups, stocks were ranked by their $\beta$ coefficients, obtained from $E_n$, and divided into seven equal subgroups. This yields 49 ($7 \times 7$) equal-sized portfolios,[13] with significant variability of the spreads as well as the betas within the spread groups. Then, we estimated $\beta$ for each portfolio from the market model regression over the months of $F_n$,

$$R^e_{pt} = \alpha_p + \beta_p R^e_{mt} + \varepsilon_{pt}, \qquad t = 1,\dots,60, \quad p = 1,\dots,49,$$

where $R^e_{pt}$ is the average[14] excess return of the securities included in portfolio $p$ in month $t$. Finally, we calculated the portfolio spread $S_{pn}$ by averaging the spreads (of the last year of $F_n$) across the stocks in portfolio $p$. Each portfolio $p$ in period $n$ is thus characterized by the pair $(\beta_{pn}, S_{pn})$ ($p = 1,2,\dots,49$, $n = 1,2,\dots,20$). Altogether, we have 980 ($= 49 \times 20$) portfolios.

(iii) *The cross-section test period* $T_n$ was used to test the relation between $R^e_{pn}$, $\beta_{pn}$ and $S_{pn}$ across portfolios, where $R^e_{pn}$ is the average monthly excess return on the stocks in portfolio $p$ in $T_n$, the last year of period $n$.[15]

Table 2 presents summary statistics for the 49 portfolio groups, classified by spread and $\beta$. Note that both $\beta$ and the excess return increase with the spread. The correlation coefficients between the portfolio excess returns $R^e_p$, the portfolio betas $\beta_p$ and the spreads $S_p$, presented in table 3, show that both $\beta_p$ and $S_p$ are positively correlated with excess returns; the correlation between $R^e_p$ and the spread over the twenty-year period is about twice as high as that between $R^e_p$ and $\beta$. Also, note the high positive correlation between $\beta$ and the spread.

### 3.2. Test methodology

We now turn to test the major hypothesis of model, namely, that expected return is an increasing and concave function of the relative spread. This is a classical case of covariance analysis and pooling of cross-section and time-series data [see Kmenta (1971, ch. 12-2), Maddala (1977, ch. 14), Judge et al. (1980, ch. 8)], where the estimation model has to allow for differences over cross-sec-

[13] The long trading-period requirement might have eliminated from our sample the riskier and higher-spread stocks, thus reducing the variability of the data. Throughout, 'equal' portfolios may differ from one another by one security due to indivisibility.

[14] Throughout, averaging means arithmetic averaging.

[15] Note that our test is predictive in nature, using estimates of risk and spread which are available at the beginning of the test period. See Fama (1976, 349–351).

Table 2

Average relative bid–ask spread, monthly excess return, relative risk ($\beta$) and firm size for the 49 portfolios for the 20 test-period years 1961–1980. Portfolios are indexed by the spread group $i$ ($i = 1$ for the smallest spread) and by the beta group $j$ ($j = 1$ for the smallest beta). Portfolio composition changes every year and the sample size ranges between 619 and 900 stocks.

The relative bid–ask spread of a stock is its dollar spread divided by the average of the bid and ask prices at year end. The portfolio spread is the average relative spread of stocks in the portfolio.

The portfolio (monthly) excess return is the 12-month arithmetic average of the monthly average returns on the stocks in the portfolio in excess of that month's Treasury-Bill rate.

The portfolio beta is the average relative risk ($\beta$) coefficient for the stocks in the portfolio, estimated over the 5 years preceding the test period. Size is the market value of the firm's equity in millions of dollars at the end of the year preceding the test period, averaged over the firms included in the portfolio.

| Spread group, $i$ | | Beta group, $j$: 1 | 2 | 3 | 4 | 5 | 6 | 7 | Mean |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Spread | 0.004765 | 0.004850 | 0.004860 | 0.004789 | 0.004878 | 0.004891 | 0.004980 | 0.00486 |
| | Excess return | 0.002706 | 0.001306 | 0.003380 | 0.004409 | 0.003427 | 0.005416 | 0.003781 | 0.00349 |
| | Beta | 0.54001 | 0.67797 | 0.75890 | 0.77867 | 0.83231 | 0.91651 | 1.08973 | 0.799 |
| | Size | 4089.8 | 3245.5 | 3231.9 | 2317.3 | 1430.0 | 1418.8 | 595.7 | 2333 |
| 2 | Spread | 0.007435 | 0.007445 | 0.007463 | 0.007414 | 0.007508 | 0.007412 | 0.007452 | 0.00745 |
| | Excess return | 0.003174 | 0.003543 | 0.003549 | 0.004995 | 0.003050 | 0.006424 | 0.011061 | 0.00511 |
| | Beta | 0.55369 | 0.71874 | 0.81652 | 0.84596 | 0.90668 | 1.02999 | 1.21992 | 0.870 |
| | Size | 780.2 | 880.3 | 741.5 | 707.6 | 656.1 | 605.9 | 282.7 | 665 |
| 3 | Spread | 0.009392 | 0.009386 | 0.009400 | 0.009375 | 0.009339 | 0.009350 | 0.009425 | 0.00939 |
| | Excess return | 0.001838 | 0.003165 | 0.006707 | 0.002619 | 0.004473 | 0.006133 | 0.005063 | 0.00429 |
| | Beta | 0.56069 | 0.67271 | 0.79543 | 0.89866 | 1.00357 | 1.04518 | 1.20940 | 0.884 |
| | Size | 476.2 | 502.1 | 695.9 | 370.1 | 363.9 | 293.3 | 227.1 | 418 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4 | Spread | 0.011470 | 0.011473 | 0.011411 | 0.011464 | 0.011449 | 0.011487 | 0.011411 | 0.01145 |
| | Excess return | 0.003217 | 0.002447 | 0.005296 | 0.004521 | 0.008505 | 0.008033 | 0.009178 | 0.00589 |
| | Beta | 0.58821 | 0.69158 | 0.84828 | 0.92208 | 0.99515 | 1.07535 | 1.26739 | 0.913 |
| | Size | 331.9 | 362.7 | 370.6 | 248.4 | 250.5 | 192.4 | 174.5 | 276 |
| 5 | Spread | 0.014015 | 0.013913 | 0.013955 | 0.013998 | 0.013883 | 0.013969 | 0.013988 | 0.01396 |
| | Excess return | 0.002583 | 0.004340 | 0.003318 | 0.006763 | 0.008076 | 0.011460 | 0.010266 | 0.00669 |
| | Beta | 0.60153 | 0.71197 | 0.82031 | 0.92906 | 1.04923 | 1.12224 | 1.28927 | 0.932 |
| | Size | 243.1 | 257.3 | 213.6 | 166.3 | 149.2 | 146.2 | 111.3 | 184 |
| 6 | Spread | 0.017662 | 0.017513 | 0.017699 | 0.017759 | 0.017789 | 0.017763 | 0.017967 | 0.01774 |
| | Excess return | 0.003637 | 0.006937 | 0.007209 | 0.007415 | 0.011254 | 0.010877 | 0.012516 | 0.00855 |
| | Beta | 0.65522 | 0.73861 | 0.87193 | 0.94479 | 1.07714 | 1.16769 | 1.33498 | 0.970 |
| | Size | 135.6 | 131.1 | 127.1 | 113.1 | 91.2 | 89.9 | 72.8 | 109 |
| 7 | Spread | 0.032890 | 0.029385 | 0.031614 | 0.031472 | 0.031647 | 0.033169 | 0.034385 | 0.03208 |
| | Excess return | 0.006683 | 0.008876 | 0.008044 | 0.007405 | 0.012335 | 0.013384 | 0.014929 | 0.01024 |
| | Beta | 0.76132 | 0.88340 | 0.99811 | 1.12656 | 1.23899 | 1.33249 | 1.46259 | 1.115 |
| | Size | 75.2 | 67.8 | 57.5 | 54.7 | 44.0 | 47.8 | 37.3 | 55 |
| Mean | Spread | 0.013947 | 0.013424 | 0.013772 | 0.013753 | 0.013792 | 0.014006 | 0.014230 | 0.01385 |
| | Excess return | 0.003405 | 0.004373 | 0.005357 | 0.005447 | 0.007303 | 0.008818 | 0.009542 | 0.00632 |
| | Beta | 0.60867 | 0.72785 | 0.84421 | 0.92083 | 1.01472 | 1.09849 | 1.26761 | 0.926 |
| | Size | 876 | 778 | 777 | 568 | 426 | 399 | 214 | 577 |

Table 3

Correlation coefficients between the annual average portfolio spread $S_p$, excess return $R_p^e$ and beta $\beta_p$ for the entire sample period 1961–1980 and for its two 10-year subperiods, 1961–1970 and 1971–1980. Portfolio spread is the average bid–ask spread as a fraction of the year-end average of the bid and ask prices for all securities in the portfolio. Excess returns are the average monthly returns in excess of the monthly T-Bill rate.

| | Correlation coefficient between | | | Number of |
|---|---|---|---|---|
| Period | $R_p^e$ and $S_p$ | $R_p^e$ and $\beta_p$ | $\beta_p$ and $S_p$ | observations |
| 1961–80 | 0.239 | 0.123 | 0.361 | 980 |
| 1961–70 | 0.179 | 0.132 | 0.163 | 490 |
| 1971–80 | 0.285 | 0.118 | 0.540 | 490 |

tional units (portfolios) and over time. This is done by employing two sets of dummy variables: The first set consists of 48 portfolio dummy variables, defined by $DP_{ij}=1$ if the portfolio is in group $(i,j)$ and zero otherwise; $i=1,2,\ldots,7$ is the spread-group index and $j=1,2,\ldots,7$ is the $\beta$-group index, with $DP_{7,7}\equiv 0$. By construction, the spread increases in $i$, and $\beta$ increases in $j$. A second set of dummy variables, defined by $DY_n=1$ in year $n$ ($n=1,2,\ldots,19$) and zero otherwise, accounts for differences in returns between years.

An important implication of our model is that the slope of the return–spread relation declines as we move to higher-spread groups. To allow for different slope coefficients across spread groups, we decomposed the spread variable $S_{pn}$ into seven variables $S_{pn}^i$ ($i=1,2,\ldots,7$) defined by $S_{pn}^i=S_{pn}$ if in spread group $i$ ($i=1,2,\ldots,7$) and zero otherwise. Due to the high correlation between $S_{pn}^i$ and $\sum_{j=1}^{7} DP_{ij}$, we constructed the mean-adjusted spread variables, $\hat{S}_{pn}^i = S_{pn}^i - \bar{S}^i$ if portfolio $(p,n)$ is in group $i$ and zero otherwise, where $\bar{S}^i$ is the mean spread for the $i$th spread group. The means of $\hat{S}_{pn}^i$ are zero and their correlations with $\sum_{j=1}^{7} DP_{ij}$ are zero. Replacing $S_{pn}^i$ by the mean-adjusted variables thus leads to a separation between the level effects among groups (captured by $DP_{ij}$) and the slope effects within spread groups (captured by $\hat{S}_{pn}^i$).

Using the above variables, we carried out the pooled cross-section and time-series estimation of our model:

$$R_{pn}^e = a_0 + a_1\beta_{pn} + \sum_{i=1}^{7} b_i\hat{S}_{pn}^i + \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}DP_{ij} + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}, \quad (8)$$

where $a_0$, $a_1$, $b_i$, $c_{ij}$ and $d_n$ are coefficients and the $\varepsilon_{pn}$ are the residuals. The slope coefficients $b_i$ measure the response of stock returns to increasing the spread *within* spread group $i$, and the dummy coefficients $c_{ij}$ measure the difference between the mean return on portfolio $(i, j)$ and that of portfolio (7, 7) which corresponds to the highest spread and $\beta$ group.

The sums $\sum_{i=1}^{7} c_{ij}$ measure the differences in mean returns between $\beta$ groups $j$, while $\sum_{j=1}^{7} c_{ij}$ measure the differences in mean returns between spread groups $i$. Thus, for any given $\beta$, model (8) represents a piecewise-linear functional form of the return–spread relation. This follows the Malinvaud (1970, pp. 317–318) and Kmenta (1971, pp. 468–469) methodology for estimating non-linear relationships, which groups the data based on the values of the explanatory variable, and fits a piecewise linear curve using two sets of variables: group dummies to capture differences between group means, and products of the explanatory variable by the group dummies to allow for the different slopes.

Estimation of the pooled model (8) using OLS is problematic due to the possibility of cross-sectional heteroskedasticity and cross-sectional correlations among residuals across portfolio groups. While the estimated OLS coefficients are unbiased and consistent, their estimated variances are not, leading to biased test statistics. This calls for a generalized least squares (GLS) estimation procedure. Given that the variance–covariance matrix of the residuals in (8) is $\sigma^2 V$, where $\sigma^2$ is a scalar and $V$ is a symmetric positive-definite matrix, the GLS procedure uses a matrix $Q$ satisfying $Q'Q = V^{-1}$ to transform all the regression variables by pre-multiplication. The variance–covariance matrix $V$ was assumed to be block diagonal (reflecting independence between years), where the diagonal blocks consist of twenty identical $49 \times 49$ positive definite matrices $U$. Then, $V = I \otimes U$, where $I$ is the $20 \times 20$ identity matrix and $\otimes$ denotes the Kronecker product. To obtain the $49 \times 49$ matrix $U$, we first estimated model (8) by OLS and then used the data month by month to obtain the residuals $\hat{\varepsilon}_{pm}$ ($p = 1, 2, \ldots, 49$) for each month $m$ ($m = 1, 2, \ldots, 240$). Then, we estimated $U$ by averaging the resulting 240 monthly variance–covariance matrices – the resulting estimate of the variance–covariance matrix $V$ is known to be consistent [cf. Kmenta (1971, ch. 12)]. The transformation matrix $Q$ was calculated using the Choleski decomposition method. The variables of model (8) were then pre-multiplied by the transformation matrix $Q$, and the transformed version of model (8) was estimated to provide the GLS results.

### 3.3. *The results*

We first ran a simple OLS regression of the excess returns on $\beta$, the spread and the nineteen-year dummy variables:

$$R^{e}_{pn} = 0.0040 + \underset{(9.17)}{0.00947}\beta_{pn} + \sum_{n=1}^{19} d_n DY_n + e_{pn},$$

and

$$R^e_{pn} = \underset{}{0.0036} + \underset{(6.18)}{0.00672\beta_{pn}} + \underset{(6.83)}{0.211 S_{pn}} + \sum_{n=1}^{19} d_n DY_n + e_{pn}.$$

($t$-statistics are in parentheses.) The results show that excess returns are increasing in both $\beta$ and the spread. The coefficient of $S_{pn}$ implies that a 1% increase in the spread is associated with a 0.211% increase in the monthly risk-adjusted excess return. The coefficient of $\beta$ declines when the spread variable is added to the equation, indicating that part of the effect which could be attributed to $\beta$ may, in fact, be due to the spread.[16] The coefficient of $\beta$ is 0.00672, very close to 0.00671, which is the average monthly excess return on common stocks for this period.

Next, we estimated the detailed model (8) using both OLS and GLS. The slope coefficients of the spread variables are presented in table 4, and the coefficients of $DP_{ij}$ are given in table 5. To estimate the pattern of the dummy coefficients, we employed the model

$$c_{ij} = \alpha + \sum_{i=1}^{6} \gamma_i DS_i + \sum_{j=1}^{6} \delta_j DB_j + e_{ij}, \tag{9}$$

where the spread dummy $DS_i$ ($i = 1,\dots,6$) is one if the portfolio is in spread group $i$ and zero otherwise, and the $\beta$ dummy $DB_j$ ($j = 1,\dots,6$) is one if the portfolio is in $\beta$ group $j$ and zero otherwise. Thus, the coefficients $\gamma_i$ in (9) measure the difference between the average return of spread group $i$ and that of the seventh (highest) spread group, and the coefficients $\delta_j$ measure the corresponding differences between $\beta$ groups.

The estimates of (8)–(9) presented in tables 4 and 6 support our two hypotheses:

(i) The coefficients $\gamma_i$ of $DS_i$ in model (9) are negative and generally increasing in $i$, implying that risk-adjusted excess returns increase with the spread. The difference in the monthly mean excess return between the two extreme spread groups is 0.857% when estimated by OLS and 0.681% when estimated by GLS.

(ii) The slope coefficients of the spreads, $b_i$, are positive and generally decreasing as we move to higher spread groups. This is consistent with the hypothesized concavity of the return–spread relation, reflecting the lower sensitivity of long-term portfolios to the spread.

[16] Given the strong positive correlation between $S_{pn}$ and $\beta_{pn}$, the omission of $S_{pn}$ from the regression equation which tests the CAPM results in an upward bias in the estimated coefficient of $\beta$; see Kmenta (1971, p. 392).

Table 4

Estimated regressions of the portfolio monthly excess returns, $R^e$, on the mean-adjusted spread variables $\hat{S}^i$ and relative risk, $\beta$, for the years 1961–1980, using ordinary least squares and generalized least squares estimation methods. The regression model (8)[a] applies pooled cross-section and time-series estimation.
The coefficient of $\hat{S}^i$ reflects the response of stock returns to an increase in the bid–ask spread within spread group $i$, where $i = 1$ corresponds to the lowest-spread group.
($t$-values are in parentheses).

| Independent variable | Ordinary least squares coefficients: Entire period 1961–1980 | Generalized least squares coefficients: Entire period 1961–1980 | Subperiod 1961–1970 | Subperiod 1971–1980 |
|---|---|---|---|---|
| $\hat{S}_1$ | 3.641 | 1.310 | 0.080 | 2.303 |
| | (2.76) | (1.16) | (0.05) | (1.27) |
| $\hat{S}_2$ | 3.242 | 1.747 | 0.975 | 2.505 |
| | (3.50) | (2.56) | (0.91) | (2.41) |
| $\hat{S}_3$ | 2.854 | 1.660 | 0.934 | 2.27 |
| | (3.93) | (3.01) | (1.10) | (2.80) |
| $\hat{S}_4$ | 1.657 | 0.482 | −0.149 | 0.983 |
| | (3.06) | (1.16) | (0.21) | (1.69) |
| $\hat{S}_5$ | 2.224 | 1.206 | 0.922 | 1.500 |
| | (5.69) | (3.84) | (1.67) | (3.47) |
| $\hat{S}_6$ | 1.365 | 0.650 | 0.838 | 0.475 |
| | (5.28) | (2.96) | (2.21) | (1.50) |
| $\hat{S}_7$ | 0.605 | 0.256 | 0.176 | 0.489 |
| | (5.28) | (2.56) | (1.49) | (2.49) |
| $\beta$ | −0.0058 | −0.000 | −0.002 | −0.003 |
| | (2.53) | (0.10) | (0.47) | (0.72) |

[a] The regression model is

$$R^e_{pn} = a_0 + a_1\beta_{pn} + \sum_{i=1}^{7} b_i\hat{S}^i_{pn} + \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}DP_{ij} + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}, \tag{8}$$

where $R^e_{pn}$ is the average excess return for portfolio $p$ in year $n$, $\beta_{pn}$ is the average portfolio relative risk, $\hat{S}^i_{pn}$ is the mean-adjusted spread within spread group $i$ (= the deviation of the spread of portfolio $p$ in year $n$ from the mean spread of its spread group, $i$), $DP_{ij}$ are the portfolio-group dummy variables (= 1 in portfolio group $(i, j)$, zero otherwise), $DY_n$ are the year dummy variables (= 1 in year $n$, 0 otherwise), and $\varepsilon_{pn}$ are the residuals. The GLS estimated coefficients of the portfolio-group dummies $DP_{ij}$ are reported in table 5.

The effect of the relative risk is measured in model (8) by both $\beta$ and the dummy variables and is further summarized by the $DB_j$ coefficients of model (9). The emerging pattern is that (spread-adjusted) excess returns increase with $\beta$ as depicted by the significant negative and increasing coefficients $\delta_j$. The effect of $\beta$ is captured mainly through the dummies rather than the coefficient $a_1$, which is highly insignificant in the GLS estimation. Finally, we estimated

Table 5

Generalized least squares estimates of the difference between the mean monthly excess return of the portfolio with the highest spread and beta – portfolio (7, 7), the 49th portfolio – and the mean monthly excess returns of each of the other 48 portfolios. These are the estimated coefficients of the 48 portfolio dummy variables $DP_{ij}$ in the pooled cross-section and time-series regression model (8), using GLS, over the entire period 1961–1980.

$t$-statistics for all unmarked table entries are greater than 1.96, implying that the estimated coefficient is significant at better than the 2.5% level (one-tail test).

| | Beta group, $j$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Spread group, $i$ | 1 (low) | 2 | 3 | 4 | 5 | 6 | 7 (high) | Mean |
| 1 (low) | −0.0117 | −0.0132 | −0.0111 | −0.0100 | −0.0111 | −0.0091 | −0.0108 | −0.0110 |
| 2 | −0.0113 | −0.0109 | −0.0109 | −0.0094 | −0.0115 | −0.0079 | −0.0033[b] | −0.0093 |
| 3 | −0.0127 | −0.0113 | −0.0078[a] | −0.0118 | −0.0100 | −0.0082 | −0.0094 | −0.0102 |
| 4 | −0.0113 | −0.0120 | −0.0091 | −0.0099 | −0.0059[a] | −0.0064 | −0.0052[b] | −0.0085 |
| 5 | −0.0120 | −0.0101 | −0.0111 | −0.0077 | −0.0062[a] | −0.0030[b] | −0.0041[b] | −0.0077 |
| 6 | −0.0108 | −0.0074[a] | −0.0072 | −0.0070 | −0.0032[b] | −0.0035[b] | −0.0020[b] | −0.0059 |
| 7 (high) | −0.0080 | −0.0049[b] | −0.0063 | −0.0068 | −0.0019[b] | −0.0013[b] | 0.0000 | −0.0042 |
| Mean | −0.0111 | −0.0100 | −0.0091 | −0.0089 | −0.0071 | −0.0056 | −0.0050 | |

[a] $1.645 < t < 1.96$, implying significance at better than the 5% level (one-tail test).
[b] $t < 1.645$, insignificant at the 5% level (one-tail test).

models (8)–(9) for the two ten-year subperiods, with generally the same pattern of results.

Detailed tests of our main hypotheses are presented in table 7. In 7(B), we test the significance of the spread effect by omitting all spread-related variables and examine the resulting increase in the unexplained variance. In 7(C) we test whether the mean excess returns of all spread groups are equal by eliminating all spread-related dummy variables. The significance of the non-linearities was tested in two ways: First we replaced all the spread-related variables (eliminating the $\hat{S}^i_{pn}$ and replacing the $DP_{ij}$ with six $\beta$ dummies) by the original spread variable $S_{pn}$ [see 7(D)]. Then we tested the equality of the slope coefficients across spread groups by re-estimating model (8), replacing the variables $\hat{S}^1$ through $\hat{S}^7$ by their sum [see 7(E)]. In all four cases, the $F$-tests for the changes in the sum of squared residuals reject the null hypotheses at better than the 0.01 level. Thus, our hypotheses are fully supported by the data.

## 4. Firm size, spread and return

The well-known negative relationship between spread and firm size suggests that our findings may bear on the 'small-firm anomaly': Banz (1981) and Reinganum (1981a, b) found a negative relation between risk-adjusted mean

Table 6

Regression estimates of the difference between the mean return of the spread and beta groups and the mean return of the highest-spread and highest-beta portfolio. The estimation model is

$$c_{ij} = \alpha + \sum_{i=1}^{6} \gamma_i DS_i + \sum_{j=1}^{6} \delta_j DB_j + e_{ij}, \tag{9}$$

where $c_{ij}$ are the dummy coefficients estimated from model (8) (table 5); $DS_i = 1$ for the $i$th spread group and zero otherwise; and $DB_j = 1$ for the $j$th beta group and zero otherwise. Spreads are increasing in $i$, and betas are increasing in $j$.
($t$-statistics are in parentheses).

| | Estimated regression coefficients | | | |
|---|---|---|---|---|
| | Entire 1961–1980 period | | Subperiods | |
| Independent variable | From OLS regression | From GLS regression | 1961–1970 GLS | 1971–1980 GLS |
| $DS_1$ | −0.00857<br>(9.05) | −0.00681<br>(7.74) | −0.00730<br>(7.46) | −0.00397<br>(3.33) |
| $DS_2$ | −0.00654<br>(6.90) | −0.00517<br>(5.88) | −0.00578<br>(5.91) | −0.00267<br>(2.24) |
| $DS_3$ | −0.00729<br>(7.70) | −0.00599<br>(6.82) | −0.00556<br>(5.69) | −0.00483<br>(4.05) |
| $DS_4$ | −0.00552<br>(5.83) | −0.00439<br>(4.99) | −0.00446<br>(4.56) | −0.00301<br>(2.53) |
| $DS_5$ | −0.00461<br>(4.86) | −0.00359<br>(4.08) | −0.00335<br>(3.42) | −0.00272<br>(2.28) |
| $DS_6$ | −0.00252<br>(2.66) | −0.00172<br>(1.95) | −0.00246<br>(2.52) | 0.00051<br>(0.42) |
| $DB_1$ | −0.00964<br>(10.18) | −0.00614<br>(6.98) | −0.00669<br>(6.84) | −0.00454<br>(3.81) |
| $DB_2$ | −0.00767<br>(8.10) | −0.00500<br>(5.68) | −0.00495<br>(5.06) | −0.00421<br>(3.53) |
| $DB_3$ | −0.00626<br>(6.61) | −0.00411<br>(4.67) | −0.00325<br>(3.32) | −0.00434<br>(3.64) |
| $DB_4$ | −0.00568<br>(6.00) | −0.00398<br>(4.53) | −0.00260<br>(2.66) | −0.00485<br>(4.07) |
| $DB_5$ | −0.00336<br>(3.55) | −0.00214<br>(2.43) | −0.00098<br>(1.00) | −0.00293<br>(2.46) |
| $DB_6$ | −0.00147<br>(1.56) | −0.00065<br>(0.74) | 0.00017<br>(0.18) | −0.00121<br>(1.01) |

returns on stocks and their market value, indicating either a misspecification of the CAPM or evidence of market inefficiency [see Schwert (1983) for a comprehensive review]. Thus, it is instructive to estimate the effects of a firm-size variable and to test its significance vis-a-vis our variables.

We re-estimated our models adding a new explanatory variable – *SIZE*, the market value of the firm's equity in millions of dollars at the end of the year

Table 7

Tests of hypotheses on the return–spread relation. All regressions are estimated by GLS.

| Model[a] | Degrees of freedom of the model | $SSR$, sum of squared residuals | Difference from model (A)[b] | | | $F$-statistic |
|---|---|---|---|---|---|---|
| | | | $DF$ | $SSR$ | $MS$ | |
| (A) | 75 | 76.7877 | – | – | – | – |
| (B) | 26 | 85.5489 | 49 | 8.7612 | 0.1788 | 2.10 |
| (C) | 33 | 83.3506 | 42 | 6.5629 | 0.1563 | 1.84 |
| (D) | 27 | 84.7339 | 48 | 7.9462 | 0.1655 | 1.95 |
| (E) | 69 | 78.4249 | 6 | 1.6372 | 0.2729 | 3.21 |

[a]The regression models are as follows.

*Model (A)* – the full model:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + \sum_{i=1}^{7} b_i\hat{S}^i_{pn} + \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}DP_{ij} + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}, \tag{8}$$

where $p = 1,2,\ldots,49$, $n = 1,2,\ldots,20$, and $DP_{77} \equiv 0$.

*Model (B)* – a restricted model for testing the existence of any spread effect:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + \sum_{j=1}^{6} \gamma_j DB_j + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

*Model (C)* – a restricted model for testing the equality of mean excess returns across spread groups:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + \sum_{i=1}^{7} b_i\hat{S}^i_{pn} + \sum_{j=1}^{6} \gamma_j DB_j + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

*Model (D)* – a restricted model for testing the non-linearity of the return–spread relation:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + a_2 S_{pn} + \sum_{j=1}^{6} \gamma_j DB_j + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

*Model (E)* – a restricted model testing the equality of the slope coefficients across spread groups:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + a_2\left(\sum_{i=1}^{7} \hat{S}^i_{pn}\right) + \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}DP_{ij} + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

The regression variables are:

$R^e_{pn}$ = average portfolio excess return (the dependent variable) for portfolio $p$ in year $n$,
$\beta_{pn}$ = average portfolio relative ($\beta$) risk,
$S_{pn}$ = average portfolio relative spread,
$\hat{S}^i_{pn}$ = mean-adjusted spread (the deviation of the spread $S_{pn}$ of portfolio $p$ in year $n$ from the mean spread of its spread group, $i$),
$DP_{ij}$ = portfolio group dummy; one in portfolio group $(i, j)$, zero otherwise,
$DY_n$ = year dummy; one in year $n$, zero otherwise,
$DB_j$ = $\beta$ group dummy; one in $\beta$ group $j$, zero otherwise. $DB_j = \sum_{i=1}^{7} DP_{ij}$ $(j = 1,2,\ldots,6)$.

[b]Data for the $F$-test on each of the restricted models:

$DF$ = difference in the number of degrees of freedom between the full and restricted model,
$SSR$ = difference in the sum of squares between the full and restricted model,
$MS$ = $SSR/DF$, the mean square.

just preceding the test period. As seen in table 2, there is a negative relationship between *SIZE* and both spread and $\beta$. The effect of firm size on stock returns was tested by incorporating *SIZE* in all our models, but its estimated effect was negligible and highly insignificant.

To allow for a possible non-linear effect (as other studies do), we replaced *SIZE* by its natural logarithm and examined the impact of adding log(*SIZE*) to our regression equations. First, we estimated the simple linear model

$$R^{e}_{pn} = \underset{}{0.0082} \; + \underset{(5.05)}{0.0060\beta_{pn}} \; + \underset{(3.44)}{0.158 S_{pn}} + \underset{(1.56)}{0.0006 \log(SIZE)_{pn}}$$

$$+ \sum_{n=1}^{19} d_n DY_n + e_{pn}.$$

The results indicate that the risk and spread effects prevail, whereas the size effect is insignificant. We then re-estimated our detailed model (8) with the added variable log(*SIZE*) using GLS over the entire sample period and its two ten-year subperiods. The results in table 8(B) suggest that the size effect is insignificant, and it remains insignificant when the only spread variable appearing in the regression equation is $S_{pn}$ [see 8(C)]. The coefficient of log(*SIZE*) becomes significant only when all the spread-related variables are altogether omitted [table 8(D)]. Finally, we performed an *F*-test for the significance of our set of spread variables given log(*SIZE*). The test produced $F = 2.02$, significant at better than the 0.01 level. Thus, while our spread variables render the size effect insignificant, they remain highly significant even with log(*SIZE*) in the regression equation. In sum, our results on the return-spread relation cannot be explained by a 'size effect' even if the latter exists. In fact, any 'size effect' may be a consequence of a spread effect, with firm size serving as a proxy for liquidity. And, rather than suggesting an 'anomaly' or an indication of market inefficiency, our return–spread relation represents a rational response by an efficient market to the existence of the spread.

A number of studies have attempted to explain the size effect in terms of the bid–ask spread. Stoll and Whaley (1983) suggested that investors' valuations are based on returns net of transaction costs, and observed that the costs of transacting in small-firm stocks are relatively higher. They thus subtracted these costs from the measured returns and tested for a small-firm effect. Using an interesting empirical procedure based on arbitrage portfolios, they found that if round-trip transactions occurred every three months, the size effect was eliminated. They thus concluded that the CAPM, applied to after-transaction-cost returns over an appropriately chosen holding period, cannot be rejected.

Table 8

Effects of firm size on portfolio returns, controlling for the effects of the bid-ask spread, over the period 1961-1980 and its two 10-year subperiods.

| Model[a] | Sample period | Definition of size variable | Estimates for the size variable: Coefficient | *t*-value | Spread variables included in the regression equation |
|---|---|---|---|---|---|
| (A) | 1961-80 | *SIZE* | $-0.23 \times 10^{-6}$ | 0.74 | all[b] |
| (B) | 1961-80 | log(*SIZE*) | $-0.000650$ | 1.52 | all[b] |
| (B) | 1961-70 | log(*SIZE*) | $-0.000916$ | 1.46 | all[b] |
| (B) | 1971-80 | log(*SIZE*) | $-0.000216$ | 0.34 | all[b] |
| (C) | 1961-80 | log(*SIZE*) | $-0.00032$ | 1.08 | $S$ ($a_2 = 0.153$, $t = 2.51$) |
| (D) | 1961-80 | log(*SIZE*) | $-0.00057$ | 2.0 | none |

[a]The models used are as follows.

*Model (A)* is obtained by adding *SIZE* to (8), i.e.,

$$R^e_{pn} = a_0 + a_1\beta_{pn} + \sum_{i=1}^{7} b_i\hat{S}^i_{pn} + \sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}DP_{ij} + \psi \cdot SIZE_{pn} + \sum_{n=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

*Model (B)* is obtained by adding log(*SIZE*) to (8), i.e., replacing $SIZE_{pn}$ in (A) by $\log(SIZE_{pn})$.

*Model (C)* includes log(*SIZE*) and the spread variable $S_{pn}$:

$$R^e_{pn} = a_0 + a_1\beta_{pn} + a_2 S_{pn} + \sum_{j=1}^{6} \gamma_j DB_j + \eta \cdot \log(SIZE_{pn}) + \sum_{j=1}^{19} d_n DY_n + \varepsilon_{pn}.$$

*Model (D)* is obtained by omitting $S_{pn}$ from model (C).

The regression variables are:

$R^e_{pn}$ = average excess return for portfolio $p$ in year $n$ (the dependent variable),
$\beta_{pn}$ = average portfolio relative ($\beta$) risk,
$S_{pn}$ = average portfolio relative spread,
$\hat{S}^i_{pn}$ = mean-adjusted spread (the deviation of the spread $S_{pn}$ of portfolio $p$ in year $n$ from the mean spread of its spread group, $i$),
$DP_{ij}$ = portfolio group dummy; one in portfolio group $(i, j)$, zero otherwise,
$DB_j$ = $\beta$-group dummy; one in $\beta$-group $j$, zero otherwise. $DB_j = \sum_{i=1}^{7} DP_{ij}$ $(j = 1, 2, \ldots, 6)$.
$DY_n$ = year dummy; one in year $n$, zero otherwise,
$SIZE_{pn}$ = average market value of the equity of firms in portfolio $p$ in the year just preceding $n$, in millions of dollars.

[b]Results obtained by adding the size variable to the full model (8).

This conclusion was challenged by Schultz (1983), who claimed that transaction costs do not completely explain the size effect. Extending Stoll and Whaley's sample to smaller AMEX firms, Schultz found that small firms earn positive excess returns after transaction costs for holding periods of one year. He thus concluded that transaction costs cannot explain the violations of the CAPM. This criticism, however, hardly settles the issue, and in fact highlights a basic problem. Given the higher returns and higher spreads of small firms' stocks, it is always possible to find an investment horizon which nullifies the abnormal return after transaction costs. But then, finding that a horizon of one year does not eliminate the size effect is insufficient to determine whether or not transaction costs are the proper explanation.

Our examination of the relation between stock returns and bid–ask spreads is based on a theory which produces well-specified hypotheses. In the context of our model, the after-transaction-cost return, as defined in the above studies, is not meaningful. Stoll–Whaley and Schultz consider this key variable to be a property of the security, and calculate it by subtracting the transaction cost from the gross return, implicitly assuming the same holding period for all stocks. By our model, the spread-adjusted return depends not only on the stock's return and spread, but also on the holding horizon of its specific clientele [see (4)]. Thus, their method is inapplicable to test our hypotheses on the return–spread relation.

The different objective guiding our empirical study has shaped its different methodology and structure. Stoll–Whaley and Schultz aim at explaining the 'small firm' anomaly through the bid–ask spread, hence their portfolio construction and test procedure are governed by firm size.[17] We start from a theoretical specification of the return–spread relation, and the objective of our empirical study is to test the explicit functional form predicted by our model. Thus, our empirical results are disciplined by the theory and in fact the test procedure is called for by the theory.

A second issue raised by Schultz (1983) is the seasonal behavior of the size effect, which is particularly pronounced in the month of January.[18] In the context of our study, there is a question whether liquidity has a seasonal. A test of this hypothesis requires data on monthly bid–ask spreads which was unavailable to us. Given our data of a single spread observation per year, we are unable to carry out a powerful test incorporating seasonality, a topic which is worthy of further research.

An empirical issue in the computation of returns on small firms is the possible upward bias due to the bid–ask spread, suggested by Blume and Stambaugh (1983), Roll (1983) and Fisher and Weaver (1985). Blume and Stambaugh estimate the bias to be $\frac{1}{4}S^2$, where $S$ is the relative spread. Given the magnitudes of the spreads and the excess returns, this difference is negligible. Indeed, we re-estimated models (8)–(9), applying the Blume–

[17]Stoll–Whaley and Schultz subordinate their study of the bid–ask effect to the small-firm classification, a procedure which is natural for studying the small-firm anomaly. Our portfolio-construction method is motivated by the prediction that stock returns are a function of the bid–ask spread and $\beta$, and is designed specifically to test this hypothesis.

[18]Lakonishok and Smidt (1984) found that the small-firm effect prevails at the turn-of-the-year when returns are measured net of transaction costs, using the high and low prices as proxies for the ask and bid prices.

Stambaugh and Fisher–Weaver approach and obtained similar results which uniformly supported our hypotheses.[19]

## 5. Conclusion

This paper studies the effect of securities' bid–ask spreads on their returns. We model a market where rational traders differ in their expected holding periods and assets have different spreads. The ensuing equilibrium has the following characteristics: (i) market-observed average returns are an increasing function of the spread; (ii) asset returns to their holders, net of trading costs, increase with the spread;[20] (iii) there is a clientele effect, whereby stocks with higher spreads are held by investors with longer holding periods; and (iv) due to the clientele effect, returns on higher-spread stocks are less spread-sensitive, giving rise to a concave return–spread relation. We design a detailed test on the behavior of observed returns, and our results support the theory. The robustness and statistical significance of our results are very encouraging, especially when compared to the Fama–MacBeth (1973) benchmark. These results do not point at an anomaly or market inefficiency; rather, they reflect a rational response by investors in an efficient market when faced with trading friction and transaction costs.

The higher yields required on higher-spread stocks give firms an incentive to increase the liquidity of their securities, thus reducing their opportunity cost of capital. Consequently, liquidity-increasing financial policies may increase the value of the firm. This was demonstrated for our numerical example in fig. 2, which depicts the relation between asset values and their bid–ask spreads. Applying our empirical results, consider an asset which yields $1 per month, has a bid–ask spread of 3.2% (as in our high-spread portfolio group) and its proper opportunity cost of capital is 2% per month, yielding a value of $50. If the spread is reduced to 0.486% (as in our low-spread portfolio group), our estimates imply that the value of the asset would increase to $75.8, about a 50% increase, suggesting a strong incentive for the firm to invest in increasing the liquidity of the claims it issues. In particular, phenomena such as 'going public' (compared to private placement), standardization of the contractual forms of securities, limited liability, exchange listing and information disclosures may be construed as investments in increased liquidity. It is of interest to examine to what extent observed corporate financial policies can be explained by the liquidity-increasing motive. Such an investigation could

[19] To illustrate, the coefficient of $DS_1$ in model (9), which reflects the difference in returns between the highest and lowest spread groups, was $-0.00765$ ($t = 8.15$) by the OLS method and $-0.00587$ ($t = 6.73$) by GLS.

[20] Recall that, in the context of our model, net returns cannot be defined as stock characteristics, since they depend on both the stock and the owning investor. Our result is that despite their higher spread, the net return on high-spread stocks to their holders is higher.

create a link between securities market microstructure and corporate financial policies, and constitutes a natural avenue for further research.

This also suggests that a more comprehensive model of the return–spread relation could consider supply response by firms. Rather than set the spread exogenously, as in our model, firms may engage in a supply adjustment, increasing the liquidity of their securities at a cost. In equilibrium, the marginal increase in value due to improved liquidity will equal the marginal cost of such an improvement. Then, differences in firms' ability to affect liquidity will be reflected in differences in bid–ask spreads and risk-adjusted returns across securities.[21]

We believe that this paper makes a strong case for studying the role of liquidity in asset pricing in a broader context. The generality of our analysis is limited in that we do not consider the difference between marginal liquidity and total liquidity, and the associated relation between liquidation uncertainty and holding period uncertainty. This issue deserves further attention. In our model, all assets are liquidated at the end of the investor's holding period. Thus, there is no distinction between the liquidity of an asset when considered by itself and its liquidity in a portfolio context, nor is it necessary to consider the dispersion of possible holding periods for each asset in the portfolio. In a more general model, each investor may be faced with a sequence of stochastic cash demands occurring at random points in time. The investor would then have to determine the quantities of each security to be liquidated at each point in time. In such a setting, an investor's portfolio is likely to include an array of assets with both low and high spreads, whose proportions will reflect both the distribution of the amounts to be liquidated and the dispersion of his liquidation times. Then, there would be a distinction between the liquidity of an asset and its marginal contribution to the liquidity of an investor's portfolio. A study along these lines should focus on the interrelationship between total and marginal liquidity and its effect on asset pricing.

Further research could also be carried out on the interplay between liquidity and risk, and on the relation between asset returns and a more comprehensive set of liquidity characteristics. And finally, it is of interest to pursue the link between corporate financial theory and the theory of exchange, possibly leading to a unified framework which will enhance our understanding of organizations and markets.

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[21] Even if some firms could issue an unlimited supply of zero-spread securities, our results show that there will still be differentials in investors' net yields.

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